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Controlling Chaotic Rössler System via Synchronization, Using Bifurcation Parameter to Choose Desirable Periodic Orbit

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Abstract: In this study, using synchronization approach, chaos control for Rössler system is investigated. Based on essential structure of synchronization approach and using bifurcation diagram, periodic Rössler systems or master systems for both period-one and period-two orbits are found. Nonlinear and linear feedback methods are used to synchronize chaotic slave system with periodic master systems. Stability conditions are discussed analytically and numerical simulation results are presented.

Key words: Chaos control, bifurcation diagram, nonlinear feedback, linear feedback, nonlinear dynamic systems

INTRODUCTION

Chaos has already had an enduring effect on science and issues of chaos control and synchronization have been extensively investigated since the early 1990s.

The pioneers of controlling chaos are Ott *et al.* (1990) proposed a controlling procedure, called OGY method. This method has been used for stabilizing Unstable Periodic Orbits (UPO) embedded within a chaotic attractor (Ott *et al.*, 1990). Since, their seminal study, control of chaos has been a challenging research subject in various sciences where different chaos control techniques have been proposed by Fradkov and Evans (2005). For example, occasional proportional feedback technique (Hunt, 1991), delayed feedback method (Pyragas, 2001), linear/nonlinear feedback control (Liu and Yang, 2008; Rafikov and Balthazar, 2008), are used to control chaos.

Chaos synchronization usually occurs in two chaotic systems consisting of a master system (drive system) and a slave system (response system), which are of identical structure and parameters except for different initial conditions. The first study on chaos synchronization was done by Pecora and Carroll (1990). Since then, various modern control methods, such as adaptive control (Chen and Lu, 2002; Park, 2005), backstepping design (Tan *et al.*, 2003; Bowong and Kakmeni, 2004), active control (Yassen, 2005) and time-delay feedback approach (Cao *et al.*, 2005) have been successfully applied to chaos synchronization. It is reasonable to expect that controlling chaos can be achieved by the method of synchronization (Gupte and Amritkar, 1993; Bai and Lonngren, 1999). Although there are too many

works in chaos control and chaos synchronization, as our knowledge no one has used the synchronization structure (i.e. Master-Slave systems) for controlling chaos in Rössler system and no one has used the bifurcation diagram for choosing various control targets.

In this study, synchronization approach is used to control chaos in Rössler system. In other words, we synchronized the chaotic slave Rössler system with a periodic master one to make it periodic (Fig. 1). First, using bifurcation diagram, a desirable periodic Rössler system (master system) is found. Then the chaotic Rössler system synchronizes as slave system with periodic master system by nonlinear and linear feedback methods.

FORMULATION AND BIFURCATION DIAGRAM

In order to control chaos, based on the structure of synchronization approach (Fig. 1), we consider slave Rössler system described by the following state Eq. 1:

$$\begin{cases} \dot{x}_1 = -x_2 - x_3 + u_1 \\ \dot{x}_2 = x_1 + ax_2 + u_2 \\ \dot{x}_3 = x_3x_1 - cx_3 + b + u_3 \end{cases} \quad (1)$$

where, u_i , $i = 1, 2, 3$, are the control inputs and a , b and c are the parameters. System (1) with parameters $a = b = 0.2$ and $c = 5.7$ without any inputs (i.e. $u_1 = u_2 = u_3 = 0$) is chaotic and we consider it as the slave system or the system we want to control (make it periodic). As Fig. 1 shows, this system is synchronized with a periodic master system. The periodic master Rössler system can be chosen as:

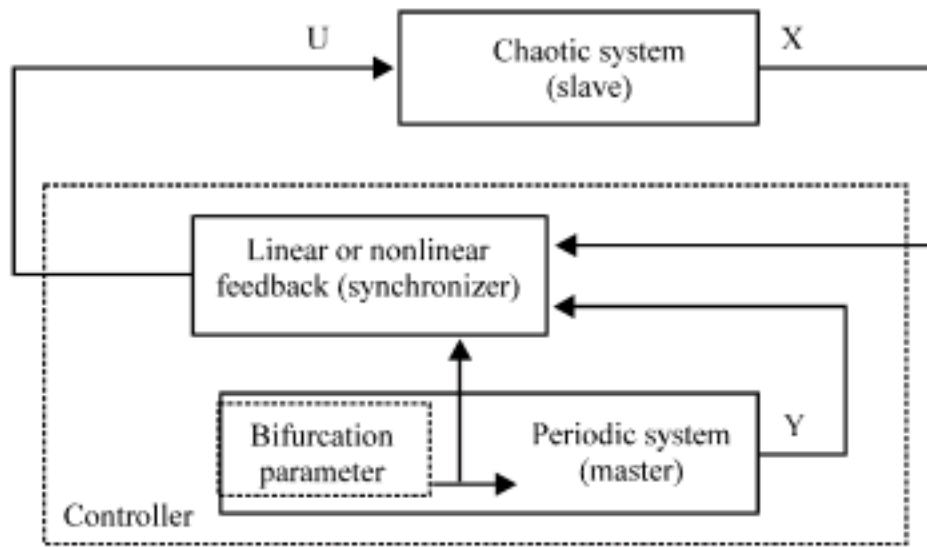


Fig. 1: Block diagram of chaos control via synchronization

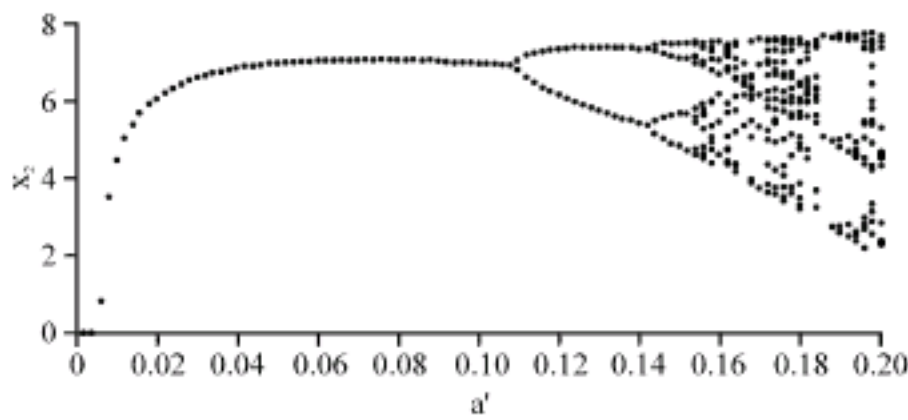


Fig. 2: Bifurcation diagram of Rössler system. Shown is the projection of Poincaré section onto x_2 (intersection at $x_1 = 0$)

$$\begin{cases} \dot{y}_1 = -y_2 - y_3 \\ \dot{y}_2 = y_1 + a'y_2 \\ \dot{y}_3 = y_3y_1 - cy_3 + b \end{cases} \quad (2)$$

In system (2), $b = 0.2$, $c = 5.7$ and a' is chosen from the bifurcation diagram (Fig. 2) so that it becomes periodic.

