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Multiple Regression Model for Compressive Strength Prediction of High Performance Concrete

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Abstract: A mathematical model for the prediction of compressive strength of high performance concrete was performed using statistical analysis for the concrete data obtained from experimental work done in this study. The multiple non-linear regression model yielded excellent correlation coefficient for the prediction of compressive strength at different ages (3, 7, 14, 28 and 91 days). The coefficient of correlation was 99.99% for each strength (at each age). Also, the model gives high correlation for strength prediction of concrete with different types of curing.

Key words: High performance concrete, compressive strength, mathematical model

INTRODUCTION

Concrete is a versatile construction material owing to the benefits it provides in term of strength, durability, availability, adoptability and economy. Great efforts have been made to improve the quality of concrete by various means in order to raise and maximize its level of performance. Using same ingredients with little adjustments in the microstructure (and probably adding specific materials), it is possible to obtain high performance concrete. The development of HPC has brought forth the need for admixtures, both mineral and chemical, to improve the performance of concrete (Zain *et al*., 2000).

There is no shortage of information on the properties of Hardened High Performance Concrete (HPC). Numerous publications are available showing that the range of outstanding properties of hardened HPC can be obtained not only in the laboratory but also in real construction. Compared to ordinary concrete, HPC has extraordinary rheological properties, especially its super-workability and flowability that make it superior to other concrete mixes (Zain *et al*., 1999).

What is high-performance concrete? According to a recent study by Aitkin (Mehta, 2004), what was known as high-strength concrete in the late 1970s is now referred to as High Performance Concrete (HPC) because it has been found to be much more than simply stronger. ACI defines HPC as a specially engineered concrete, one or more specific characteristics of which have been enhanced through the selection of component materials and mix proportions. Note that this definition does not cover a single product but a family of high-tech concrete products whose properties have been tailored to meet specific engineering needs, such as high workability, very-high early strength (e.g., 30-40 MPa compressive strength in 24 h), high toughness and high durability to exposure conditions.

In construction industry, strength is a primary criterion in selecting a concrete for a particular application. Concrete used for construction gains strength over a long period of time after pouring the characteristic strength of concrete is defined as the compressive strength of a sample that has been aged for 28 days.

Neither waiting 28 days from such a test would serve the rapidity of construction, nor neglecting, it would serve the quality control process on concrete in large construction sites. Therefore, rapid and reliable prediction for the strength of concrete would be of great significance. For example, it provide a chance to do the necessary adjustment on the mix proportion used to avoid situation where concrete does not reach the required design strength or by avoiding concrete that is unnecessarily strong and also, for more economic use of raw materials and fewer construction failures, hence reducing construction cost (Kheder *et al*., 2003).

Prediction of concrete strength, therefore, has been an active area of research and a Considerable number of studies have been carried out.

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Many attempts have been made to obtain a suitable mathematical model which is capable of predicting strength of concrete at various ages with acceptable (high) accuracy.

MATERIALS AND METHODS

Prediction methods for strength of concrete: Under the currently quicker pace of construction, there was a great need for more production of concrete with persisting on the conformability of the quality of the produced concrete with the standards and specifications. The compliance of any produced concrete with these specifications consider to be significant evidence for good concrete. The specifications, generally, include a statement of physical and chemical requirements. Among all, strength tests are prescribed by all specifications, because compressive strength of concrete in the hardened condition is very important and perhaps it is the most obviously required for structural use.

Specifications usually specify test method as well as age of test. Strength of concrete, as specified by all the standards, is very important (from 1 to 28 days), because the early development of strength (early gain in strength) is very important. But, as early strength of concrete is important, strength at later ages is more important, because after all, it is this property which is relied upon in structural design of concrete as a construction material. The traditional 28 days standard test has been found to give general index of the overall quality (used in quality control process) and acceptance of concrete and has served well for so many years. Neither waiting for the result of such a test would serve the rapidity of construction, nor, neglecting it would serve the quality control process of the concrete. Moreover, rapid and reliable prediction of the results of 28 days strength test as early as possible would be of satisfaction for all parties instead of waiting for the traditional 28 days results (Kheder *et al*., 2003).

A number of improved prediction techniques have been proposed by including empirical or computational modeling, statistical techniques and artificial intelligence approaches.

Computational modeling: Many attempts have been made for modeling this process through the used of computational techniques such as finite element analysis. These techniques often based on the complex thermodynamic equations that underpin the aging of concrete in addition the computational complexity of the models is prohibiting in many cases, requiring non-proprietary mathematical tools.

