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An Application of Fuzzy Numbers Ranking in Performance Analysis

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Abstract: Data Envelopment Analysis (DEA) is a mathematical optimization technique that measures the relative efficiency of Decision Making Units (DMUs) with multiple input-output. In traditional DEA models, the data of different DMUs are assumed to be measured by precise values. But, in many real applications there are some imprecise data which represented by fuzzy numbers. In this study, an application of ranking fuzzy numbers is introduced and CCR model with fuzzy inputs and outputs in DEA is extended to propose an innovative version of fuzzy DEA (FDEA). In fact, we transform a fuzzy DEA model to a conventional crisp model by applying ranking fuzzy numbers method. Three numerical examples including an application to bank branches assessment at capital city of Iran are finally applied using the proposed fuzzy CCR model to show its applications and the differences from the other fuzzy DEA models.

Key words: Data envelopment analysis, fuzzy data, fuzzy mathematical programming, fuzzy numbers ranking

INTRODUCTION

Data Envelopment Analysis (DEA) is a data-oriented approach for relatively evaluating the performance of a set of homogenous entities referred as Decision Making Units (DMUs) by using a ratio of the weighted sum of outputs to the weighted sum of inputs. DEA models first proposed by Charnes *et al.* (1978) are formulated to measure the efficiency of DMUs for crisp data. Although traditional DEA models such as CCR (Charnes-Cooper-Rhodes) and BBC (Banker-Charnes-Cooper) are a powerful tool for efficiency measurement, but are only developed for crisp data (Charnes *et al*., 1978; Banker *et al*., 1984; Banker and Morey, 1986; Zhou *et al*., 2007). Whereas, in many real situations, inputs and outputs in DEA models are imprecise and inaccurate. Therefore, how to evaluate the management or operation efficiency of a set of DMUs in fuzzy environments is a worth-studying problem. Fuzzy set theory has been proposed as a way to challenge imprecise data in DEA models (Bellman and Zadeh, 1970).

Fuzzy DEA model is a fuzzy linear programming that provides a technique to deal with the uncertainty in fuzzy objectives and/or fuzzy constraints (Buckley, 1988; Rommelfanger *et al*., 1989; Julien, 1994). We can meet various fuzzy DEA models to evaluate the efficiency of DMUs in the DEA literature (Guo and Tanaka, 2001; Jahanshahloo *et al*., 2004; León *et al*., 2003; Saati *et al*., 2002; Guo, 2008; Kao and Liu, 2000, 2003; Soleimani-Damaneh *et al*., 2006). Most of the earlier studies in the fuzzy DEA included the computational complexity, hence, the decision maker cannot be clearly realized the procedure of assessing DMUs. Therefore, we try to propose a simple approach for imprecise environment to obtain a method that is usable and practicable for real-world problems such as performance of bank branches.

The main subject in using fuzzy theory in DEA is the notion of ranking fuzzy numbers. Several different approaches have been proposed for ranking fuzzy numbers. These include methods based on the Coefficient of Variation (CV index), distance between fuzzy sets, centroid point and original point and weighted mean value. In this study, the main interest is in the approach proposed by Asady and Zendehnam (2007) which uses a modification of a distance method called sign distance. Asady and Zendehnam's (2007) method constructed a ranking for fuzzy numbers is very realistic and its method can be used for ordering all types of the fuzzy numbers. Their method led to the crisp point, which is the best related to a certain measure of distance between the fuzzy number and a crisp point of support function. Some advantages of their method are the following:

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- In that approach used the nearest point of support function for ranking fuzzy numbers and the calculation is very simple. It is independent of family of the fuzzy numbers, horizon, upper and/or lower horizons
- A distance is applied in space fuzzy number
- The distance minimization is used for ordering of fuzzy numbers

In this study, by using ranking fuzzy numbers, we present a new defuzzification approach to solve fuzzy CCR model. For the case of fuzzy data in the form of trapezoidal fuzzy numbers, DEA models are linear programming and can be solved by usual methods. For demonstrating the applicability of the proposed method, three numerical examples including an application to bank branches at Tehran city in Iran are represented. The banking industry has grown into profitable competitive. Improving the efficiency of bank branches is now considered to be important to the banking industry and this case study aims to help clarify this subject by applying the proposed method.