The bifurcation diagram for system (2) with respect to parameter a' is shown in Fig. 2. As this Fig. 1 shows, Rössler system has period-one solutions for $0.01 \leq a' \leq 0.1$ and period-two solutions for $0.11 \leq a' \leq 0.14$.

We choose $a' = 0.0886$ for the period-one master system with period $T_1 \approx 6$ sec and $a' = 0.136$ for the period-two master system with period $T_2 \approx 12$ sec. Figure 3 shows period-one, period-two and chaotic orbits for Rössler system.

If the error states of the master and slave systems are defined as:

$$\begin{cases} e_1 = x_1 - y_1 \\ e_2 = x_2 - y_2 \\ e_3 = x_3 - y_3 \end{cases} \quad (3)$$

Then the dynamic equations of these errors can be determined directly by subtracting Eq. 2 from Eq. 1, to yield:

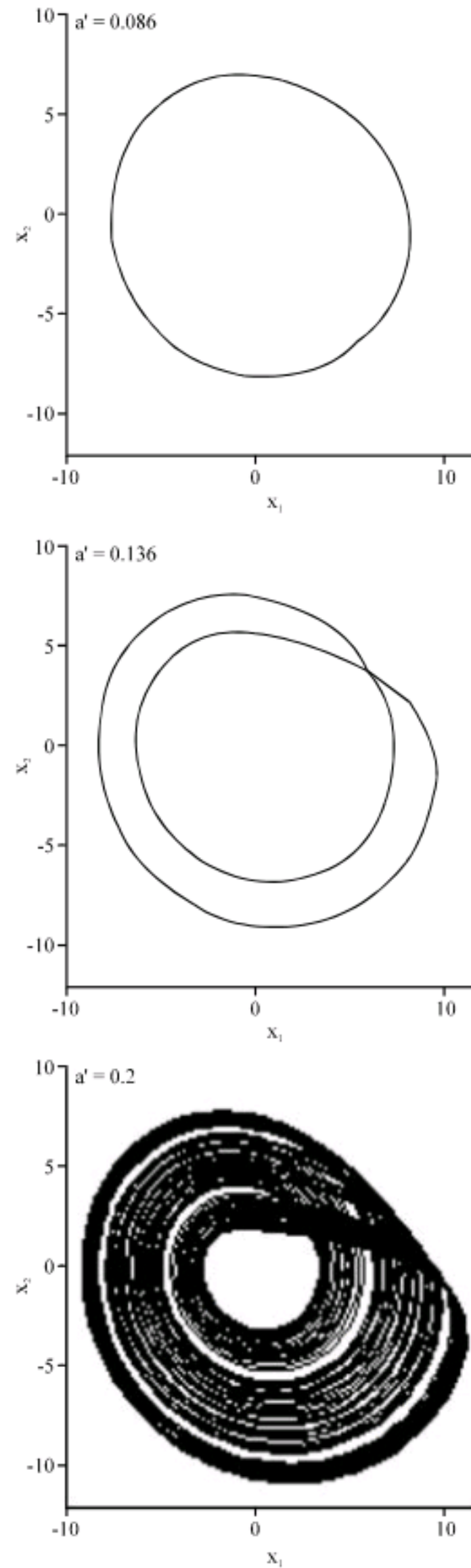


Fig. 3: Period-one, period-two and chaotic orbits of Rössler system

$$\begin{cases} \dot{e}_1 = -e_2 - e_3 + u_1 \\ \dot{e}_2 = e_1 + ax_2 - a'y_2 + u_2 \\ \dot{e}_3 = x_3x_1 - y_3y_1 - ce_3 + u_3 \end{cases} \quad (4)$$

Here, the goal is to make synchronization between two systems by using control inputs μ_i , i.e., all error states go toward zero by time, or:

$$\lim_{t \rightarrow \infty} |e_i(t)| = 0, \quad i = 1, 2, 3 \quad (5)$$

NONLINEAR FEEDBACK SYNCHRONIZER (FEEDBACK LINEARIZATION)

Feedback linearization is an approach to nonlinear control design which transforms a nonlinear system dynamics into a linear one. This differs entirely from conventional linearization (i.e., Jacobian linearization) and in that feedback linearization is achieved by exact state transformation and feedback, rather than by linear approximations of the dynamics (Slotine and Li, 1991).

Nonlinear feedback control scheme: The following control law is considered for the system Eq. 4:

$$\begin{cases} u_1 = 0 \\ u_2 = -ax_2 + a'y_2 + \alpha e_2 \\ u_3 = -x_3x_1 + y_3y_1 \end{cases} \quad (6)$$

where, α is a new parameter.

Lemma 1: Master-slave systems (Eq. 1, 2) with control law Eq. 6 would achieve synchronization in the sense of Eq. 5, if $c > 0$ and $\alpha < 0$.

Proof: Substitution of Eq. 6 in Eq. 4 yields:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \alpha & 0 \\ 0 & 0 & -c \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \quad (7)$$

By definition:

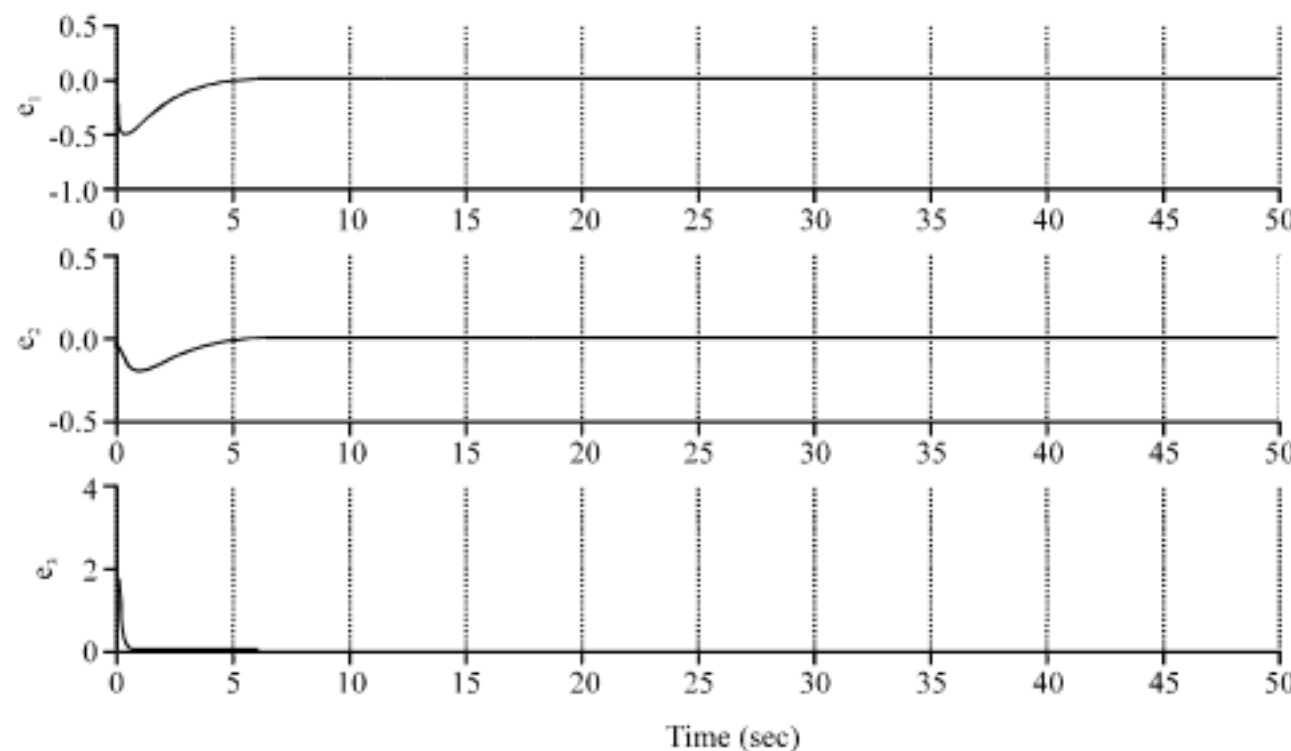


Fig. 4: Synchronization errors of synchronizing Rössler system and period-one system by nonlinear method

$$A_{cl} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \alpha & 0 \\ 0 & 0 & -c \end{bmatrix}$$

the eigenvalue of the matrix A_{cl} can be written as:

$$\lambda_{1,2} = \frac{\alpha \pm \sqrt{\alpha^2 - 4}}{2}, \quad \lambda_3 = -c \quad (8)$$

If $c > 0$ and $\alpha < 0$ all eigenvalue of matrix A_{cl} will have negative real parts and the linearized system Eq. 7 will be stable i.e., it's states will tend to zero by time or:

$$\lim_{t \rightarrow \infty} |e_i(t)| = 0, \quad i = 1, 2, 3$$

So, the master-slave systems (1) and (2) will achieve synchronization in sense of (5).

For example for $\alpha = -2$ and $C = 5.7$ the eigenvalue of matrix A_{cl} are: -1, -1 and -5.7

Numerical simulation results: Setting $\alpha = -2$, the results of synchronizing chaotic slave system and period-one master system are shown in Fig. 4-6. As shown in Fig. 4, the error states tend to zero after about 6 sec.

Figure 5 shows the control inputs and Fig. 6 shows periodicity errors for the slave system, the periodicity errors for period-one orbits are defined as:

$$\begin{cases} e_1^p = x_1(t) - x_1(t - T_1) \\ e_2^p = x_2(t) - x_2(t - T_1) \\ e_3^p = x_3(t) - x_3(t - T_1) \end{cases} \quad (9)$$

Figure 7-9, show the results of synchronizing chaotic system and period-two master system. As shown in Fig. 7, the error states tend to zero after about 5 sec. The periodicity errors for period-two orbits are defined as:

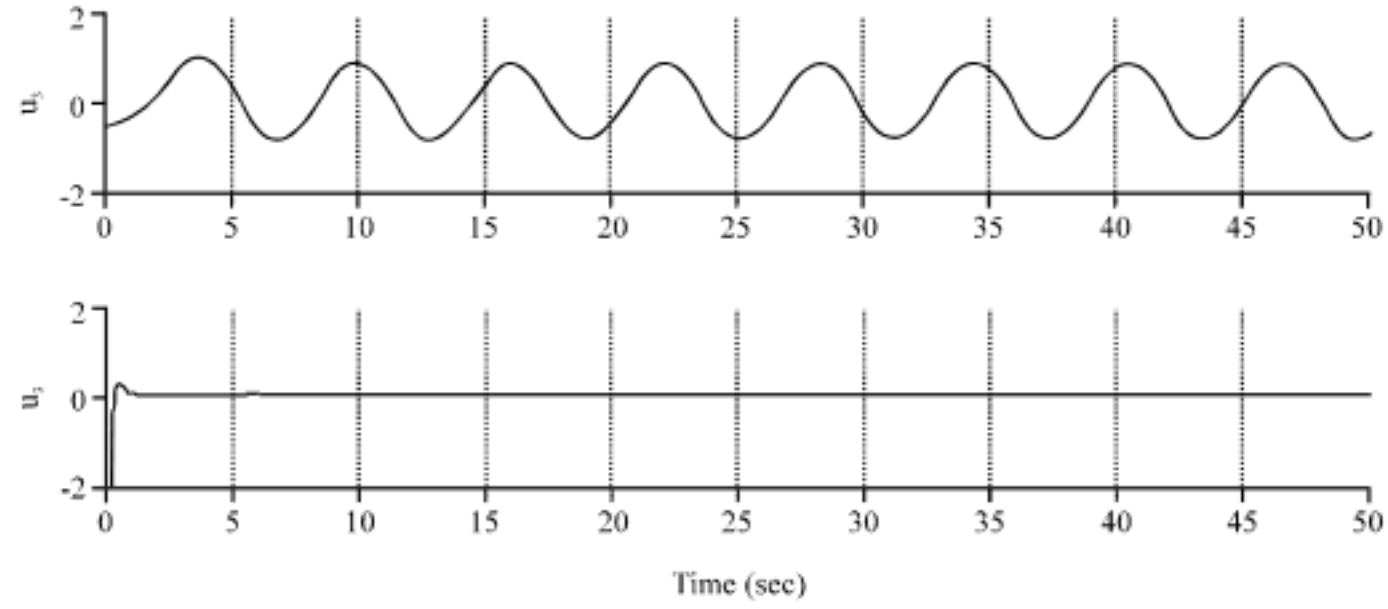


Fig. 5: Control inputs of synchronizing Rössler system and period-one system by nonlinear method

