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A Cash Flow Oriented EOQ Model with Deteriorating Items Under Permissible Delay in Payments

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Abstract: This study develops an inventory model to determine an optimal ordering policy for deteriorating items with delayed payments permitted by the supplier under inflation and time discounting. This study applies the discounted cash flows approach for problem analysis. Mathematical models have been derived for obtaining the optimal cycle time and optimal payment time for item so that the annual total relevant cost is minimized. The present value of the annual total relevant cost in this inventory system is developed first, then an optimal number of replenishment, cycle time and order quantity are obtained by a solution procedure. Finally, a numerical example is given to illustrate the results.

Key words: Inventory, deterioration, delay in payments, cash flow, inflation

INTRODUCTION

The traditional EOQ model assumes that retailer must pay for the items as soon as the items are received. However, in real-life situations, the supplier may offer the retailer a delay period, that is trade credit period, in paying for the amount of purchasing cost. The retailer can sell the goods to accumulate revenue and earn interest before the end of trade credit period. But if the payment is delayed beyond that period, a higher interest will be charged. Such a convenience is likely to motivate customer to order more quantities because paying later indirectly reduces the purchase cost. Therefore, trade credit is an important source of financing for intermediate purchasers of goods and services and plays a large role in our economy. Recently, Hou and Lin (2008) considered an ordering policy with a cost minimization procedure for deterioration items under trade credit and time discounting. Other many related articles can be found by Goyal (1985), Aggarwal and Jaggi (1995), Jamal *et al.* (1997), Sarker *et al.* (2000), Chung *et al.* (2005), Chung and Liao (2006), Ouyang *et al.* (2006) and Huang (2007). On the other hand, many researchers have studied inventory models for deteriorating items such as volatile liquids, blood banks, medicines, electronic components and fashion goods. The analysis of deteriorating inventory problems began with Ghare and Schrader (1963), who developed a simple Economic Order Quantity (EOQ) model with a constant rate of deterioration. Since then, many related articles could be found by Heng *et al.* (1991), Sarker *et al.* (1997) and Balkhi and Benkherouf (2004). While determining the optimal ordering policy, the effects of inflation and time value of money cannot be ignored. The pioneer research in this direction was Buzacott (1975), who developed an EOQ model with inflation subject to different types of pricing policies. Other related articles can be found in Misra (1977) and Ray and Chaudhuri (1997), Liao *et al.* (2000) and Chung and Lin (2001).

This study develops a deterministic inventory model when a delay in payments is permissible. The effects of the inflation, deterioration and delay in payments are discussed. Moreover, an algorithm is developed to obtain the optimal the number of replenishment, cycle time and order quantity.

NOTATION AND MODEL

The following notation is used throughout the study:

- H : Length of planning horizon
- T : Replenishment cycle time
- n : No. of replenishment during the planning horizon; n = H/T
- Q : Order quantity, units/cycle
- D : Demand rate per unit time, units/unit time
- A : Ordering cost at time zero, \$/order

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c : Per unit cost of the item, \$/unit
h : Inventory holding cost per unit per unit time excluding interest charges, \$/unit/unit time
θ : Deterioration rate, a constant fraction of the on-hand inventory, units/unit time
r : Discount rate represent the time value of money
f : Inflation rate
R : The net discount rate of inflation
R : r-f
I_e : The interest earned per dollar per unit time
I_c : The interest charged per dollar in stocks per unit time by the supplier, $I_c \geq I_e$
M : The permissible delay in settling account (i.e., the trade credit period)

Let I(t) be the inventory level during the first replenishment cycle. This inventory level is depleted by the effects of demand and deterioration. So, the variation of I(t) with respect to t is governed by the following differential equation:

$$\frac{dI(t)}{dt} = -D - \theta I(t), \qquad 0 \leq t \leq T \tag{1}$$

with the boundary condition I(T) = 0. The solution of (1) can be represented by:

$$I(t) = \frac{D}{\theta}\left[e^{\theta(H/n-t)} - 1\right], \quad 0 \leq t \leq T = H/n \tag{2}$$

