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Precise Formulation of Electrical Capacitance for a Cylindrical Capacitive Sensor

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Abstract: In this study a more precise formulation of electrical capacitance for a cylindrical capacitive sensor is reported. By using different theoretical models such as Coulomb law, Gauss law and Laplace equation the electrical capacitance is calculated. Based on the given models the relation between the capacitance and the geometrical parameters (e.g., cylindrical length) is formulated and by using suitable software capacitance variation is computed and compared for different methods. In Coulomb method, the electrical potential is first solved numerically by using Mathematica and then the electrical capacitance is computed. It is found that the active capacitor length is crucial parameter in the calculations and therefore variation of this parameter is considered in our calculations. It is noted that the capacitance value is very sensitive to the length according to the method and it is deviated sharply for small length (about 20 cm) from the Gauss approximation. By comparing obtained results one recognizes that there is a pronounced error difference using approximated laws in short capacitor length range while for long length a negligible difference is noted between the tested models.

Key words: Capacitance, potential, Coulomb law, Gauss law, Laplace equation

INTRODUCTION

In recent years various reports on the capacitance measurements for different liquids (one phase and two phases) and possible applications of capacitive sensors have been given by Golnabi and Azimi (2008a), Zadeh and Sawan (2005) and Ahn *et al.* (2005). Physical and chemical effects have been used to develop nanometer-scale capacitors for various applications (Stepanic and Bilalbegovic (1999). Recently some authors investigate on the capacitance of the open-ended charged tube, it is a fundamental and important problem in potential theory because it exhibits the physical features of finiteness, edge and curvature (Scharstein, 2007). Butler (1980) summarized some of the earlier approximations for the capacitance and compares these with his numerical results. The potential problem for the cylindrical capacitive sensor with finite-length is a truly complicated problem and unlike the simpler geometries considered by Hernandes and Asis (2005). For example, has not surrendered to an exact solution via separation of variables. Several moment method solutions for a capacitance measurement can be found in literature, such as Chakraborty *et al.* (1993). The basic geometry of the open-ended charged tube is augmented with dielectric coatings by Chakrabarty *et al.* (2002) and Monzon (2003). The charged tube problem is one version of the thin wire problem that is the subject of continuing debate (Jackson, 2002). The primary contribution of the present study is the analytical calculation of the cylindrical capacitive sensor capacitance in different ways.

CAPACITANCE CALCULATION OF CYLINDRICAL CAPACITIVE SENSOR

The proposed capacitive probe is shown in Fig. 1 consists of a three-part coaxial capacitive sensor in which the middle one is acting as the main sensing probe and the other two capacitors considered as the guard rings in order to reduce the stray capacitance effect and the source of errors in measurements (it must be mentioned that such design is more useful for the case of small capacitance change measurements). In this arrangement a cylindrical geometry is chosen and aluminum materials are used as the capacitor tube electrodes. The diameter of the inner electrode is about 12 mm and the inner diameter of the outer electrode is about 23 mm and has a thickness wall diameter of about 3.5 mm. The overall height of the probe is about 12 cm while the active probe has a length of about 4 cm. The radial gap between the two tube

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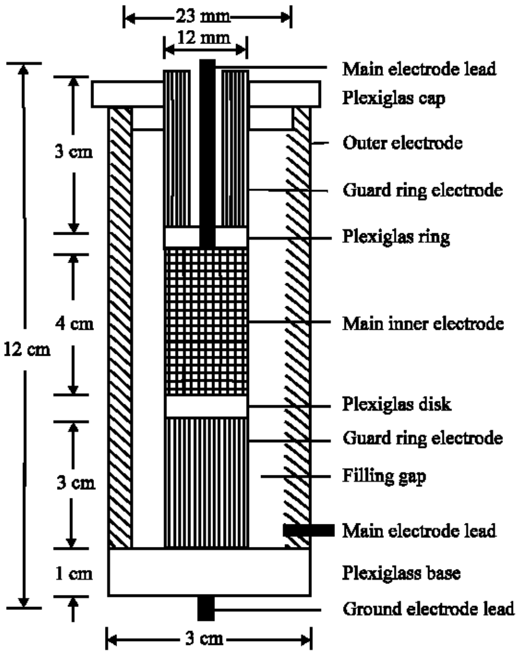

Fig. 1: Diagram of the designed cylindrical capacitive sensor

electrodes is about 5.5 mm and the overall diameter of the probe is about 3 cm. The length of the employed wire connection to the inner active electrode is about 5 cm. As shown in Fig. 1, the middle active part of the probe has a length of 4 cm and the outer guard electrodes have a length of about 3 cm.