Statistical techniques: A number of research efforts have concentrated on using multivariable regression models to improve the accuracy of predictions.

Statistical models have the attraction that once fitted they can be used to perform predictions much more quickly than other modeling techniques and are correspondingly simpler to implement in software.

Popovics augments Abrams model, a widely accepted equation relating the water cement ratio w/c of concrete to its strength with additional variables such as slump and uses least square regression to determine equation coefficients (Popovics and Ujhelyi, 2008). Apart of its speed, statistical modeling has the advantage over other techniques that it is mathematically rigorous and can be used to define confidence interval for the predictions. This is especially true when comparing statistical modeling with artificial intelligence techniques. Statistical analysis can also provide insight into the key factors influencing 28 days compressive strength through correlation analysis. For these reasons statistical analysis was chosen to be technique for strength prediction of this study.

Artificial neural networks: Because strengthening of concrete is a complex non-linear process dependent on many variables, it is a problem well suited to the artificial intelligence concept known as Artificial Neural Networks (ANNs). Much of the current research into concrete strength prediction recognizes that neural nets are appropriate for the problem.

In the last years, Artificial Neural Networks (ANN) technology, a sub-field of artificial intelligence, are being used to solve a wide variety of problems in civil engineering applications (Bai *et al*., 2003; Topcu *et al*., 2008a, b, 2007; Pala *et al*., 2007; Adhikary *et al*., 2006). The most important property of ANN in civil engineering problems are their capability of learning directly from examples.

Modeling the prediction of compressive strength of concrete: The most popular regression equation used in the prediction of compressive strength is:

$$f = b_0 + b_1 \, w/c \tag{1}$$

Where:

f = Compressive strength of concrete

w/c = Water/cement ratio

b_0, b_1 = Coefficients

The earlier equation is the linear regression equation. The origin of this equation is Abram's Law

(Popovics and Ujhelyi, 2008) which relate compressive strength of concrete to the w/c ratio of the mix and according to this law, increasing w/c ratio will definitely lead to decrease in concrete strength. The original formula for Abram is:

$$f = \frac{A}{B^{w/c}} \tag{2}$$

Where:
f = Compressive strength of concrete
A, B = Empirical constants

Lyse (Jee *et al*., 2004) made a formula similar to Abram but he relate compressive strength to cement /water ratio and not water /cement ratio. According to Lyse strength of concrete increase linearly with increasing c/w ratio .the general form of this popular model was:

$$f = A + B\,c/w \tag{3}$$

Where:
f = Compressive strength of concrete
c/w = Cement /water ratio
A, B = Empirical constants

The quantities of cement, fine aggregate and coarse aggregate were not included in the model and not accounted for the prediction of concrete strength. So, for various concrete mixes were their w/c ratio is constant, the strength will be the same and this is not true. Therefore, efforts should be concentrating on models taken into account the influence of mix constituents on the concrete strength in order to have more reliable and accurate results for the prediction of concrete strength.

For this reason, Eq. 1 which referred to Abrams Law was extended to include other variables in the form of multiple linear regression equation and used widely to predict the compressive strength of various types of concrete as below:

$$f = b_0 + b_1\, w/c$$

Eq. 1 linear least square regression (referred to Abram) and Eq. 4 is multiple linear regression:

$$f = b_0 + b_1 w/c + b_2 CA + b_3 FA + C \tag{4}$$

Where:
f = Compressive strength of concrete
w/c = Water/cement ratio
C = Quantity of cement in the mix
CA = Quantity of coarse aggregate in the mix
FA = Quantity of fine aggregate in the mix

According to Eq. 4 all the variables related to the compressive strength in a linear fashion, but this is not always true because the variables involved in a concrete mix and affecting its compressive strength are interrelated with each other and the additive action is not always true. Here, it appears that there is a need to another type of mathematical model can reliably predicts strength of concrete with acceptable high accuracy. So, if we took the general form of the multiple linear regressions as below:

$$Y = a_0 + a_1X_1 + a_2X_2 + a_3X_3 + \ldots.a_mX_m$$

For situations where the multiple dependencies are curvilinear (non-linear), the logarithmic transformation can be applied to this type of regression:

$$\log(y) = \log(a_0) + a_1\log(X_1) + a_2\log(X_2) + a_3\log(X_3) + \ldots\ldots.a_m\log(X_m) \tag{5}$$

This equation could be transformed back to a form that predicts the dependent variable (Y) by taking the antilogarithm to yield an equation of the type:

$$Y = a_0X_1^{a_1}.X_2^{a_2}.X_3^{a_3}\ldots\ldots.X_m^{a_m} \tag{6}$$

This equation called the multivariable power equation and in engineering, variables are often dependent on several independent variables, this functional dependency is best characterized by the equation mentioned earlier and is said to give results that are more realistic too. This equation has been used successfully to predict the compressive strength for ordinary Portland cement also (Kheder *et al*., 2003).

