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A New Lyapunov-Based Design Scheme for Adaptive Friction Compensation

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Abstract: A new method for adaptive friction compensation in mechanical control systems is developed. The design is based on Lyapunov technique and attempts to compensate for frictional force by estimating the unknown Coulomb friction coefficient. The contribution of this paper is to generalize the Friedland and Park's method and show that it is possible to include their scheme as a special case of the proposed method. More specifically, it is shown that for asymptotic stability of the error dynamics the constraint on the velocity is removed in both cases of time varying (without any constraint on frequency) and time invariant friction coefficient. Furthermore, an analytical procedure is developed for designing a general nonlinear friction estimator. Simulation results confirm the advantages of the proposed methods for a simple single-mass system as well as a more complicated system such as a two-link planar rigid robot manipulator.

Key words: Friction coefficient, adaptive compensation, Lyapunov technique, robot manipulator

INTRODUCTION

Friction is an unavoidable force which exists in all mechanical systems involving moving parts; so it is constantly a hot topic in control community. In certain systems such as braking system it is a desirable property, whereas, as shown in Fig. 1, for servomechanism systems the effects of the friction must be compensated in order to get a good performance.

Although by using lubricating materials the effect of friction can be reduced, this remedy is not generally feasible and practical. Consequently, different nonlinear and adaptive control strategies have been used in the past decade for friction compensation. Friction models may be categorized into two main classes, namely, static and dynamic models. From a practical point of view, dynamical effects of friction are often small and difficult to measure (Friedland and Mentzelopoulou, 1993). On the other hand, static models with a nonlinear map from velocity to frictional force play an important role in control system analysis and design. Once the effects of Coulomb, viscous and other types of friction are considered, one may develop different nonlinear maps from the velocity variable to the frictional force.

The problem of friction modeling and estimating has been considered in the past in many research works. Since the friction compensation concept was introduced by Haessig and Friedland (1991) several adaptive friction compensation controllers have been designed in many works (Friedland and Park, 1991; Friedland and

Mentzelopoulou, 1992; Yazdizadeh and Khorasani, 1996a, b; Liao and Chien, 2000; Kelly *et al.*, 2004; Suraneni *et al.*, 2005; Hua-Xia *et al.*, 2006; Shang *et al.*, 2008). After the pioneering works by Friedland and Park (1991) and Friedland and Mentzelopoulou (1992) several modifications were introduced by Yazdizadeh and Khorasani (1996a, b) and Liao and Chien (2000). They have focused on designing new adaptation laws by proposing new forms of the tuning function $g(|v|)$ where, v is the velocity. Mainly, their considerations were to design more stable nonlinear adaptive controller. In the work by Friedland and Park (1991), the asymptotically stable controller with restrictive conditions was designed. The contribution in the work by Yazdizadeh and Khorasani (1996a, b) which are more elaborated in this paper was to relax those restrictive conditions imposed in the study by Friedland and Park (1991).

Due to the recent advances in intelligent control, research in friction compensation using an intelligent control scheme has appeared in the literature. Suraneni *et al.* (2005) proposed an adaptive tracking control scheme based on dynamic fuzzy logic system. The proposed method is an online identification and indirect adaptive control, in which the control input is adjusted

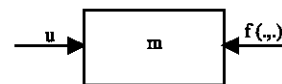


Fig. 1: A single mass servomechanism system

adaptively to compensate the effects of the nonlinearities. A robust adaptive compensation technique for tracking issue in a 1-DOF mechanical system with stick-slip friction is proposed by Lee and Kim (1995). However, most of the conventional adaptive control technique relies on linear parameterizations that usually tend to be restrictive.

In all above mentioned analytical efforts in attacking friction effects, the Coulomb friction coefficient is assumed to be constant. In other words, the existing stability analysis is all restricted to the time invariant friction coefficient. But in reality, the magnitude of friction coefficient may depend on velocity which practically is not a constant. Having considered this fact, Ahn and Chen (2004) focused on the applications, where the reference position and reference velocity are periodically time varying and so the friction is also a periodic variable. An adaptive friction compensation controller with a time varying friction coefficient was designed. In another study by Ahn and Chen (2005), the situation in which the friction force be related to the state is considered. The past information of the trajectory along the state axis was used to update the current adaptation since the friction is state-periodic.

In this study, we first introduce and more elaborate on the original method proposed by Yazdizadeh and Khorasani (1996a, b) that relaxes the constraint imposed by previous works and then a situation in which the frictional force is considered as a disturbance that may depend on velocity, position, state or an external source is investigated. The variation of the friction is not assumed to be restricted to periodic cases. The proposed approach can be used to adaptively compensate the frictional force in many practical applications including a mobile robot moving on a floor composed of different materials with different friction coefficients that experiences different friction forces depending on position. It can be seen from the simulation results that the proposed approach makes more precision as well as higher speed of the convergence.