Consequently, initial inventory after replenishment becomes:

$$I(0) = Q = \frac{D}{\theta}\left[e^{\theta H/n} - 1\right] \tag{3}$$

The present value of the total replenishment costs is given by:

$$\begin{aligned} C_R &= A\sum_{j=0}^{n-1} e^{-jRT} \\ &= A\frac{1-e^{-RH}}{1-e^{-RH/n}} \end{aligned} \tag{4}$$

and the present value of total purchasing costs is given by:

$$\begin{aligned} C_P &= c\sum_{j=0}^{n-1} I(0)e^{-jRT} \\ &= \frac{cD}{\theta}(e^{\theta H/n} - 1)\frac{1-e^{-RH}}{1-e^{-RH/n}} \end{aligned} \tag{5}$$

The present value of the total holding costs over the time horizon H is given by:

$$C_h = \frac{hD}{\theta}\left\{\frac{(e^{\theta H/n} - e^{-RH/n})}{\theta + R} + \frac{(e^{-RH/n} - 1)}{R}\right\}\frac{1-e^{-RH}}{1-e^{-RH/n}} \tag{6}$$

Case I: $M \leq T = H/n$

In this case, the present value of the interest payable during the first replenishment cycle is:

$$I_{p1}^{1} = cI_c\int_M^T I(t)e^{-Rt}dt = \frac{cI_cD}{\theta(\theta+R)}\left[e^{\theta(H/n-M)-RM} - e^{-RH/n}\right] + \frac{cI_cD}{\theta R}(e^{-RH/n} - e^{-RM}) \tag{7}$$

hence, the present value of the total interest payable over the time horizon H is:

$$\begin{aligned} I_{p1}^{H} &= \sum_{j=0}^{n-1} I_{p1}^{1} e^{-jRT} \\ &= \left\{\frac{cI_cD}{\theta(\theta+R)}\left[e^{\theta(H/n-M)-RM} - e^{-RH/n}\right]\right. \\ &\quad \left. + \frac{cI_cD}{\theta R}(e^{-RH/n} - e^{-RM})\right\}\frac{1-e^{-RH}}{1-e^{-RH/n}} \end{aligned} \tag{8}$$

Next, the present value of the interest earned during the first replenishment cycle is:

$$I_{e1}^{1} = cI_e\int_0^T Dte^{-Rt}dt = \frac{cI_eD}{R}\left[\frac{1}{R}(1-e^{-RH/n}) - Te^{-RH/n}\right] \tag{9}$$

hence, the present value of the total interest earned over the time horizon H is:

$$I_{e1}^{H} = \sum_{j=0}^{n-1} I_{e1}^{1} e^{-jRT} = \left\{\frac{cI_eD}{R}\left[\frac{1}{R}(1-e^{-RH/n}) - \frac{H}{n}e^{-RH/n}\right]\right\}\frac{1-e^{-RH}}{1-e^{-RH/n}} \tag{10}$$

Therefore, the total present value of the costs over the time horizon H is:

$$TVC_1(s,n) = C_R + C_P + C_h + I_{p1}^{H} - I_{e1}^{H} \tag{11}$$

Case II: $M > T = H/n$

The interest earned in the first cycle is the interest earned during the time period (0, T) plus the interest earned from the cash invested during the time period (T, M) after the inventory is exhausted at time T and it is given by:

$$\begin{aligned} I_{e2}^{1} &= cI_e\left[\int_0^T Dte^{-Rt}dt + (M-T)e^{-RT}\int_0^T Ddt\right] \\ &= \frac{cI_eD}{R^2}(1-e^{-RH/n}) - \frac{cI_eDH}{Rn}e^{-RH/n} \\ &\quad + \left[MH/n - (H/n)^2\right] cI_eDe^{-RH/n} \end{aligned} \tag{12}$$

hence, the present value of the total interest earned over the time horizon H is:

$$I_{e2}^{H} = \sum_{j=0}^{n-1} I_{e2}^{1} e^{-jRT}$$
$$= \left\{ \frac{cI_eD}{R^2}(1 - e^{-RH/n}) - \frac{cI_eDH}{Rn} e^{-RH/n} + [MH/D - (H/n)^2] cI_e De^{-RH/n} \right\} \frac{1 - e^{-RH}}{1 - e^{-RH/n}} \quad (13)$$

Since, the replenishment cost, purchasing cost and inventory holding cost over the time horizon H are the same as Case I, the total present value of the costs, $TVC_2(s, n)$, is given by:

$$TVC_2(n) = C_R + C_P + C_h - I_{e2}^{H} \quad (14)$$

At M = T = H/n, we find $TVC_1(n) = TVC_2(n)$. Consequently, we have:

$$TVC(n) = \begin{cases} TVC_1(n) & \text{if } T = H/n \geq M \\ TVC_2(n) & \text{if } T = H/n \leq M \end{cases}$$

where, $TVC_1(n)$ and $TVC_2(n)$ as expressed in Eq. 11 and 14, respectively.