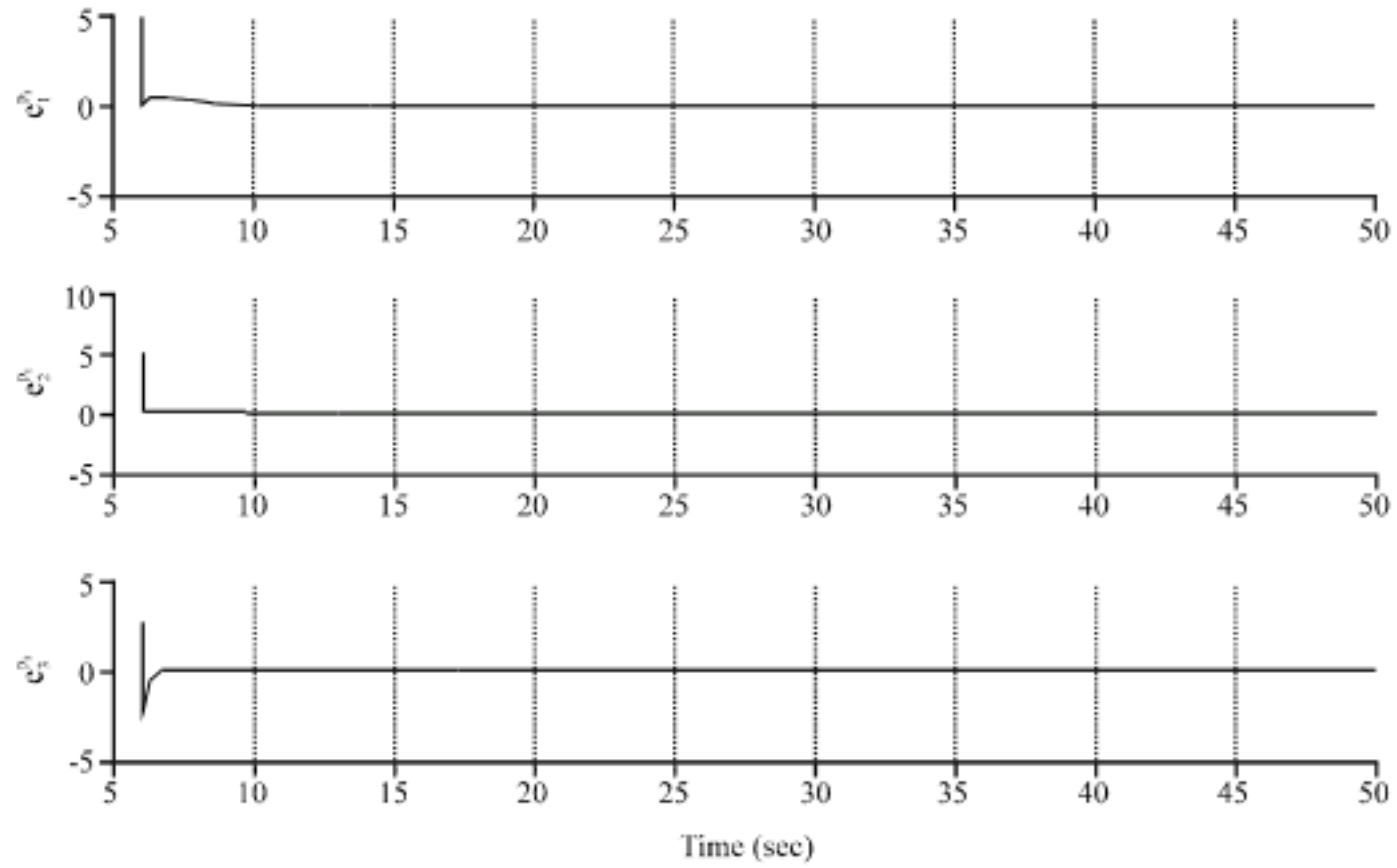


Fig. 6: Periodicity errors of synchronizing Rössler system and period-one system by nonlinear method

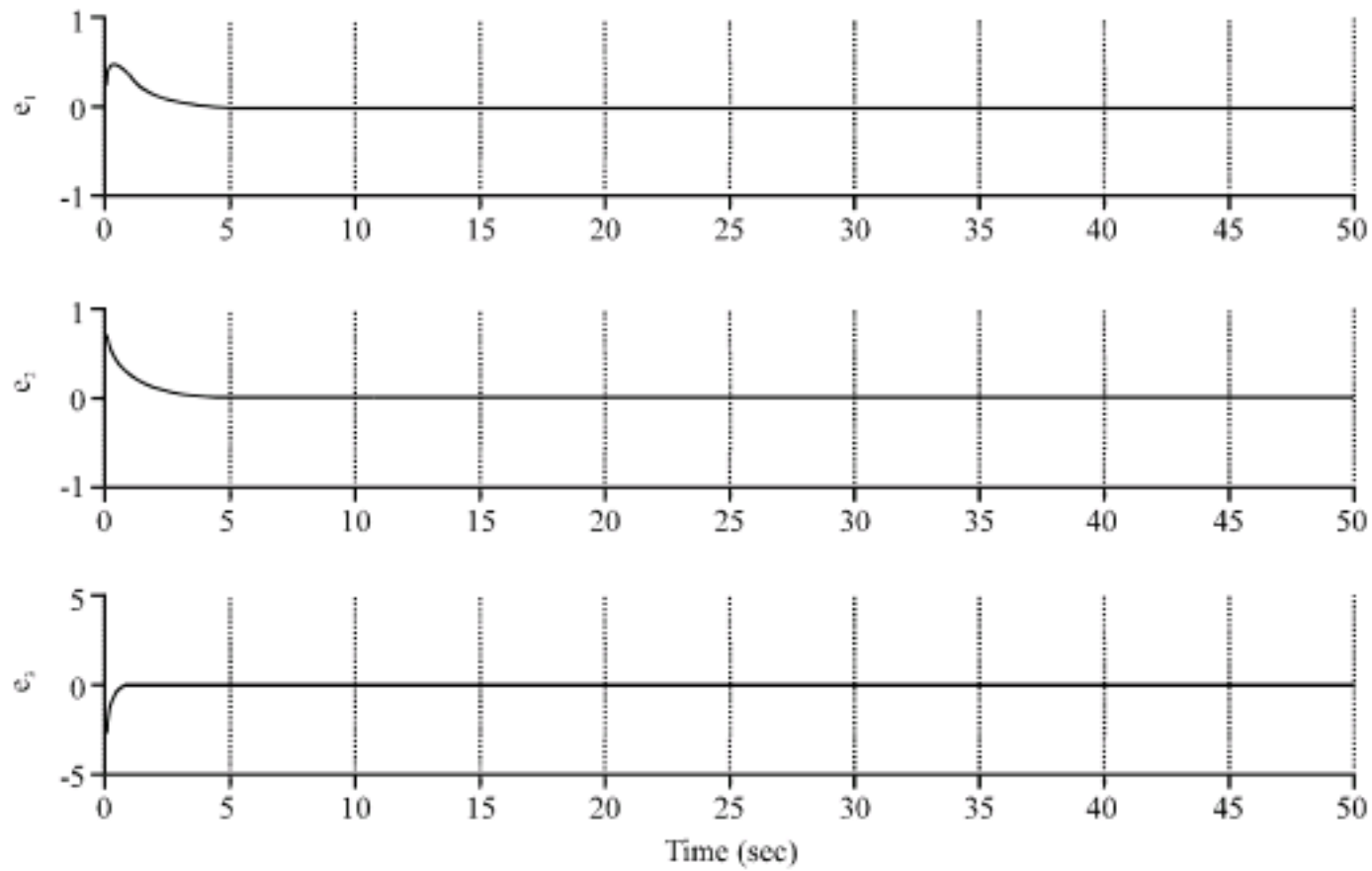


Fig. 7: Synchronization errors of synchronizing Rössler system and period-two system by nonlinear method