MATERIALS AND METHODS

The original lyapunov-based strategy for friction compensation: Generally, if the nonlinearities in a system are known precisely, a control law based on feedback linearization method can be constructed (under certain conditions) and by proper selection of the controller gains, stability and desired performance of the closed-loop system can be ensured. In the case where for example friction is unknown, the parameters are first estimated and based on the estimates control command

can be constructed and applied to the system. Recursive Least-Squares (RLS) and Least Mean-Squares (LMS) methods for parameter estimation for friction compensation have been reported in the study by Canudas and Praly (2000). Basically, the RLS and LMS algorithms are first used to estimate unknown parameters such as viscous and Coulomb coefficients and subsequently the control command is constructed based on the estimates. The other common approach reported in the literature is based on Model Reference Adaptive Control (MRAC) strategy (Gilbert and Winstone, 1974). In this approach the objective is to minimize the error between the states of the system and the model states whose trajectories characterize the desired signals to be followed.

Among different parameter estimation methods, the Lyapunov-based method has shown promising results (Friedland and Park, 1992). In the Lyapunov-based method the control command is designed (or constructed) such that the derivative of a Lyapunov function candidate along the trajectories of the system enjoys certain properties. For the sake of clarity and illustration, the steps that are involved in the design of an adaptive friction compensation for a single-mass system are worked out in detail.

Dynamic equation of a single-mass system (Fig. 1) is given by:

$$ma = -f(v, k_c) + u \tag{1}$$

Where:

$$f(v, k_c) = k_c \operatorname{sgn}(v) \tag{2}$$

is the Coulomb friction model, u is the effect of all forces except the friction, a is the acceleration of the mass and m is the mass. Without loss of generality and for the sake of simplicity we assume that $m = 1$, so, the dynamic equation is written as:

$$\dot{v} = -k_c \operatorname{sgn}(v) + u \tag{3}$$

or equivalently in the state space representation as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -k_c \operatorname{sgn}(x_2) + u \end{cases} \tag{4}$$

where, $x_1 = x$ is the position of the mass and $x_2 = \dot{x}$ is its velocity. A linearizing feedback control for this system is given by:

$$u = k_c \operatorname{sgn}(x_2) + g_1 x_1 + g_2 x_2 \tag{5}$$

that results in the closed loop system as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = g_1 x_1 + g_2 x_2 \end{cases} \quad (6)$$

where, g_1 and g_2 are chosen such that the system is asymptotically stable and desired performance specifications are satisfied. The main problem that we are faced with is that k_c is unknown.

Therefore, the above controller is rendered not implementable. Due to this difficulty, it is proposed that the estimated value of k_c should be used instead based on the following nonlinear expression:

$$\hat{k}_c = z - g(|x_2|) \quad (7)$$

where, z is an intermediate variable whose dynamics is to be specified shortly based on a Lyapunov function candidate.

Defining the estimation error by $e = k_c - \hat{k}_c$, the Lyapunov function candidate is now chosen as:

$$V = \frac{1}{2} e^2 \quad (8)$$

Consequently, the time derivative of V along the trajectories of the system becomes:

$$\dot{V} = -\hat{k}_c (k_c - \hat{k}_c) = -[z - g(|x_2|) \hat{x}_2 \operatorname{sgn}(x_2)] (k_c - \hat{k}_c) \quad (9)$$

The term \dot{z} is at our disposal and is now selected in such a way as to make \dot{V} negative definite. By a simple manipulation, one may show that if \dot{z} is governed by:

$$\dot{z} = g(|x_2|) [u - \hat{k}_c \operatorname{sgn}(x_2)] \operatorname{sgn}(x_2) \quad (10)$$

then we get, $\dot{e} = -g(|x_2|)e$ and $\dot{V} = -g(|x_2|)e^2$. Clearly, \dot{V} is negative definite if and only if $0 < g(|x_2|) < K_{\max}$. Therefore, the conditions for asymptotic stability of the error system may now be written as follows:

- $g(|x_2|)$ is positive definite and $g(|x_2|)$ is monotonically increasing
- $g(|x_2|)$ is bounded

The nonlinear function $g(|x_2|)$ is selected based on the above two criteria. There are many functions that will satisfy these conditions. As an example, let:

$$g(|x_2|) = \ln \frac{1}{1 + e^{-|x_2|}}$$

a monotonically increasing function in $|x_2|$ with a derivative:

$$\dot{g}(|x_2|) = \frac{e^{-|x_2|}}{1 + e^{-|x_2|}}$$

bounded by 0.5, so, the nonlinear estimator is constructed according to:

$$\hat{k}_c = z - \ln \frac{1}{1 + e^{-|x_2|}} \quad (11)$$

$$\dot{z} = \frac{e^{-|x_2|}}{1 + e^{-|x_2|}} [u - \hat{k}_c \operatorname{sgn}(|x_2|)] \operatorname{sgn}(|x_2|) \quad (12)$$