ALGORITHM

The following algorithm is developed to derive the optimal n, T, Q and TVC(n) values:

Step 1: Start by choosing a discrete variable n, where n is any integer number equal or greater than 1.

Step 2: If $T = H/n \geq M$ for different integer n values, derive $TVC_1(n)$ from (11); If $T = H/n \leq M$ for different integer n values, derive $TVC_2(n)$ from Eq. 14.

Step 3: Repeat Step 1 and 2 for all possible n values with $T = H/n \geq M$ until the minimum $TVC_1(n)$ is found from Eq. 11 and let $n_1^* = n$. For all possible n values with $T = H/n \geq M$ until the minimum $TVC_2(n)$ is found from Eq. 14 and let $n_2^* = n$. The n_1^*, n_2^*, $TVC_1(n^*)$ and $TVC_2(n^*)$ values constitute the optimal solution and the satisfy the following conditions:

$$\Delta TVC_1(n_1^* - 1) < 0 < \Delta TVC_1(n_1^*) \quad (15)$$

$$\Delta TVC_2(n_2^* - 1) < 0 < \Delta TVC_2(n_2^*) \quad (16)$$

Where:

$$\Delta TVC_1(n_1^*) = TVC_1(n_1^* + 1) - TVC_1(n_1^*)$$

and

$$\Delta TVC_2(n_2^*) = TVC_2(n_2^* + 1) - TVC_2(n_2^*)$$

Step 4: Select the optimal number of replenishment n* such that:

$$TVC(n^*) = \min \begin{cases} TVC_1(n_1^*) & \text{if } H/n_1^* \geq M \\ TVC_2(n_2^*) & \text{if } H/n_2^* \leq M \end{cases}$$

Hence, optimal order quantity Q^* is obtained by substituting n* into (3) and optimal cycle time, T*, is T* = H/n*.

NUMERICAL RESULTS

An example is devised here to illustrate the results of the general model developed in this study with the following data.

The demand rate, D = 600 unit /year, the replenishment cost, A =\$80/order, the holding cost excluding interest charges, h = \$2.4/unit/year, the per unit item cost, c = \$15/unit, the constant rate of deterioration, $\theta = 0.15$, the net discount rate of inflation, R = \$0.12/\$/year, the interest charged per \$ in stocks per year by the supplier, I_c = \$0.18/\$/year, the interest earned per \$ per year, I_e = \$0.16/\$/year and the planning horizon, H, is 5 year. The permissible delay in settling account, M = 60 days = 60/360 years (assume 360 days per year). Using the solution procedure, we have the computational results shown in Table 1. We find the Case I is optimal option in credit policy. From the case, the minimum total present value of costs is found when the number of replenishment, n, is 23. With 23 replenishments, the optimal cycle time T is 0.217 year, the optimal order quantity, Q = 132.59 units and the optimal total present value of costs, TVC = \$36296.70.

Table 1: The numerical results

Case	Order No. (n)	Cycle time (T) (year)	Order quantity (Q) (units)	Total costs (TVC) (n)
I	22	0.227	138.72	36297.33
	23*	0.217*	132.59*	36296.70*
	24	0.208	126.97	36302.66
II	30	0.167	101.26	36439.13
	31	0.161	97.95	36444.90
	32	0.156	94.86	36454.07

*Optimal solution

CONCLUSION

This study develops a deterministic inventory model for deteriorating items over a finite planning horizon when the supplier provides a permissible delay in payments. The model considers the effects of deterioration, inflation and permissible delay in payments. Based on the DCF approach we permit a proper recognition of the financial implication of the opportunity cost in inventory analysis. In addition, we have presented an optimal solution procedure to find the optimal number of replenishment, cycle time and order quantity to minimize the total present value of costs. Finally, a numerical example is given to illustrate the results. Further research can be done for case with stochastic market demand when the supplier provides a permissible delay in payments and cash discount.

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