PRELIMINARY DEFINITIONS OF FUZZY DATA

Some basic definitions of fuzzy sets are reviewed by Kauffman and Gupta (1991), Zimmermann (2005) and Dubois and Prade (1978).

Definition 1: Let U be a universe set. A fuzzy set $\tilde{A}$ of U is defined by a membership function $\mu_{\tilde{A}}(x) \to [0,1]$, where, $\mu_{\tilde{A}}(x), \forall x \in U$, indicates the degree of membership of $\tilde{A}$ to U.

Definition 2: A real fuzzy number $\tilde{A}$ denoted by $\tilde{A}$ = (a, b, c, d; w) is described as any fuzzy subset of the real line R with membership function $\mu_{\tilde{A}}$ which satisfies the following properties:

- $\mu_{\tilde{A}}$ is a semi continuous mapping from R to the closed interval [0, w], $0 \le w \le 1$
- $\mu_{\tilde{A}}$ (x) = 0, for all x∈[-∞, a]
- $\mu_{\tilde{A}}$ is increasing on [a, b]
- $\mu_{\tilde{A}}$ (x) = w for all x∈[b, c], where w is a constant and $0 < w \le 1$
- $\mu_{\tilde{A}}$ is decreasing on [c, d]
- $\mu_{\tilde{A}}$ (x) = 0, for all x∈[d, ∞]

where, a, b, c and d are real numbers.

Unless elsewhere specified, it is assumed that $\tilde{A}$ is convex and bounded; i.e., -∞<a, d<∞. If w = 1, $\tilde{A}$ is a normal fuzzy number and if 0<w<1, $\tilde{A}$ is a non normal fuzzy number.

The membership function $\mu_{\tilde{A}}$ of $\tilde{A}$ can be expressed as:

$$\mu_{\tilde{A}}(x) = \begin{cases} f^L(x), & a \le x \le b, \\ w, & b \le x \le c, \\ f^R(x), & c \le x \le d, \\ 0, & \text{O.W.} \end{cases}$$

where, f^L: [a, b]→[0, w] and f^R: [c, d]→[0, w].

In addition, a fuzzy number $\tilde{A}$ in parametric form is denoted by $(\underline{a}, \overline{a})$ of function $\underline{a}(r), \overline{a}(r), 0 \le r \le 1$, which satisfies the following requirements:

- $\underline{a}(r)$ is a bounded increasing left continuous function
- $\overline{a}(r)$ is a bounded decreasing right continuous function
- $\underline{a}(r) \le \overline{a}(r)$, where, $0 \le r \le 1$

Definition 3: A trapezoidal fuzzy number is widely best used for solving practical problems. Trapezoidal fuzzy number is determined by quadruples $\tilde{u} = (\underline{m}, \overline{m}, \sigma, \beta)$ of crisp numbers with two defuzzifier $\underline{m}, \overline{m}$ and left fuzziness σ>0 and right fuzziness β>0, which membership function can be defined as follows:

$$\mu_{\tilde{u}}(x) = \begin{cases} \frac{1}{\sigma}(x - \underline{m} + \sigma) & \underline{m} - \sigma \le x \le \underline{m}, \\ 1 & \underline{m} \le x \le \overline{m}, \\ \frac{1}{\beta}(\overline{m} - x + \beta) & \overline{m} \le x \le \overline{m} + \beta, \\ 0 & \text{O.W.} \end{cases} \quad (1)$$

Trapezoidal fuzzy number in its parametric form can be obtained as follows:

$$\underline{u}(r) = \underline{m} - \sigma + \sigma r \\ \overline{u}(r) = \overline{m} + \beta - \beta r \quad (2)$$

Note that the trapezoidal fuzzy number is a triangular fuzzy number if $\underline{m} = \overline{m} = m$, denoted by a triple (m, σ, β).