Using the above update rule, the following equations are obtained for \dot{e} and \dot{V}

$$\dot{e} = -\frac{e^{-|x_2|}}{1 + e^{-|x_2|}} e \quad (13)$$

$$\dot{V} = -\frac{e^{-|x_2|}}{1 + e^{-|x_2|}} e^2 \leq -\frac{1}{2} e^2 \quad (14)$$

Implying that the error dynamics is asymptotically stable (Note that $e = 0$ is an equilibrium point, so, $e \rightarrow 0$ as $t \rightarrow \infty$). In order to have control over the rate of parameter convergence, the estimator structure is now modified to:

$$\hat{k}_c = z - k \ln \frac{1}{1 + e^{-k|x_2|}} \quad (15)$$

$$\dot{z} = k\mu \frac{e^{-|x_2|}}{1 + e^{-|x_2|}} [u - \hat{k}_c \operatorname{sgn}(|x_2|)] \operatorname{sgn}(|x_2|) \quad (16)$$

where, the parameters k and μ are design variables that are selected to achieve proper transient and convergence performances.

For the sake of comparison, note that the estimator by Friedland and Park (1991) may be considered as a special case of our systematic and general strategy in which

$$g(|x_2|) = k |x_2|^\mu \quad (17)$$

$$\hat{k}_c = z - k |x_2|^\mu \quad (18)$$

$$\dot{z} = k\mu |x_2|^{\mu-1} [u - \hat{k}_c \operatorname{sgn}(|x_2|)] \operatorname{sgn}(|x_2|) \quad (19)$$

The above choice results in conditional asymptotic stability of the error dynamics. In other words, the error dynamics is asymptotically stable provided that velocity

is always maintained to be bounded away from zero (Friedland and Park, 1992). It is precisely this specific condition that is now relaxed by our proposed approach.

It is worth noting that the principal behind our approach, Friedland and Park's, RLS, LMS and MRAC methods are all the same. In all the above cases the parameter estimation is proportional to the integral of the acceleration error. For example, in the Lyapunov-based approach, \hat{k}_c is proportional to z and \dot{z} is proportional to $[u - \hat{k}_c \text{sgn}(|x_2|)]$ which implies that \hat{k}_c is proportional to the integral of $[u - \hat{k}_c \text{sgn}(|x_2|)]$. This is also the case for the RLS, LMS and MRAC (Armstrong-Helouvy *et al.*, 1994) methods and is an important property that one should consider for designing other nonlinear estimators.

It is also worth noting that in all the above cases k_c is assumed to be an unknown parameter which is constant. But as mentioned before it is known that in many applications this parameter is not constant and may vary by variation of the system position or velocity. In the next section we propose a new method in which the constant constraint on k_c is also relaxed.

The new proposed controller for time varying friction coefficient: Similar to the previous case, we consider a single-mass system with Coulomb friction in the state space representation which is given by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_2) + u \end{cases} \quad (20)$$

where, x_1 is the position of the mass and x_2 is the velocity, $f(x_2)$ is the frictional force and may be considered as:

$$f(x_2) = -k_c(t) \text{sgn}(x_2)$$

where, $k_c(t)$ is Coulomb friction coefficient. Again u is the control input representing the effect of all forces except the friction. Using feedback u , it is possible to make the closed loop control system asymptotically stable. In the new proposed controller for time varying cases, we consider the feedback control input as follow:

$$u = \hat{k}_c(t) \text{sgn}(v) - \alpha E_1(t) - \lambda e_v(t) + \ddot{x}_d \quad (21)$$

where, a combinational definition of error is given by:

$$E_1(t) = e_v(t) + \lambda e_x(t) \quad (22)$$

And the error on position and velocity are given by:

$$e_x(t) = x(t) - x_d(t) \quad (23)$$

$$e_v(t) = v(t) - \dot{x}_d(t) \quad (24)$$

where, $\alpha > 0$ and $\lambda > 0$ are constant gains, $e_x(t)$ and $e_v(t)$ are position and velocity errors in which x_d is the desired position trajectory. Again $\hat{k}_c(t)$ is an estimation of the Coulomb friction coefficient. In the proposed new method, the adaptation law is designed as follows:

$$\dot{\hat{k}}_c(t) = -P \text{sgn}(v) E_1(t) \quad (25)$$

where, P is a positive design parameter.