In this study, the multivariable power equation was found to be very suitable for prediction strength of high performance concrete (as a dependent variable). Factors affecting this strength were the elements of the concrete mix itself.

EXPERIMENTAL WORK

This study has been conducted at UKM University-Malaysia 2008. The main characteristics of materials and procedures used for the purpose of this research are as explained below:

Materials: Locally available crushed stone granite aggregate, mining sand and type I normal Portland

Table 1: Physical properties of the materials

Materials	Properties
Coarse Aggregate(CA)	Specific gravity: 2.62 Absorption: 0.9%
Crushed granite stone	Size: ≤ 19 mm Fineness modulus: 6.82
Fine aggregate: Mining	Specific gravity: 2.62 Absorption: 1.2%
Sand (MS)	Size: ≤ 4.75 mm Fineness modulus: 3.01
Cement (C)	Specific gravity: 3.15
Silica Fume (SF)	Specific gravity: 2.20 Average size: 0.1 μm
Fly Ash (FA)	Specific gravity: 2.26
Super Plasticizer Agent (SP)	Specific gravity:1.21 Solid content: 41%
Air Entraining Agent (AEA)	Specific gravity:1.02 Solid content: 8%
Water (W)	pH:6.9 Dissolved solids:<2000 ppm

Table 2: Mix proportions

	CA	FA	C	SF	FA	W	SP	AEA
Type of mix	(kg m^{-3})						(%B)	
Water/binder ratio = 0.35								
NPC	1013	675	530			185.5	1.75	0.04
SF	1002	667	477	53		185.5	2.25	0.07
FA	1003	668	477		53	185.5	1.75	0.09
SFFA	1002	668	477	26.5	26.5	185.5	2.15	0.08
Water/binder ratio = 0.5								
NPC	1055	707	400			200	1.0	0.06
SF	1046	697	360	40		200	2.0	0.07
FA	1047	698	360		40	200	1.0	0.09
SFFA	1046	698	360	20	20	200	1.5	0.08

cement were used in this study. Class F Malaysian fly ash and Elkem silica fume have been used as mineral admixtures. Sulfonated naphthalene condensate-based Super Plasticizer (SP) and Darex Air Entraining Admixture (AEA) were also used as liquid chemical admixtures. Normal tap water (pH = 6.9) was used as mixing water. The physical properties of the materials are shown in Table 1.

Mix proportions: Four types of high performance concrete with two water / binder ratios were designed including the control mix. These are Normal Portland Cement (NPC), Silica Fume (SF), Fly Ash (FA) and Silica Fume-Fly Ash (SFFA) concrete. The proportions of the constituent materials obtained from mix design were based on saturated surface dry condition. Thus, necessary corrections were made to get the weight of materials in air dry basis. The details of various mix proportions are given in Table 2.

Curing: Three types of curing were adopted. These are dry air (after demoulding, the specimens were marked weight and stored in an air conditioned room, maintaining temperature at 20°C), wrapped (after demoulding, the specimens were marked wrapped and stored inside an air conditioned room, keeping temperature at 20°C) and water curing (after demoulding, the specimens were marked weight and stored inside the water tank suited in an air conditioned room, maintaining temperature at 20°C).

Table 3: Slump and unit weight of concrete mixes

Type of mix	Slump (S)	Unit weight
Water/binder = 0.35		
NPC	25.0	2412
SF	25.0	2370
FA	23.5	2365
SFFA	24.0	2367
Water/binder = 0.50		
NPC	24.0	2392
SF	24.0	2355
FA	23.0	2362
SFFA	23.5	2358

RESULTS AND DISCUSSION

The properties of freshly mixed high performance concrete were determined with respect to slump and unit weight for each type of concrete (Table 3).

The compressive strength test specimens were determined at 3, 7, 14, 28 and 91 days after casting under the curing temperature of 20 °C for different types of concrete and for different types of curing. The results for compressive strength test are given in (Table 4).