Definition 4: In fuzzy linear programming, the min T-norm is usually applied to assess a linear combination of fuzzy quantities. Therefore, for a given set of trapezoidal fuzzy numbers $\tilde{a}_j = (\overline{m}_j, \underline{m}_j, \sigma_j, \beta_j)$, j = 1, 2, ..., n, a linear combination is defined as follows:

$$\sum_{j=1}^{n} \lambda_j \tilde{a}_j = \left(\sum_{j=1}^{n} \lambda_j \underline{m}_j, \sum_{j=1}^{n} \lambda_j \overline{m}_j, \sum_{j=1}^{n} \lambda_j \sigma_j, \sum_{j=1}^{n} \lambda_j \beta_j \right), \ \lambda_j \ge 0 \quad (3)$$

where, $\sum_{j=1}^{n}\lambda_j\tilde{a}_j$ denotes the combination $\lambda_1\tilde{a}_1 \oplus \lambda_2\tilde{a}_2 \oplus \ldots \oplus \lambda_n\tilde{a}_n$.

Ranking fuzzy numbers is important in decision-making, data analysis, artificial intelligence and fuzzy linear programming. Many researchers have investigated various ranking methods (Asady and Zendehnam, 2007; Wang *et al*., 2006; Huijun and Jianjun, 2006; Cheng, 1998; Wang and Kerre, 2001a, b). Recently, Asady and Zendehnam (2007) propose a method for minimizing distance to order the fuzzy numbers. Interval EI ($\tilde{A}$) of a fuzzy number $\tilde{A}$ is introduced independently by Dubois and Prade (1987) and Heilpern (1992) as follows:

$$EI(\tilde{A}) = \left[\int_0^1 \underline{a}(r)dr, \int_0^1 \overline{a}(r)dr\right] \tag{4}$$

The middle point of interval EI ($\tilde{A}$) can be defined as follows (Asady and Zendehnam, 2007):

$$M(\tilde{A}) = \frac{1}{2}\int_0^1 (\underline{a}(r) + \overline{a}(r))dr \tag{5}$$

M ($\tilde{A}$) is used to rank fuzzy numbers. Therefore, for any two fuzzy numbers $\tilde{A}_i$ and $\tilde{A}_j$, if $M(\tilde{A}_i) > M(\tilde{A}_j)$, then $\tilde{A}_i \succ \tilde{A}_j$. If $M(\tilde{A}_i) < M(\tilde{A}_j)$, then $\tilde{A}_i \prec \tilde{A}_j$. Finally, if $M(\tilde{A}_i) = M(\tilde{A}_j)$, then $\tilde{A}_i \cong \tilde{A}_j$.

In particular, if $\tilde{u} = (\underline{m}, \overline{m}, \sigma, \beta)$ be a trapezoidal fuzzy number, then:

$$M(\tilde{u}) = \frac{1}{2}(\underline{m} + \overline{m}) + \frac{\beta - \sigma}{4} \tag{6}$$

The properties of this ranking method have been expressed by Asady and Zendehnam (2007).

THE CCR MODEL WITH FUZZY COEFFICIENTS

The most frequently used DEA model is the CCR model, called after Charnes *et al*. (1978). Suppose that there are n DMUs, each consumes the same type of inputs and produces the same type of outputs. Let m be the number of inputs and r is the number of outputs.

The CCR model is formulated as the following linear program:

$$\begin{aligned} \text{Min} \quad & Z_p = \theta \\ \text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j x_{ij} - \theta x_{ip} \le 0 \quad \forall i, \\ & \sum_{j=1}^{n}\lambda_j y_{rj} - y_{rp} \ge 0 \quad \forall r, \\ & \lambda_j \ge 0 \quad \forall j. \end{aligned} \tag{7}$$

where, y_{rp} (r = 1, ..., s) and x_{ip} (i = 1, ..., m) are the output and input values of DMU_p, p = 1, ..., n, the DMU under consideration, respectively, u_r (r = 1, ..., s) and v_i (i = 1,..., m) the weights associated with rth output and ith input, respectively. All inputs and outputs are assumed to be nonnegative, but at least one input and one output are positive.