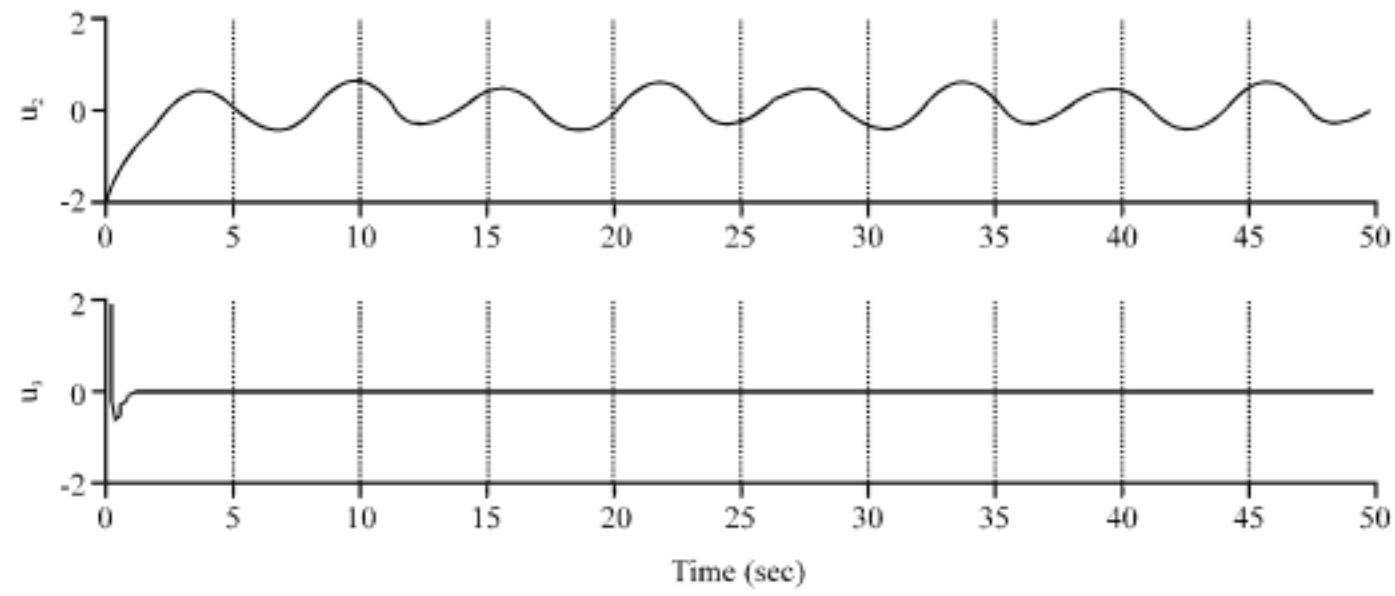


Fig. 8: Control inputs of synchronizing Rössler system and period-two system by nonlinear method

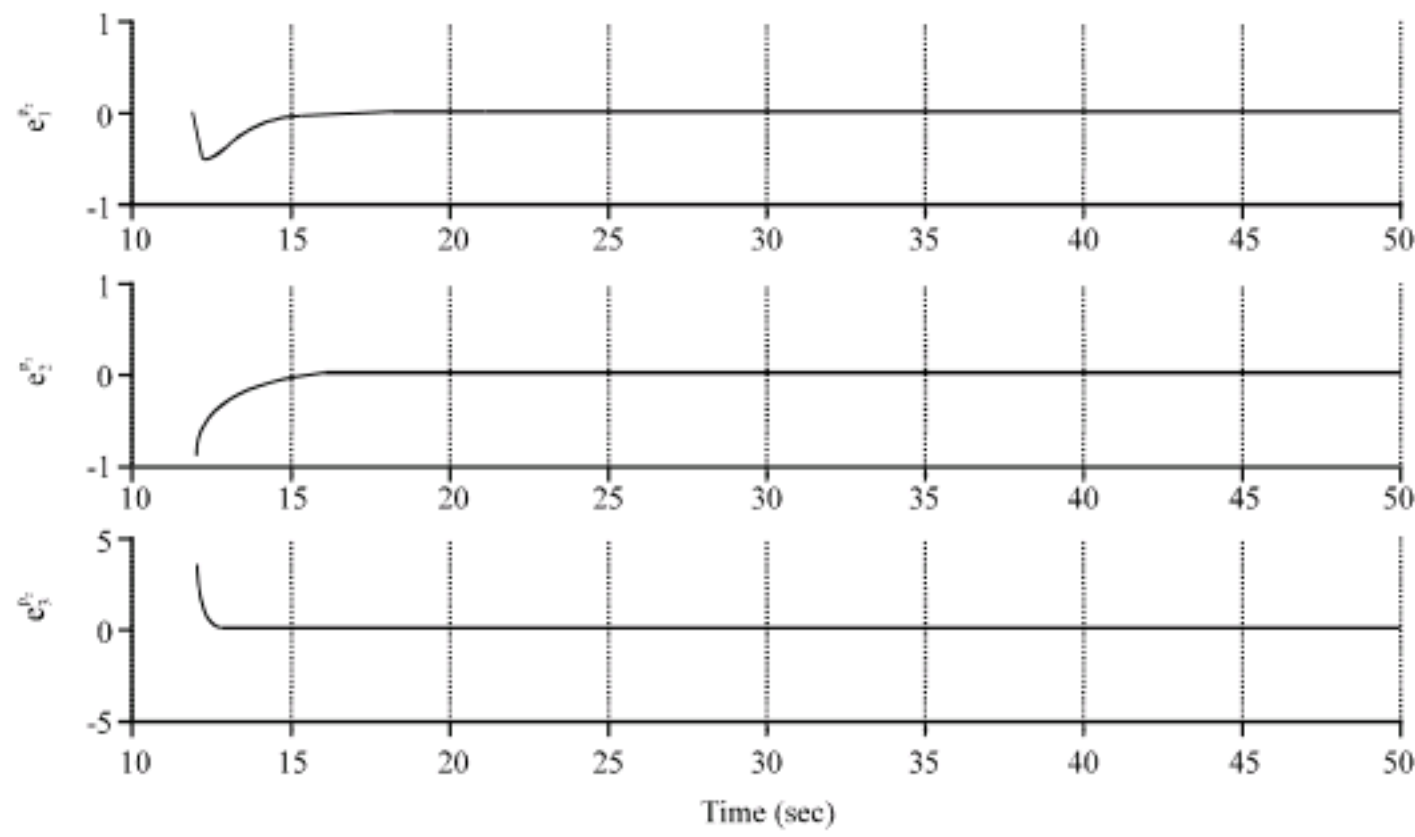


Fig. 9: Periodicity errors of synchronizing Rössler system and period-one system by nonlinear method