Gauss law: Using Gauss's law the capacity of a long cylindrical capacitor can be obtained from:

$$C = \frac{2\pi\varepsilon L}{\text{Ln}\frac{b}{a}} \quad (1)$$

where, ε is the permittivity of the gap dielectric medium, a is the inner electrode radius, b outer electrode radius and L is the capacitor length.

However Eq. 1 is only valid when L>> a,b. Several problems such as edge effect can cause a deviation in the actual capacity from the given formula in Eq. 1. For this reason, various attempts have been made to reduce errors due to limited size effects. One simple remedy has been the use of a Kelvin guard-ring (Golnabi, 2000) in which the main inner electrode is shielded by a grounded guard-ring electrode.

Coulomb law: The capacitance of the cylindrical capacitive sensor of Fig. 2 is an interesting problem in the experimental calculations of liquids capacitance in the

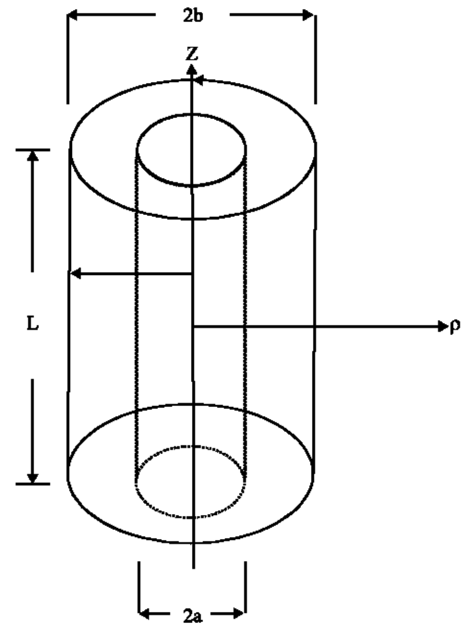

Fig. 2: Schematic diagram of the cylindrical capacitive sensor

precise measurements (Golnabi and Azimi, 2008b). So, it is shown the obvious calculations of the capacitance by Coulomb method.

In this method, we use an electrical potential according to:

$$\varphi(r) = \frac{1}{4\pi\varepsilon}\int \frac{\sigma' ds'}{|\vec{r} - \vec{r}'|} \quad (2)$$

In the cylindrical coordinates, we have:

$$ds' = \rho' d\varphi' dz' \quad (3)$$

$$|\vec{r} - \vec{r}'|^2 = \rho^2 + \rho'^2 + (z - z')^2 - 2\rho\rho' \cos(\varphi - \varphi') \quad (4)$$

$$\varphi_\rho^{(\rho')} = \frac{1}{4\pi\varepsilon}\int \frac{\sigma' \rho' d\varphi' dz'}{\sqrt{\rho^2 + \rho'^2 + (z - z')^2 - 2\rho\rho' \cos(\varphi - \varphi')}} \quad (5)$$

For simplicity we replace φ = 0 in Eq. 4. In regard to

$$\sigma' = \frac{Q}{2\pi\rho' L}$$

We derive electrical potential in:

$$\varphi_\rho^{(\rho')} = -\frac{Q}{4\pi\varepsilon(2\pi\rho' L)}\left(\int \rho' \text{Ln}\left(z - z' + \sqrt{\rho'^2 + (z - z')^2 - 2\rho\rho' \cos\varphi' + \rho^2}\right)\Big|_{-\frac{L}{2}}^{\frac{L}{2}} d\varphi'\right) \quad (6)$$

$$\varphi_{\rho}^{(\rho')} = -\frac{Q}{8\pi^2\varepsilon L}\left(\int \mathrm{Ln}\left(\frac{z-\frac{L}{2}+\sqrt{\rho'^2+(z-\frac{L}{2})^2-2\rho\rho'\cos\varphi'+\rho^2}}{z+\frac{L}{2}+\sqrt{\rho'^2+(z+\frac{L}{2})^2-2\rho\rho'\cos\varphi'+\rho^2}}\right)d\varphi'\right) \quad (7)$$