In recent years, fuzzy set theory has been proposed as a way to quantify imprecise and vague data in DEA models. The CCR model with fuzzy coefficients is as follows:

$$\begin{aligned} \min \quad & Z_p = \theta \\ \text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j\tilde{x}_{ij} - \theta\tilde{x}_{ip} \le 0 \quad \forall i, \\ & \sum_{j=1}^{n}\lambda_j\tilde{y}_{rj} - \tilde{y}_{rp} \ge 0 \quad \forall r, \\ & \lambda_j \ge 0 \quad \forall j. \end{aligned} \tag{8}$$

Many researchers are used triangular and trapezoidal fuzzy numbers in their studies (Guo and Tanaka, 2001; Jahanshahloo *et al*., 2004; León *et al*., 2003; Saati *et al*., 2002). In this study also, inputs and outputs are assumed to be fuzzy numbers with trapezoidal membership function.

When input-output data are fuzzy numbers, Eq. 8 can be expressed as the following fuzzy LP problem:

$$\begin{aligned} \text{Min} \quad & Z_p = \theta \\ \text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j(\underline{x}_{ij}, \overline{x}_{ij}, \sigma_{ij}, \beta_{ij}) - \theta(\underline{x}_{ip}, \overline{x}_{ip}, \sigma_{ip}, \beta_{ip}) \le 0 \quad \forall i, \\ & \sum_{j=1}^{n}\lambda_j(\underline{y}_{rj}, \overline{y}_{rj}, \xi_{rj}, \tau_{rj}) - (\underline{y}_{rp}, \overline{y}_{rp}, \xi_{rp}, \tau_{rp}) \ge 0 \quad \forall r, \\ & \lambda_j \ge 0 \quad \forall j. \end{aligned} \tag{9}$$

Equation 9 is a fuzzy linear programming. There are several fuzzy approaches to solve it in the literature which mentioned in introduction part. In this study the proposed ranking fuzzy numbers method by Asady and Zendehnam (2007) would be used to solve this fuzzy linear programming problem. Therefore, based on definition 4, Eq. 9 can be transformed as follows:

$$\begin{aligned} \min \quad & Z_p = \theta \\ \text{s.t.} \quad & 2\sum_{j=1}^{n}\lambda_j(\underline{x}_{ij} + \overline{x}_{ij}) - 2\theta(\underline{x}_{ip} + \overline{x}_{ip}) + \sum_{j=1}^{n}(\sigma_{ij} + \beta_{ij}) - \theta(\sigma_{ip} + \beta_{ip}) \le 0 \quad \forall i, \\ & 2\sum_{j=1}^{n}\lambda_j(\underline{y}_{rj} + \overline{y}_{rj}) - 2(\underline{y}_{rp} + \overline{y}_{rp}) + \sum_{j=1}^{n}(\xi_{rj} + \tau_{rj}) - (\xi_{rp} + \tau_{rp}) \ge 0 \quad \forall r, \\ & \lambda_j \ge 0 \quad \forall j \end{aligned} \tag{10}$$

It is obvious that Eq. 10 is a linear programming model. Therefore, the conventional LP method can be applied to solve it.

It should be noted that by crisp data, Eq. 10 will be converted to standard CCR model.

To avoid complex operation in ranking of DMUs, the linear scale transformation is used to transform the different efficiency values into comparable scale. The normalized efficiencies can be represented as:

$$W_p = \frac{Z_p}{\max_j\{Z_j\}} \qquad (11)$$

Therefore, DMU_p, p = 1, ..., n is an efficient DMU if $W_p = 1$, otherwise it is an inefficient one. It is evident that there exists at least one efficient DMU.