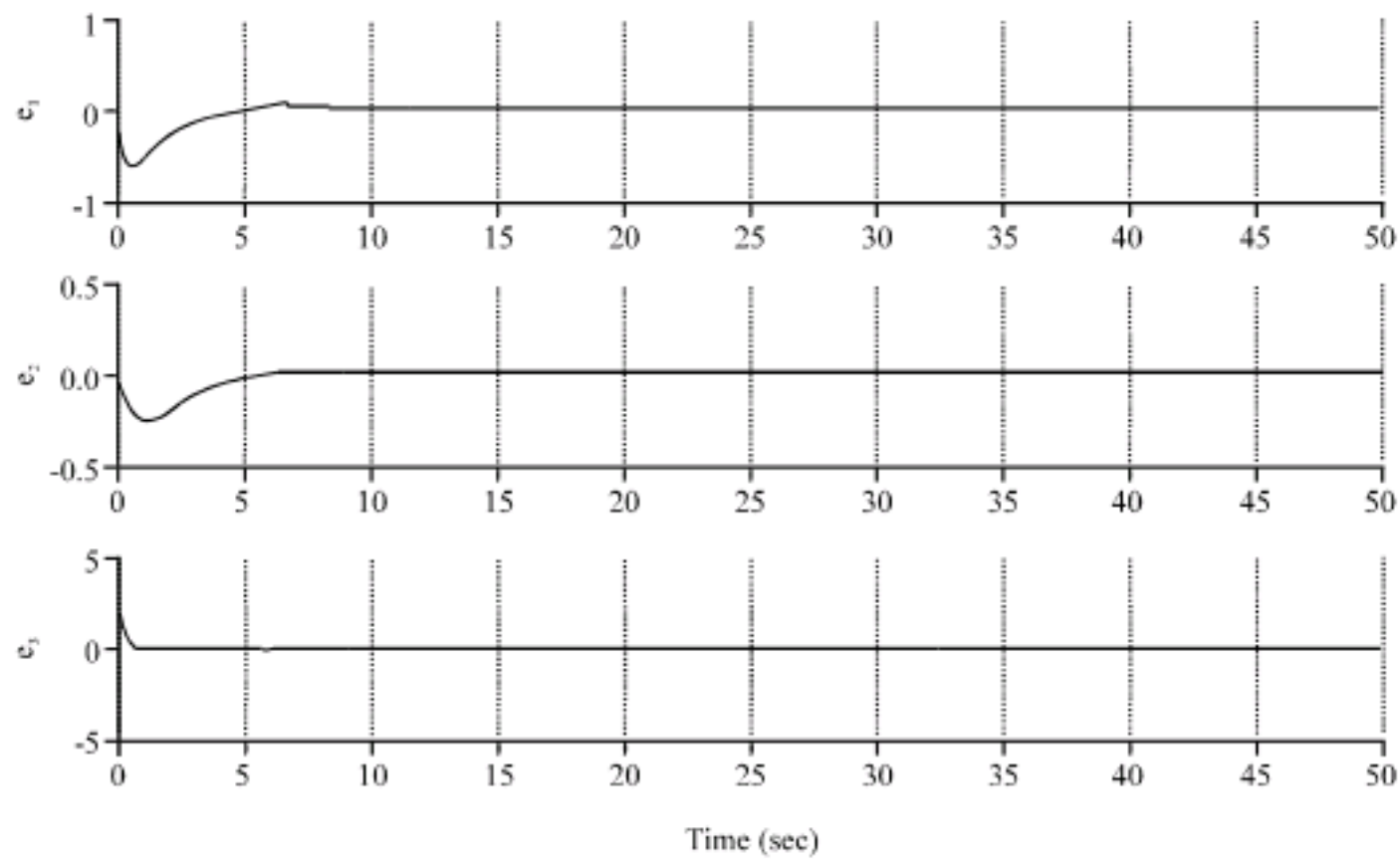


Fig. 10: Synchronization errors of synchronizing Rössler system and period-one system by linear method

$$\begin{cases} e_1^{p_2} = x_1(t) - x_1(t - T_1) \\ e_2^{p_2} = x_2(t) - x_2(t - T_1) \\ e_3^{p_2} = x_3(t) - x_3(t - T_1) \end{cases} \quad (10)$$

$$\begin{cases} u_1 = 0 \\ u_2 = -ax_2 + a'y_2 + \alpha e_2 \\ u_3 = 0 \end{cases} \quad (11)$$

Linear feedback synchronizer: As shown in Fig. 5 and 8, the control input 3 is nonzero only in a few seconds, then it maybe possible to ignore it and use the following linear feedback method:

Setting $\alpha = -2$ in Eq. 11 the corresponding results of synchronizing the chaotic system with the period-one master system and the period-two master system are shown in Fig. 10-12 and in Fig. 13-15, respectively.

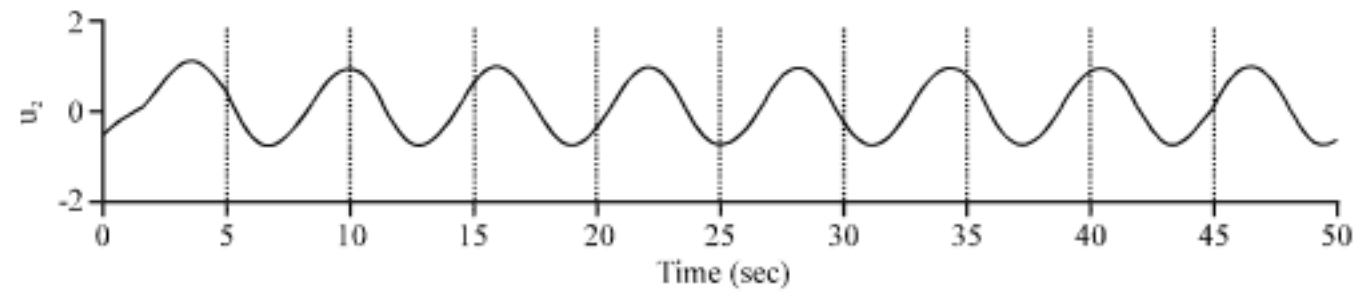


Fig. 11: Control input of synchronizing Rössler system and period-one system by linear method