To prove asymptotic stability, the Lyapunov stability theorem and LaSalle theorem are adopted herein. If the Lyapunov function candidate is chosen as:

$$V(t) = \frac{1}{2} E_1^2(t) + \frac{1}{2P} \int_{t-T_g}^t e_k^2(\tau) d\tau \quad (26)$$

Where:

$$e_k := k_c(t) - \hat{k}_c(t) \quad (27)$$

Then the time variation of the $V(t)$ on a period of time along the trajectory of the closed-loop system leads to:

$$\Delta V(t) = V(t) - V(t - T_g) = \int_{t-T_g}^t -(\alpha + \frac{P}{2}) E_1^2(\tau) d\tau \quad (28)$$

It can be easily shown that if $\alpha + P/2 > 0$, then $\Delta V \leq 0$. Using the Invariant set theorem, it can be seen that the control law (Eq. 21) and the periodic adaptation law (Eq. 25) guarantee the asymptotically stability of the equilibrium points as $t \rightarrow \infty$.

RESULTS AND DISCUSSION

Simulation results: Here, the original proposed method is first applied to a simple single-mass system in and then applied to a more complicated system such as a two-link robot manipulator in, to compensate for friction. For the sake of comparison between the Friedland and Park's scheme and our proposed scheme and in addition to see how the selection of $g(|x_2|)$ affects the results, single-mass system will be simulated with different desired trajectories (different amplitudes) and different Coulomb friction coefficients. It will be shown that friction compensation by our original method is robust to

amplitude variation of the desired trajectory and to the variation of Coulomb friction coefficients (different environments and surfaces), whereas the Friedland and Park's method is not robust to amplitude variation of the desired trajectory and to the variation of Coulomb friction coefficients. The capabilities of the new proposed controller for time varying cases are also shown by performing some simulation. The simulation results in this part are compared with the recent works for periodic time varying case presented by Ahn and Chen (2004, 2005).

A single-mass system with the original proposed method:

The dynamic nonlinear equations for this system and the linearizing feedback control law are as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -k_c \text{sgn}(x_2) + u \end{cases} \quad (29)$$

with

$$u = \hat{k}_c \text{sgn}(x_2) + g_1(x_1 - x_d) + g_2 x_2$$

where, g_1 and g_2 are chosen so that the damping ratio and the natural frequency of the closed-loop system are given by $\xi = 0.707$ and $\omega = 10 \text{ rad sec}^{-1}$. This results in $g_1 = 200$ and $g_2 = 20$. Also k_c is selected as $k_c = 50$.

To see the advantage of the original proposed scheme, the closed-loop system is first simulated with no compensation for the cases with and without friction. Figure 2 shows the result. It is clear that without friction the output x_1 follows the desired trajectory quite satisfactorily, whereas when friction is present the same control results in an unacceptable output tracking due to the presence of large error. In other words, there is not only a steady-state tracking error but also a considerable time delay. Our design goal is to guarantee that the output tracks the desired trajectory even when the friction is present.

Figures 3a, b depict the simulation results for the case when the proposed friction compensation is implemented.

As shown almost perfect tracking is achieved. The nonlinear function $g(|x_2|) = -ke^{-\mu|x_2|}$ that satisfies the two aforementioned criteria has been used for these simulations. The desired trajectory is a square wave lower bounded by -1 and upper bounded by 1 with a period of 2 sec.

It is possible to adjust the parameters k and μ so, that almost the same results are achieved by the Friedland and Park's method. Figure 4a, b show the simulation results for the same system using Friedland and Park's method.

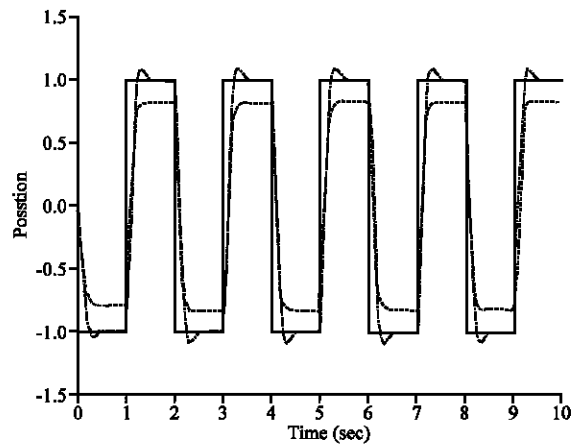


Fig. 2: Performance of the closed-loop system with (----) and without (——) friction compensation

Now to demonstrate the robustness of our proposed method subject to variation in the friction coefficient as well as different desired trajectories, the adaptive system is first simulated for a new desired trajectory, namely, the same square wave but with amplitude of 0.1. Figure 3c, d depict the simulation results for this case. As shown almost perfect tracking is achieved in this case too. However, when Friedland and Park's method is used the output does not track the desired trajectory perfectly. Figure 4c, d show that the output is almost zero for the first 5 sec and the parameter estimate converges to its true value very slowly.

The second advantage of our proposed method is its robustness to variation in the Coulomb friction coefficient. Figure 3e, f depict the simulation results for the case when the friction coefficient is 500. As shown almost perfect tracking is achieved by our method confirming its robustness to Coulomb friction coefficient variation. However, as shown in Fig. 4e and f, the tracking result by the Friedland and Park's method is not satisfactory.