NUMERICAL EXAMPLES

Here, we examine three numerical examples using the proposed fuzzy DEA model. The first two examples are taken from Soleimani-Damaneh *et al.* (2006) and León *et al.* (2003) for the purpose of comparison and the last example is an application of the proposed model to 16 bank's branches at Capital city of Iran.

Example 1: Consider the example provided by Soleimani-Damaneh *et al.* (2006) for 6 DMUs with single fuzzy input and single fuzzy output. First two rows of Table 1 provide the data of this example. The efficiency scores obtained by Soleimani-Damaneh *et al.* (2006) and by the proposed approach in this paper are recorded in the last two columns of Table 1.

It can be seen that A and F are became efficient units in Soleimani-Damaneh *et al.* (2006) method. Indeed when using the proposed approach, A is inefficient and fell in second rank. Therefore, the proposed approach can be used for assessing and ranking of efficient DMU in Soleimani-Damaneh *et al.* (2006) approach.

Example 2: Consider the example investigated by León *et al.* (2003), in which eight DMUs are evaluated against single fuzzy input and single fuzzy output. The data are presented in the columns 1 and 2 of Table 2, respectively. According to (2003), A, B, C and G are efficient, while the proposed approach only expressed C as efficient and other DMUs can finally be ranked as A, B, H, D, D, E, G and F.

Example 3: Consider the collected data of sixteen branches at Tehran city in Iran (2004) which each branch consumes three crisp and two fuzzy inputs to produce two crisp outputs. The data for each DMU is depicted in Fig. 1.

There are used linguistic variables shown in Fig. 2 to determine fuzzy inputs. Data of sixteen bank branches are shown in Table 3 and 4.

Table 1: Data and obtained efficiency scores by Soleimani-Damaneh *et al.* (2006) and proposed approach in example 1

DMUs	Input	Output	Efficiency values by Soleimani-Damaneh *et al.* (for h = 1)	Efficiency values by proposed approach
A	(2, 4, 3, 5)	(6, 8, 1, 4)	1.00	0.70
B	(3, 7, 1, 2)	(5, 9, 2, 5)	0.67	0.61
C	(3, 6, 2, 3)	(4, 9, 3, 6)	0.71	0.61
D	(2, 4, 2, 5)	(4, 8, 2, 4)	0.65	0.61
E	(4, 7, 3, 6)	(3, 7, 1, 3)	0.45	0.45
F	(1, 2, 3, 5)	(1, 3, 1, 4)	1.00	1.00

Table 2: Data and obtained efficiency scores by León *et al.* (2003) and proposed approach in example 2

DMUs	Input	Output	Efficiency values by León *et al.* (2003) (for h = 1)	Efficiency values by proposed approach
A	(3, 3, 2, 2)	(3, 3, 1, 1)	1.00	0.87
B	(4, 4, 0.5, 0.5)	(2.5, 2.5, 1, 1)	1.00	0.82
C	(4.5, 4.5, 1.5, 1.5)	(6, 6, 1, 1)	1.00	1.00
D	(6.5, 6.5, 0.5, 0.5)	(4, 4, 1.25, 1.25)	0.75	0.61
E	(7, 7, 2, 2)	(5, 5, 0.5, 0.5)	0.64	0.56
F	(8, 8, 0.5, 0.5)	(3.5, 3.5, 0.5, 0.5)	0.61	0.43
G	(10, 10, 1, 1)	(6, 6, 0.5, 0.5)	1.00	0.49
H	(6, 6, 0.5, 0.5)	(2, 2, 1.5, 1.5)	0.56	0.69