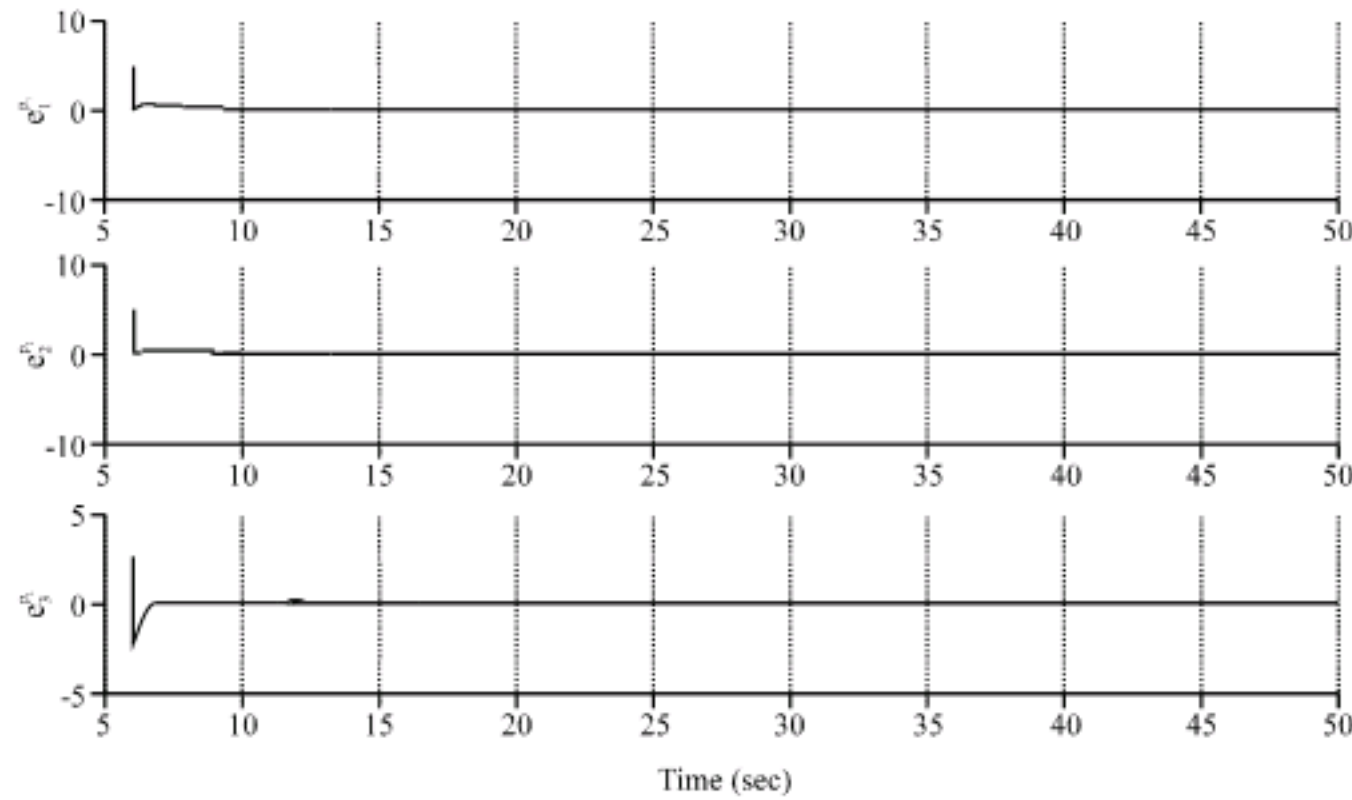


Fig. 12: Periodicity errors of synchronizing Rössler system and period-one system by linear method

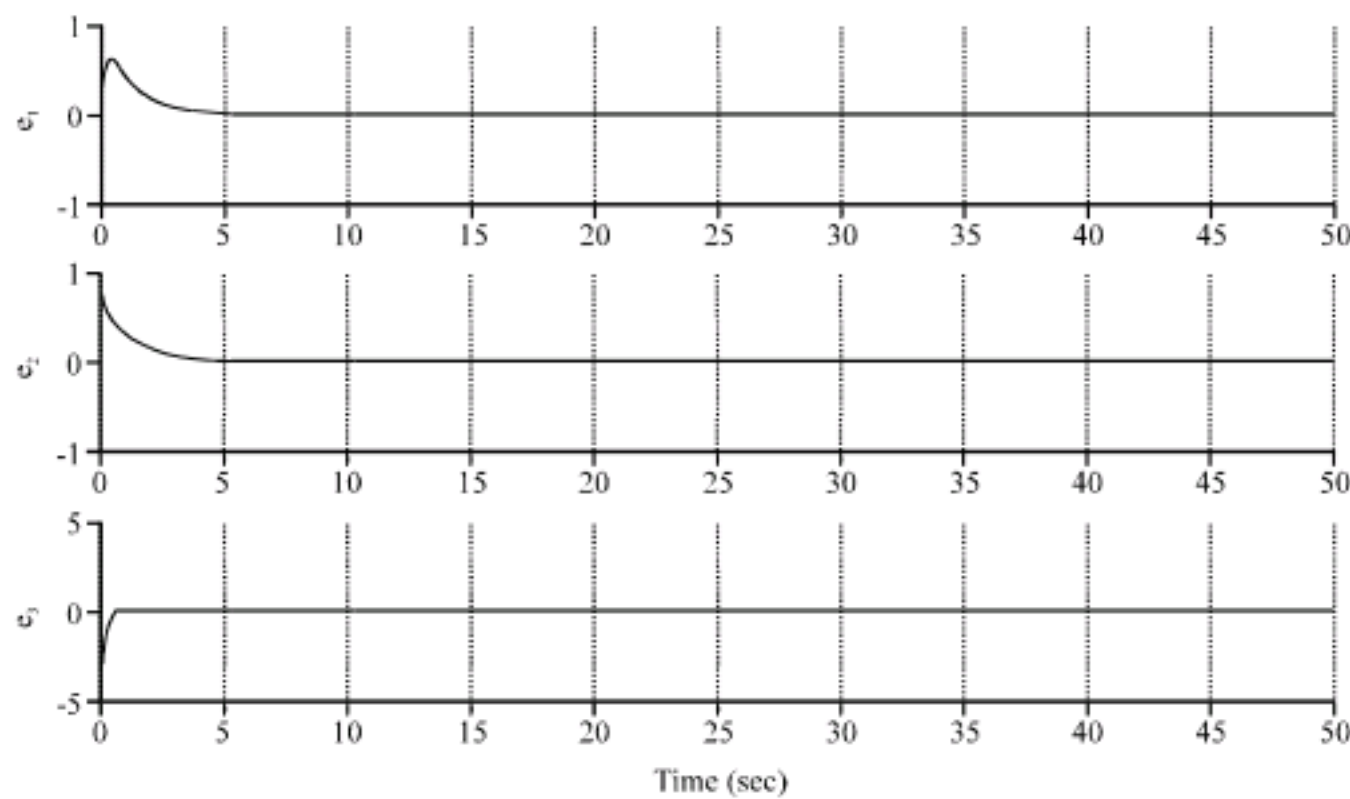


Fig. 13: Synchronization errors of synchronizing Rössler system and period-two system by linear method