A two-link planar robot manipulator with the original proposed method:

The dynamic equations of a two-link rigid manipulator are given by Craig (1989) as:

$$\tau = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta})\dot{\theta} + G(\theta) + F(\theta) \quad (30)$$

where, τ is the applied torque, θ is the position vector, that is:

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

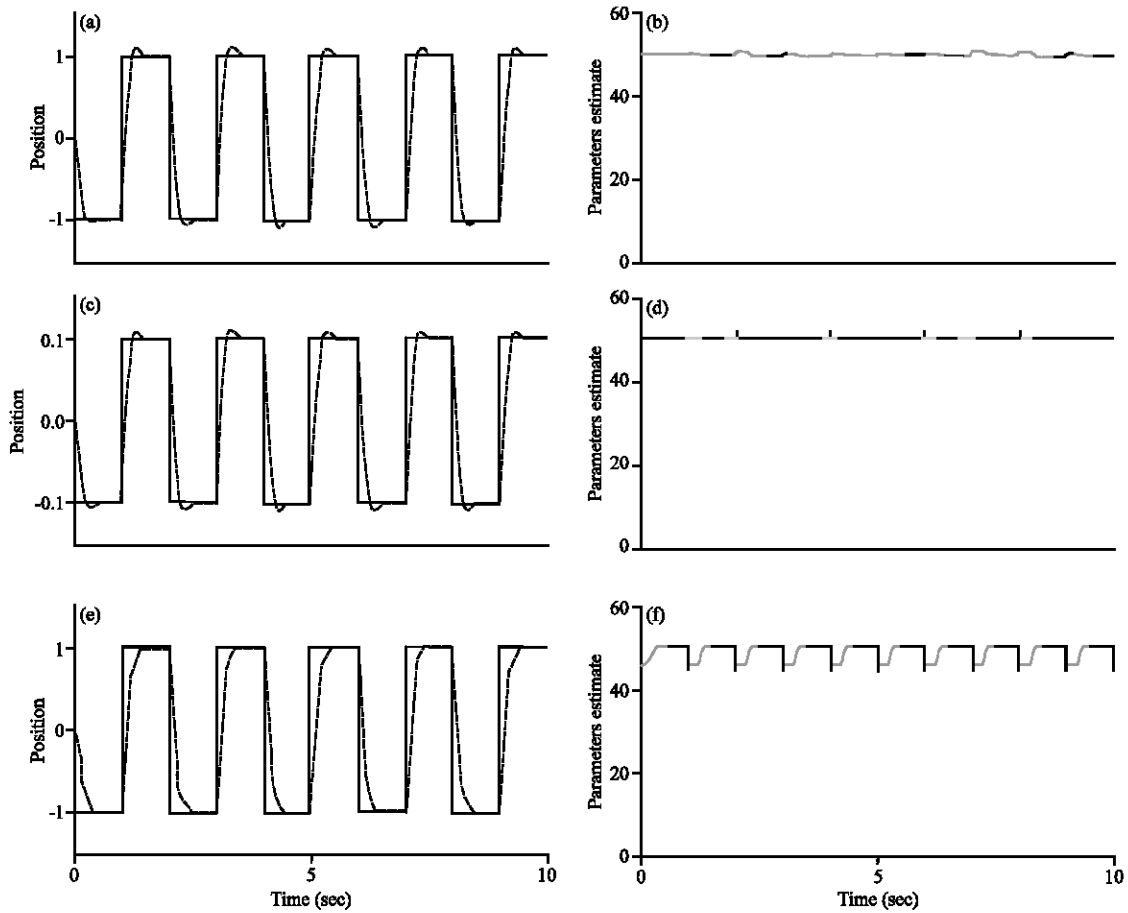


Fig. 3: The results for the original proposed scheme. (a, c, e) Depict the performance of the closed-loop system and (b, d, f) Depict the parameter estimate for $(k_c = 50, x_d = 1)$, $(k_c = 50, x_d = 0.1)$ and $(k_c = 500, x_d = 1)$ respectively. In all cases $k = 135$ and $\mu = 1$

$M(\theta)$ is the symmetric positive definite mass matrix:

$$M(\theta) = \begin{bmatrix} l_2^2 m_2 + 2l_1 l_2 m_2 c_2 + l_1^2 (m_1 + m_2) & l_2^2 m_2 + l_1 l_2 m_2 c_2 \\ l_2^2 m_2 + l_1 l_2 m_2 c_2 & l_2^2 m_2 \end{bmatrix} \quad (31)$$