The electrical potential difference between two cylinders with radius a,b is:

$$\varphi_{ab} = -\frac{Q}{8\pi^2\varepsilon L}\left(\begin{aligned}&\int \mathrm{Ln}\left(\frac{z-\frac{L}{2}+\sqrt{a^2+(z-\frac{L}{2})^2-2ba\cos\varphi'+b^2}}{z+\frac{L}{2}+\sqrt{a^2+(z+\frac{L}{2})^2-2ba\cos\varphi'+b^2}}\right)d\varphi' + \\ &\int \mathrm{Ln}\left(\frac{z-\frac{L}{2}+\sqrt{b^2+(z-\frac{L}{2})^2-2b^2\cos\varphi'+b^2}}{z+\frac{L}{2}+\sqrt{b^2+(z+\frac{L}{2})^2-2b^2\cos\varphi'+b^2}}\right)d\varphi'\end{aligned}\right) \quad (8)$$

$$+\frac{Q}{8\pi^2\varepsilon L}\left(\begin{aligned}&\int \mathrm{Ln}\left(\frac{z-\frac{L}{2}+\sqrt{b^2+(z-\frac{L}{2})^2-2ba\cos\varphi'+a^2}}{z+\frac{L}{2}+\sqrt{b^2+(z+\frac{L}{2})^2-2ba\cos\varphi'+a^2}}\right)d\varphi' + \\ &\int \mathrm{Ln}\left(\frac{z-\frac{L}{2}+\sqrt{a^2+(z-\frac{L}{2})^2-2a^2\cos\varphi'+a^2}}{z+\frac{L}{2}+\sqrt{a^2+(z+\frac{L}{2})^2-2a^2\cos\varphi'+a^2}}\right)d\varphi'\end{aligned}\right) \quad (9)$$

Finally we can calculate the electrical capacitance from Eq. 9:

$$C_c = \frac{Q}{\varphi_{ab}} = \frac{8\pi^2\varepsilon L}{\left(\begin{aligned}&\int \mathrm{Ln}\left(\frac{z-\frac{L}{2}+\sqrt{a^2+(z-\frac{L}{2})^2-2a^2\cos\varphi'+a^2}}{z+\frac{L}{2}+\sqrt{a^2+(z+\frac{L}{2})^2-2a^2\cos\varphi'+a^2}}\right)d\varphi' - \\ &\int \mathrm{Ln}\left(\frac{z-\frac{L}{2}+\sqrt{b^2+(z-\frac{L}{2})^2-2b^2\cos\varphi'+b^2}}{z+\frac{L}{2}+\sqrt{b^2+(z+\frac{L}{2})^2-2b^2\cos\varphi'+b^2}}\right)d\varphi'\end{aligned}\right)} \quad (10)$$

One can solve Eq. 10 numerically and then shows the capacitance variation as a function of the length in defined azimuth angle.

Laplace equation: The exact capacitance value of the cylindrical capacitive sensor shown in Fig. 2 is an important problem in the real applications of capacitive sensors. In an alternate method the potential theory can be employed to calculate the capacitance value theoretically and compare with the experimental results. The canonical geometry is simple enough to attract the attention of analysis. If the conducting surfaces are held to the potential φ_a and φ_b for the invasive and non-invasive cases then one seeks the axisymmetric solution to the Laplace's equation given by:

$$\nabla^2\varphi(\rho,z) = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\varphi}{\partial\rho}\right)+\frac{\partial^2\varphi}{\partial z^2} = 0 \quad (11)$$

In the cylindrical coordinates, subject to the boundary condition one can write:

$$\varphi(a,z) = \varphi_a \qquad \varphi(b,z) = \varphi_b \qquad (|z| \le \frac{L}{2}) \quad (12)$$

where, the harmonic function φ also vanishes at infinity. The calculation starts from the electrical potential $\varphi_{ab}(\rho, z)$ in the space between two coaxial cylinders of radii a and b (a<b) for a finite length L in the z direction:

$$\varphi_{ab}(\rho,z) = V\frac{4}{\pi}\sum_{k=1,3,5,...}\frac{\sin\nu_k z}{k}\frac{I_0(\nu_k\rho)K_0(\nu_k a)-I_0(\nu_k a)K_0(\nu_k\rho)}{I_0(\nu_k b)K_0(\nu_k a)-I_0(\nu_k a)K_0(\nu_k b)} \quad (13)$$