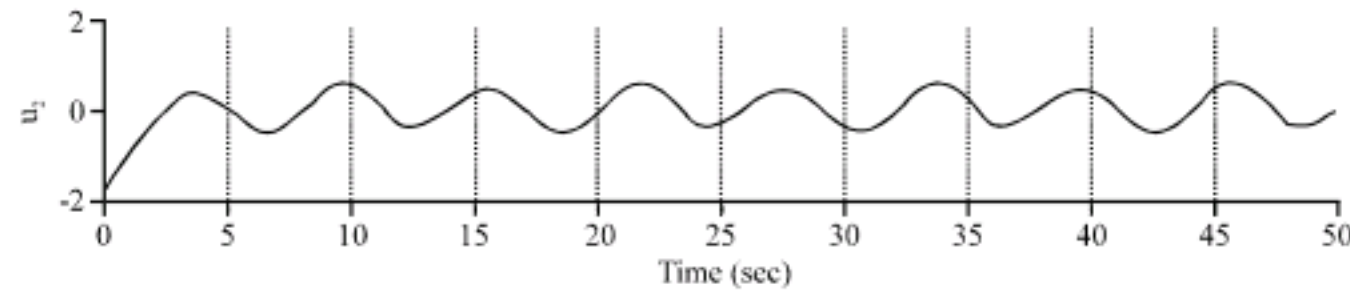


Fig. 14: Control input of synchronizing Rössler system and period-two system by linear method

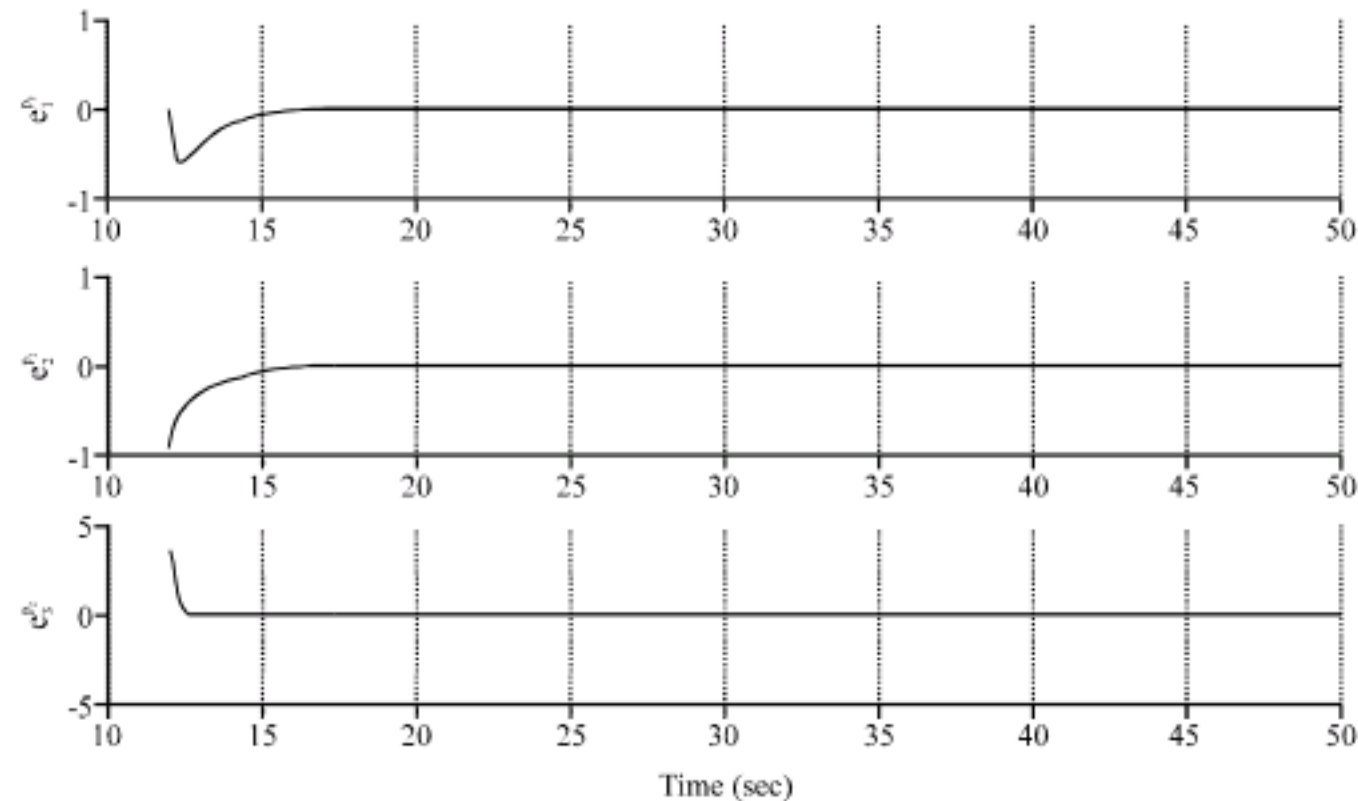


Fig. 15: Periodicity errors of synchronizing Rössler system and period-two system by linear method

By comparing the results of linear feedback method and nonlinear feedback method, it is realized that there is only a little difference between them.

CONCLUSION

Controlling chaos using bifurcation diagram and synchronization is a useful scheme so that, the desirable periodic orbits can be chosen through the bifurcation diagram as control targets.

In this study nonlinear and liner feedback method have been used to control Rössler system. It should be noted that other control methods can be used as synchronizer and other chaotic systems can be considered too.

Using nonlinear feedback method, a complete synchronization is achieved in a short time, although this method is complex and needs full states measurement and two control inputs.

Linear feedback method is simpler and needs one control input and measurement of one state only, but it needs a little more time for synchronization.

From chaos control point of view, periodicity errors defined in Eq. 9 and 10 are more meaningful than synchronization errors defined in Eq. 3, because the goal of chaos control is to make chaotic system periodic.

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