$V(\theta, \dot{\theta})$ is the centrifugal and Coriolis forces:

$$V(\theta, \dot{\theta}) = \begin{bmatrix} -m_2 l_1 l_2 s_2 \dot{\theta}_2^2 - 2m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 \\ m_2 l_1 l_2 s_2 \dot{\theta}_1^2 \end{bmatrix} \quad (32)$$

$G(\theta)$ is the term due to gravity:

$$G(\theta) = \begin{bmatrix} m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{bmatrix} \quad (33)$$

and $F(\dot{\theta})$ is the coulomb friction that is modeled by:

$$F(\dot{\theta}) = \begin{bmatrix} K_{c1} \text{sgn}(\dot{\theta}_1) \\ K_{c2} \text{sgn}(\dot{\theta}_2) \end{bmatrix} \quad (34)$$

By using partitioned controller design, τ is chosen as:

$$\tau = \alpha \tau_1 + \beta + F(\theta) \quad (35)$$

with $\alpha = M(\theta)$, $\beta = V(\theta, \dot{\theta}) + G(\theta)$ and $\tau_1 = \ddot{\theta}_d + K_v \dot{E} + K_p E$ where, $E = \theta_d - \theta$ and $\ddot{\theta}_d, \dot{\theta}_d$ and θ_d are desired trajectories with $\theta_{1d} = \sin(\pi t)$ and $\theta_{2d} = 5 \sin(\pi t)$. The gains k_v and k_p are chosen such that desired performance specifications are satisfied. Since, $F(\theta)$ is assumed to be unknown, the estimates of k_{c1} and k_{c2} based on the results of the previous section are used in the controller. In other words, the modified controller is given by:

$$\tau = \alpha \tau_1 + \beta + \hat{F}(\theta) \quad (36)$$

By simple manipulations, the output error equations are now derived as:

$$\ddot{E} + K_v \dot{E} + K_p E = M(\theta)^{-1} \begin{bmatrix} \hat{K}_{c1} - K_{c1} & 0 \\ 0 & \hat{K}_{c2} - K_{c2} \end{bmatrix} \begin{bmatrix} \text{sgn}(\dot{\theta}_1) \\ \text{sgn}(\dot{\theta}_2) \end{bmatrix} \quad (37)$$

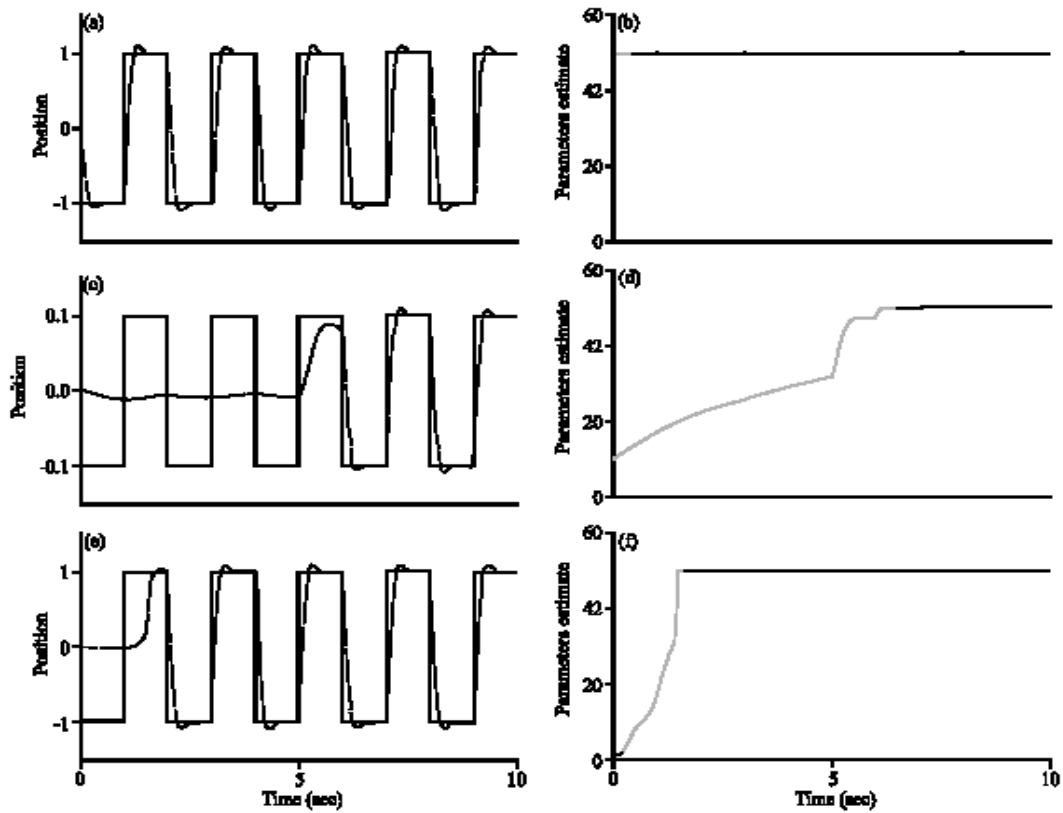


Fig. 4: The results for the Friedland and Park's scheme, Fig. a, c and e depict the performance of the closed-loop system and Fig. b, d and f depict the parameter estimate for $(k_c = 50, x_d = 1)$, $(k_c = 50, x_d = 0.1)$ and $(k_c = 500, x_d = 1)$, respectively. In all cases $k = 10$ and $\mu = 2$