Rapid determination or prediction of the strength of concrete could be attained by Suitable mathematical model (with variables affecting strength development of concrete) capable of predicting strength of concrete at different ages . The final form of the regression model proposed in this study was:

$$Y = a_0 X_1^{a_1}.X_2^{a_2}\, X_3^{a_3}.......X_m^{a_m}$$

The variables used in the mathematical model in this study are:

Table 4: Compressive strength of concrete at different ages and different curing conditions

Comp.str.3 days			Comp. str. 7 days			Comp. str. 14 days			Comp. str. 28 days			Comp. str. 91 days		
DAC	WAC	WRC	DAC	WAC	WRC	DAC	WAC	WRC	DAC	WAC	WRC	DAC	WAC	WRC
Water/binder = 0.35														
26.0	36.1	33.1	34.7	42.3	36.0	38.5	49.0	44.5	45.3	54.2	50.0	44.0	55.2	52.3
33.2	39.3	37.0	42.0	47.4	46.5	46.5	51.8	49.2	48.0	56.6	55.5	51.6	64.8	62.0
28.9	36.8	34.0	35.0	44.0	37.2	39.0	50.3	46.5	47.0	55.6	52.6	46.6	58.6	55.3
32.8	37.0	35.4	40.7	47.1	42.9	43.3	50.7	47.3	47.2	56.0	54.6	49.6	61.8	61.0
Water/binder = 0.50														
15.0	19.7	18.0	22.0	25.6	24.7	23.5	28.0	26.9	25.1	31.4	31.0	26.4	35.8	32.3
18.0	20.9	20.4	25.8	28.1	26.2	27.4	31.0	28.2	29.8	34.4	32.4	31.0	41.1	40.2
16.5	20.0	19.1	22.9	25.7	24.8	25.0	29.1	27.3	28.2	32.2	31.0	28.8	36.5	34.7
17.4	20.6	19.7	23.6	26.3	25.3	26.9	30.8	28.0	29.0	33.7	32.0	30.7	39.1	37.4

Table 5: Correlation coefficients between compressive strength at different ages and selected variables for the proposed model

Variable	3 days comp.str.	7 days comp.str.	14 days comp.str.	28 days comp.str.	91 days comp.str.
Ca	-0.99*	-0.99*	-0.99*	-0.99*	-0.99*
Fa	-0.98*	-0.98*	-0.99*	-0.99*	-0.98*
C	0.92*	0.89*	0.91*	0.91*	0.85*
SF	0.22	0.28	0.22	0.20	0.36
FA	0.10	0.09	0.12	0.12	0.08
W	-0.99*	-0.98*	-0.99*	-1.0*	0.97*
S	0.59	0.57	0.56	0.56	0.58
Ñ	0.26	0.20	0.24	0.25	0.13

*: Marked correlations are significant

- Mix proportions elements, i.e., cement, fly ash, silica fume, water and coarse aggregate
- Slump test results
- Density of concrete

The basic concept of this model is that, it produces a reliable relationship between strength of concrete and its own characteristics (the proposed model uses the mix proportions which believed to have significant effect on the characteristics of the produced concrete and one of the most important indications on the properties of the freshly mixed concrete which is slump test results). Also, the unit weight of the concrete was used as variable in this model for its important role in the explanation of strength development process of concrete. These factors were considered to be independent variables in the equation .The multi-variable power equation was used to relate all these variables with the strength of concrete at the specified ages until getting the final and best form of the mathematical model.

Table 5 shows the relationship between the compressive strength of high performance concrete at different ages (3, 7, 14, 28 and 91 days) with the selected variables that are going to be used in the proposed model for the type of water curing. This relationship is represented by the correlation coefficient between each variable and each strength. Also, from this table, it can be seen that some variables have significant correlation with the predicted strength at the specified age. The highest significant correlations were with cement content, water content, fine aggregate and coarse aggregate. These significant correlations were for compressive strength at all ages.

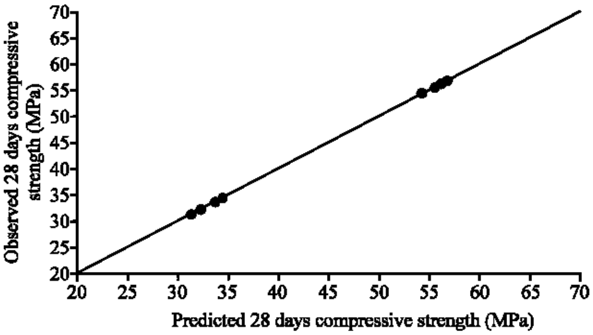

Fig. 1: Relation ship between observed and predicted compressive strength of high performance concrete at 28 days