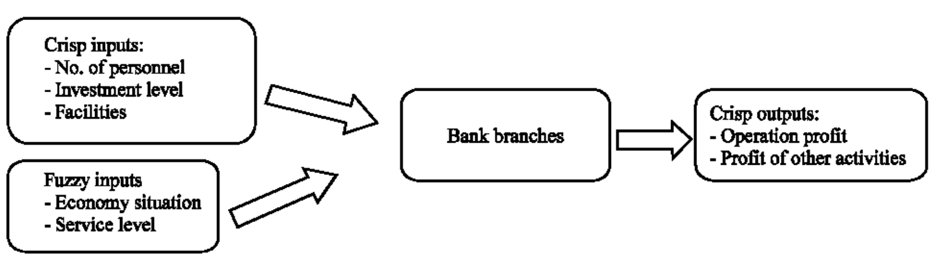

Fig. 1: Conceptual model

Table 3: Data of crisp inputs and outputs in example 3

	Inputs			Outputs	
Branches	No. of personnel	Investment level	Facilities	Operation profit	Profit of other activities
1	16	4.41	51.35	90.37	23.22
2	13	31.36	46.30	97.60	60.46
3	11	152.70	24.53	54.34	40.92
4	13	83.02	56.25	83.02	50.59
5	15	74.38	75.94	104.60	44.46
6	12	26.10	25.11	36.63	7.48
7	3	0.90	0.74	2.44	5.71
8	11	2.21	33.22	49.00	42.91
9	11	51.95	7.77	70.57	4.53
10	48	280.39	279.18	26.54	993.07
11	23	401.41	179.30	630.48	1500.69
12	41	241.00	233.76	384.94	1193.82
13	30	202.60	206.64	278.08	53.28
14	25	335.67	322.73	473.77	567.00
15	22	251.95	238.35	135.45	3.00
16	26	237.91	191.18	359.09	567.27

Table 4: Data of fuzzy inputs in example 3

Bank branches	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Service level	H	M	VH	H	MH	ML	M	VL	M	ML	H	M	L	VH	M	MH
Economy situation	L	M	M	H	VH	MH	L	M	H	MH	L	ML	M	M	H	VH

Table 5: Obtained results by proposed model

Bank branches	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Efficiency	0.217	0.15	0.316	0.01	0.11	0.189	0.305	1	0.178	0.272	0.385	0.279	0.274	0.168	0.179	0.14
Rank	8.000	13.00	3.000	16.00	15.00	9.000	4.000	1	11.000	7.000	2.000	5.000	6.000	12.000	10.000	14.00

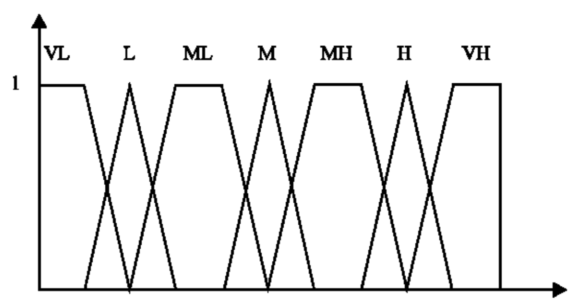

Fig. 2: Linguistic variable

The results of the proposed method for ranking the branches are presented in Table 5. It can be shown that branch 8 is ranked at the first place by suggested method.

CONCLUSIONS

Since, Data Envelopment Analysis (DEA) was proposed in 1978, it has been got comprehensive attention both in theory and application. Most of the studies assume that all inputs and outputs are precisely known. But in more general cases, uncertainty is an attribute of data. Hence, it may be more suitable to interpret the experts' understanding of the data as fuzzy numerical data which can be represented by means of fuzzy numbers. In this study, an application of ranking fuzzy numbers is introduced and CCR model with fuzzy inputs and outputs is extended to propose a new fuzzy DEA (FDEA) model for evaluating the efficiencies of DMUs. This approach transforms the fuzzy model into crisp LP model by ranking fuzzy numbers. Also, the obtained efficiencies from the proposed model reflect the inherent fuzziness in assessment problems. We compare the results of proposed model with Soleimani-Damaneh *et al.* (2006) results and León *et al.* (2003) by representing a numerical example introduced by them. Furthermore, a case study for ranking of bank's branches in Iran is discussed. This application demonstrates that fuzzy DEA models are quite powerful in evaluating real problems under uncertainty. Because uncertainty always exists in human thinking and judgment, fuzzy DEA models can play an important role in the real-world problems.

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