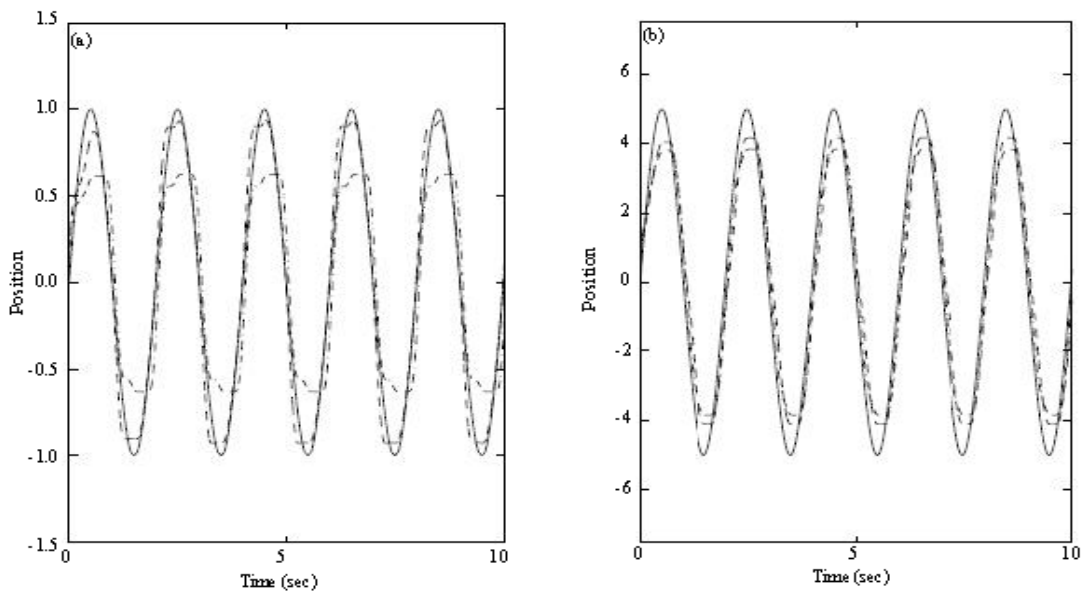


Fig. 5: Performance of the closed-loop system with (- - -) and without (-.-) friction compensation. The solid line is the desired trajectory. Figure a shows the first joint position θ and Fig. b shows the second joint position θ_2 with $k_1 = 10, \mu_1 = 1, k_2 = 100$ and $\mu_2 = 1$

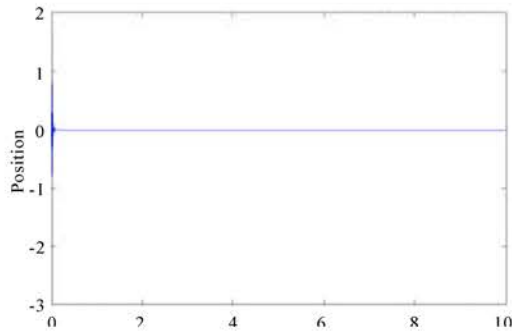


Fig. 6: Performance of the new proposed controller for position tracking error where,

$$k_c(t) = \begin{cases} 50 + 5\sin \pi t & 0 \leq t \leq 4 \\ 50 + 25\sin \pi t & 4 < t \leq 10 \end{cases}$$

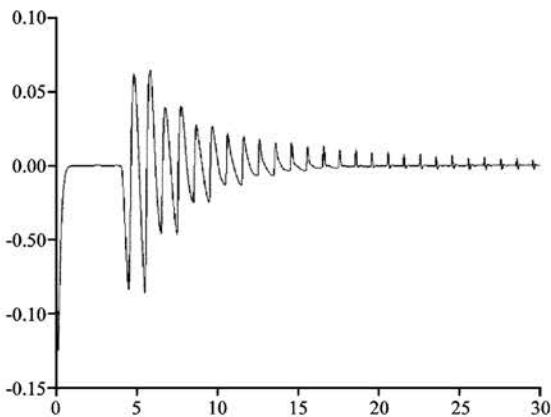


Fig. 7: Performance of the proposed method by Ahn and Chen (2004) for position tracking error, where,

$$k_c(t) = \begin{cases} 50 + 5\sin \pi t & 0 \leq t \leq 4 \\ 50 + 25\sin \pi t & 4 < t \leq 10 \end{cases}$$

Clearly, if $\hat{k}_{c1} \rightarrow k_{c1}$ and $\hat{k}_{c2} \rightarrow k_{c2}$, then it follows that $e_1 \rightarrow 0$ and $e_2 \rightarrow 0$ as $t \rightarrow \infty$. Simulation results are performed using the following parameters: $m_1 = 2, m_2 = 1, l_1 = 2, l_2 = 1, k_{v1} = k_{v2} = 20, k_{p2} = 200, k_{c1} = 500$ and $k_{c2} = 100$.