Where:

$$\nu_k = \frac{k\pi}{L} \text{ and } \frac{-L}{2} \le z \le \frac{L}{2}$$

The parameter V is the potential difference applied between the two cylinders, whereas $I_i(x)$ and $K_i(x)$ are the modified Bessel functions of i order. The capacitance: C_1 (a, b, L) of the structure is obtained:

$$C_1 = \frac{2\pi a\varepsilon}{V}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(\frac{\partial\varphi}{\partial\rho}\right)_{\rho=a} dz \quad (14)$$

And the final result is given by:

$$C_1 = 16a\varepsilon\sum_{k=1,3,5,...}\frac{1}{k}\frac{I_1(\nu_k a)K_0(\nu_k a)+I_0(\nu_k a)K_1(\nu_k a)}{I_0(\nu_k b)K_0(\nu_k a)-I_0(\nu_k a)K_0(\nu_k b)} \quad (15)$$

So, by using Eq. 15 we can calculate the capacitance of a cylindrical capacitive sensor. We consider the capacitance variations of a cylindrical sensor as a function of the active length and compare the computed results.

COMPUTATIONAL RESULTS

As described, the capacitance value strongly depends on geometrical configuration, length and dimension of capacitance, so Fig. 3 shows the results of our theoretical computation of the capacitance divided by length as a function of the cylindrical length by two methods.

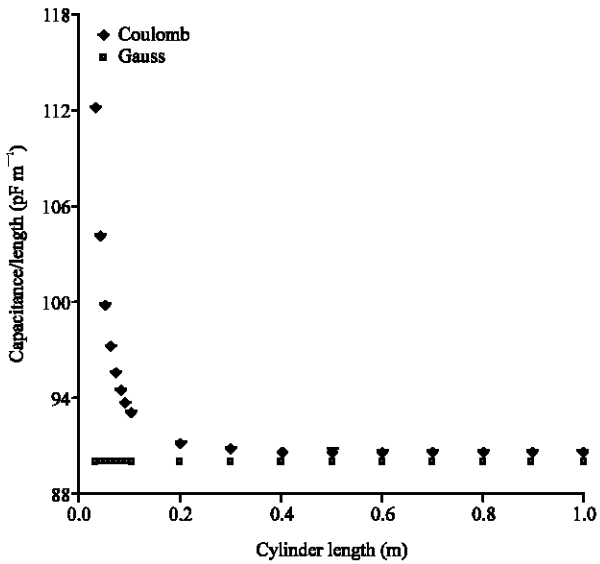

Fig. 3: Division of capacitance per length as a function of the cylindrical length

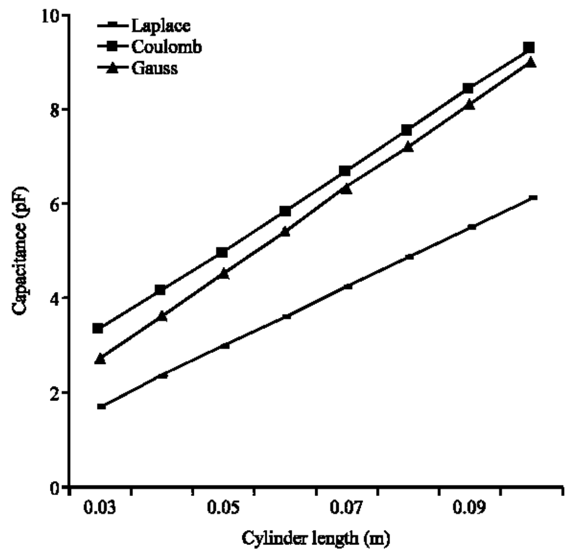

Fig. 4: Capacitance as a function of cylindrical length in, Coulomb, Gauss and Laplace methods