Depending on the above mentioned variables, the form of the proposed model will be:

$$f_{age} = a_0 C^{a_1}.W^{a_2}.Fa^{a_3}.Ca^{a_4}.SF^{a_5}.FA^{a_6}.\rho^{a_7}.S^{a_8}$$

where, f_{age} is the compressive strength of concrete at specific age, for example f_{28} is the compressive strength of high performance concrete at the age of 28 days. The coefficient of correlation for the 28 days compressive strength prediction was 99.99%. Figure 1 shows the relationship between the predicted and observed 28 days compressive strength for high performance concrete and the high correlation between the two set of data is very clear. Using the same model to predict the compressive strength of high performance concrete at different ages, i.e., 3, 14 and 91 days give coefficient of correlation of 99.99% for each strength. Also, the proposed model proved its validity to be use for predicting the compressive strength of high performance concrete for different types of curing yielding high coefficient of correlation for each type of curing and strength at specified age.

CONCLUSION

Earlier and accurate estimation of concrete strength are valuable to the construction industry. The presence of such model would possibly obtain the hard balance and equality between controlling the quality (quality control process) and economics (saving time and expense, i.e., this model could be used in construction to make the necessary adjustments on mix proportion used, to avoid situations where,concrete does not reach the required design strength or by avoiding concrete that is unnecessarily strong.

This methodology allows a fast and accurate prediction of values for compressive strength on site. Common methods for estimation of in place strength requires extensive use of curing of mortar cubes at constant temperatures or the use of databases containing a large number of compressive strength values made at many ages and cured at different temperatures. These databases have to be fed with a statistical relevant number of data before a reliable estimation of the strength can be made. Furthermore all of these methods requires many hours of lab and field time for testing, collecting and analyzing data.

Furthermore, the existing variables in the model yielded good reasonable results. Also, it is not preferred to load the prediction model with large number of variables, because it is preferred to use a model with lesser number of variables with most higher possible accuracy to assure the rapid and easy use of the model.

REFERENCES

Adhikary, B.B. and H. Mutsuyoshi, 2006. Prediction of shear strength of steel fiber RC beams using neural networks. Construct. Build. Mater., 20: 801-811.

Bai, J., S. Wild, J.A. Ware and B.B. Sabir, 2003. Using neural networks to predict workability of concrete incorporating metakaolin and fly ash. Adv. Eng. Software, 34: 663-669.

Jee, N., Y. Sangchun and C. Hongbum, 2004. Prediction of compressive strength of *in situ* concrete based on mixture proportions. J. Asian Architect. Build. Eng., 3: 9-16.

Kheder, G.F., A.M. Al-Gabban and M.A. Suhad, 2003. Mathematical model for the prediction of cement compressive strength at the ages of 7 and 28 days within 24 hours. Mater. Struct., 36: 693-701.

Mehta, P.K., 2004. High performance, high-volume fly ash concrete for sustainable development. Proceedings of the International Workshop on Sustainable Development and Concrete Technology CTRE Iowa State University, May 20-21, Beijing, China, pp: 3-14.

Pala, M., E. Özbay, A. Özta° and M.I. Yüce, 2007. Appraisal of long-term effects of fly ash and silica fume on compressive strength of concrete by neural networks. Construct. Build. Mater., 21: 384-394.

Popovics, S. and J. Ujhelyi, 2008. Contribution to the concrete strength versus water-cement ratio relationship. J. Mater. Civil Eng., 20: 459-463.

Topcu, I.B. and M. Saridemir, 2008a Prediction of compressive strength of concrete containing fly ash using artificial neural network and fuzzy logic. Comput. Mater. Sci., 41: 305-311.

Topcu, I.B. and M. Saridemir, 2008b Prediction of rubberized concrete properties using artificial neural network and fuzzy logic. Construct. Build. Mater., 22: 532-540.

Topçu, Ý.B. and M. Sarýdemir, 2007. Prediction of properties of waste AAC aggregate concrete using artificial neural network Comput. Mater. Sci., 41: 117-125.

Zain, M.F.M., M. Safiuddin and K.M. Yusof, 1999. Study on the properties of freshly mixed high performance concrete. Cem. Concrete Res., 29: 1427-1432.

Zain, M.F.M., M. Safiuddina and H. Mahmudb, 2000. Development of high performance concrete using silica fume at relatively high water-binder ratios. Cem. Concrete Res., 30: 1501-1505.