The estimator is constructed by taking $g(|x_2|) = -ke^{-\mu|x_2|}$. Figures 5a, b show the tracking performance of the system with and without friction compensation. Despite the large magnitude of the friction in the first link, the adaptive estimator successfully compensates for the friction.

A single-mass system with the new proposed controller for time varying case: In this section the new proposed controller for time varying case is applied to a simple single-mass system. The simulation results of the proposed method as compared to the methods by Ahn and Chen (2004, 2005) are presented in Fig. 6-9 for two

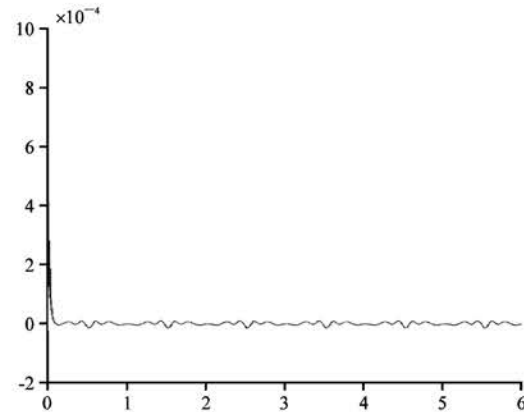


Fig. 8: Performance of the new proposed controller for position tracking error where, $k_c(t) = [50 + 5\sin(2\pi X(t)) + 2\sin(4\pi X(t)) + 2\sin(4\pi X(t)) + \sin(6\pi X(t))] \text{sgn}(v)$

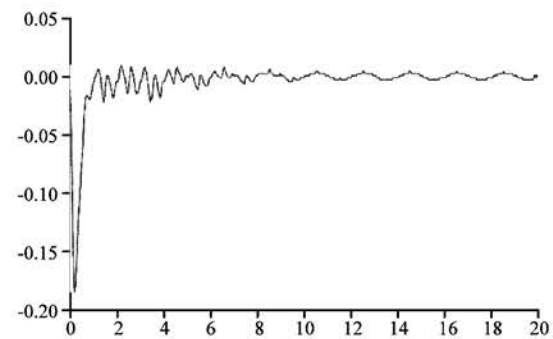


Fig. 9: Performance of the proposed method by Ahn and Chen (2005) for position tracking error where, $k_c(t) = [50 + 5\sin(2\pi X(t)) + 2\sin(4\pi X(t)) + 2\sin(4\pi X(t)) + \sin(6\pi X(t))] \text{sgn}(v)$

different cases, namely, $k_c(t)$ as a periodic signal and $k_c(t)$ as a state depended parameter. Desired trajectory of the position is $d_x(t) = \sin \pi t$ and α, λ are selected as $\alpha = 10$ and $\lambda = 10$.

In the first case, the coulomb friction coefficient $k_c(t)$ is considered as a periodic signal. The position tracking error signal achieved by the proposed new method is shown in Fig. 6 and the result of the proposed method by Ahn and Chen (2004) is given in Fig. 7.

In the second case, the coulomb friction coefficient $k_c(t)$ is considered as a state depended parameter. The position tracking error signal achieved by the new proposed controller is shown in Fig. 8 and the result of the proposed method by Ahn and Chen (2005) is given in Fig. 9. As can be seen from the Fig. 9, the proposed approach has more precision and higher rate of convergence.

CONCLUSION

The new nonlinear adaptive scheme based on Lyapunov analysis developed in the work by Yazdizadeh and Khorasani (1996a, b) to estimate and to compensate for friction are more elaborated in this paper. The method is applied to a simple mass system and a two-link rigid robot manipulator. The proposed strategy adaptively compensates for unknown static Coulomb friction nonlinearities. The method is general and systematic in construction. It was shown analytically that the estimation error dynamics is asymptotically stable without requiring a constraint on the velocity. Simulation results confirm the robustness and the advantages of the proposed scheme compared to the other similar works in the literature. The shortage and drawback of the proposed method is in its assumption on k_v . The proposed original method, similar to many other references, assumes the friction coefficient as a constant. To remove this assumption another new controller is proposed in this study. Simulation results of the new proposed method confirm the advantages of the proposed scheme compared to the other main similar works in the literature.

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