Two major points can be drawn from Fig. 3. It is considered in Coulomb method by increasing the cylindrical length from 0.03 to 20 cm. The capacitance decreases sharply from 112.138 to 91.184 pF and then it becomes constant accordingly, on the other hand in regard to this subject, the Gauss formula is for the cylindrical capacitance with long lengths, so we recognize division of capacitance by lengths is constant (90.1 pF) for all the higher length, Fig. 3 shows the Coulomb and Gauss formulas are compatible with each other in large lengths. Figure 4 shows us the capacitance value as a function of length in the three methods. Figure 4 shows the Coulomb and Gauss methods that the capacitance is increased as a function of cylindrical length from 3.364 to 9.308 pF in finite length, 10 cm, but in Laplace method the capacitance is increased from 1.720 to 6.101 pF, which is a little different because of the boundary condition in cylindrical surface.

To see the capacitance difference (C_c-C) divided by length as a function of the cylinder length in Coulomb method , a series of calculations are shown in Fig. 5. As can be seen in Fig. 5, results of calculated capacitance values for different length show the capacitance difference is decreased significantly in short length (20 cm) from 21.9 to 0.947 pF that is compatible with Fig. 3. And then it reaches to a nearly constant value 0.32 pF.

To understand the importance of the capacitance variations as a result of different cylindrical lengths in a cylindrical sensor, we consider the calculated values for

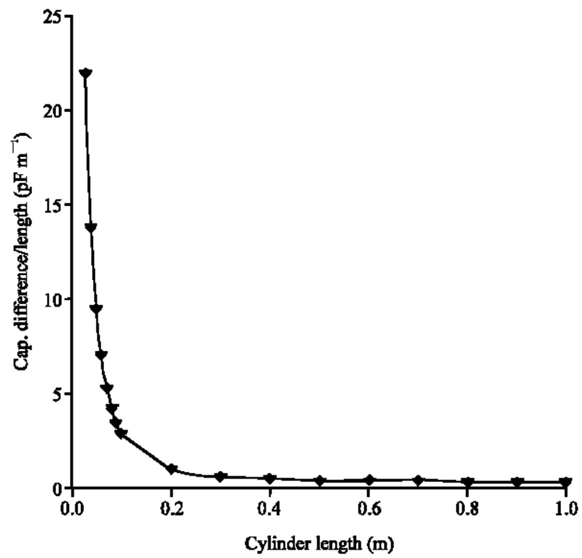

Fig. 5: Capacitance difference as a function of cylindrical length

the length of 1 m in 10 cm increments. As indicated in Fig. 6. The average capacitive value for different lengths is 31.423 pF as shown in Fig. 5 the highest calculated capacitance value is 90.327 pF for the length 100 cm while the lowest calculated capacitance is 2.707 pF.

Precision is an important parameter to calculate the capacitance of cylindrical sensor. Figure 7 shows the relative error percentage for a series of data that is calculated from:

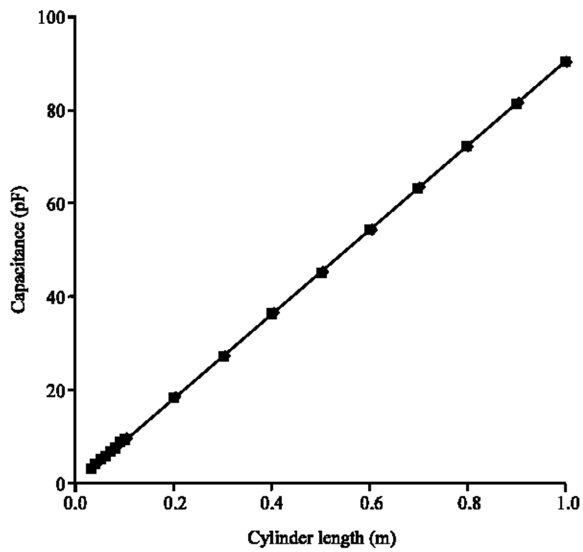

Fig. 6: Capacitance as a function of cylindrical length in Coulomb method

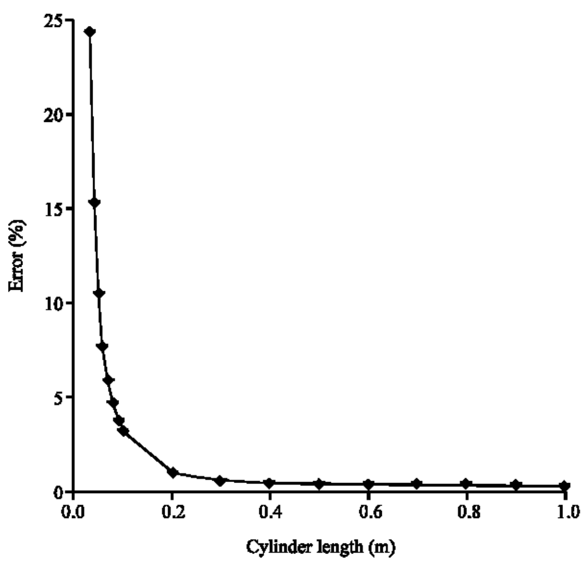

Fig. 7: Capacitance error percentage as a function of cylindrical length in cylindrical sensor

$$\text{Relative error (\%)} = \frac{C - C_c}{C_c} \times 100 \qquad (16)$$

Figure 7 shows the error percentage is high in small lengths about 2 cm and then decreases sharply in about 8 cm, finally it reaches to a minimum value of 1% in 60 cm after that, it becomes constant in higher length.

CONCLUSION

As described, we derived the capacitance of the cylindrical sensor in three methods, in regard to the calculated values from these formulas, we recognize that the Coulomb method is more precise than the other ways, on the other hand the calculated values obtained from the Gauss formula are compatible with Coulomb method in large lengths, also Laplace method is a strong way to calculate the capacitance of a cylindrical sensor but the data that is considered in this method have a little deviation from the obtained values from the Coulomb and Gauss methods which depend on the boundary condition on the surfaces of the cylindrical sensor. By knowing these formulas we can compare the experiment results with theoretical results and then find the precise value of physical quantities.

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REFERENCES

Ahn, H.J., I.H. Kim and D.C. Han, 2005. Nonlinear analysis of cylindrical capacitive sensor. Measure. Sci. Technol., 16: 699-706.

Butler, C.M., 1980. Capacitance of a finite-length conducting cylindrical tube. J. Applied Phys., 51: 5607-5609.

Chakrabarty, S.B., S. Das and B.N. Das, 2002. Capacitance of dielectric coated cylinder of finite axial length and truncated cone isolated in free space. IEEE Trans. Electromagn. Comp., 44: 394-398.

Chakraborty, C., D.R. Poddar, A. Chakraborty and B.N. Das, 1993. Electrostatic charge distribution and capacitance of isolated cylinders and truncated cones in free space. IEEE Trans. Electromagn. Comp., 35: 98-102.

Golnabi, H., 2000. Guard-ring effects on capacitive transducer systems. Sci. Iran., 7: 25-31.

Golnabi, H. and P. Azimi, 2008a. Design and performance of a cylindrical capacitive sensor to monitor the electrical properties. J. Applied Sci., 8: 1699-1705.

Golnabi, H. and P. Azimi, 2008b. Simultaneous measurements of the resistance and capacitance using a cylindrical sensor system. Mod. Phys. Lett. B., 22: 595-610.

Hernandes, J.A. and A.K.T. Asis, 2005. Electric potential due to an infinite conducting cylinder with internal or external point charge. J. Electrostatics, 63: 1115-1131.

Jackson, J.D., 2002. Charge density on a thin straight wire. Am. J. Phys., 70: 409-410.

Monzon, C., 2003. The capacitance of a dielectric coated finite cylindrical duct. IEEE Trans. Electromagn. Comp., 45: 636-638.

Scharstein, R., 2007. Capacitance of a tube. J. Electrostatics, 65: 21-29.

Stepanic, J. and G. Bilalbegovic, 1999. Nanometer-scale capacitors. Fizika A., 8: 261-266.

Zadeh, E.G. and M. Sawan, 2005. High accuracy differential capacitive circuit for bioparticles sensing applications. Proceedings of the 48th Midwest Symposium on Circuits and Systems, Aug. 7-10, IEEE Xplore, pp: 1362-1365.