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Endurance Time Method-Application in Nonlinear Seismic Analysis of Single Degree of Freedom Systems

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Abstract: Endurance Time (ET) method has been introduced as a time-history based dynamic analysis procedure. In this method, structures are subjected to a gradually intensifying acceleration function. Performance of the structures is assessed based on the length of the time interval that they can satisfy required performance objectives. In this study, some fundamental concepts of ET method are explained and the potentials and limitations of this procedure in nonlinear seismic analysis of SDOF structures are investigated. A numerical optimization procedure for generating ET acceleration functions that are compatible with ground motions are explained. Results of ET analysis for inelastic SDOF systems are compared with ground motions analysis results for different strength ratios, ductilities and damping ratios. The accuracy of ET method in predicting the response of SDOF systems with stiffness degradation and strength deterioration is also investigated application of ET method in performance based earthquake engineering is described by an example of a single degree of freedom system. The results show that the approximations of ET method are in good agreement with the exact response history results of the similar ground motions for different nonlinear systems. It is shown that ET acceleration functions optimized in linear range considering long periods can be used in nonlinear analysis with reasonable accuracy.

Key words: Time-history analysis, inelastic deformation, endurance time method, nonlinear seismic analysis, performance based earthquake engineering, accuracy

INTRODUCTION

Advances in computational technology have stimulated the application of numerically intensive analysis and design procedures in the field of earthquake engineering. These advances have provided powerful tools for analysis and design of complex structures based on their realistic seismic performance. Major objective of Performance-Based Earthquake Engineering (PBEE) is to establish measures of acceptable performance and to develop design and evaluation procedures that take into account different performance objectives. At the core of PBEE lies the accurate estimation of the seismic demand and capacity of structures, a task that several methods have been proposed to tackle.

So far, the most popular analysis and performance evaluation method for practical applications seems to be simplified nonlinear procedure. Examples of simplified nonlinear methods are: Capacity Spectrum method, Coefficient method, the N2 method, the modal pushover analysis and some other procedures (ATC, 1996; Otani *et al.*, 2000; FEMA, 2000; Fajfar, 2000; Chopra and

Goel, 2002; SEAOC, 1999; Aschheim and Black, 2000; Panagiotakos and Fardis, 2001; Priestley and Calvi, 1997; Aydinoglu, 2003; Casarotti and Pinho, 2007). Different procedures often give different estimates for displacement demand of the same structure and ground motion. Furthermore, current procedures for addressing the degradation of stiffness and strength deterioration in structures are often ambiguous and unclear (FEMA, 2005). On the other hand, the predicted response of short-period structures with these procedures seems to be extreme when compared with observed performance. As most of these methods are based on a SDOF approximation, they may not reliably predict important response parameters for some MDOF structures (FEMA, 2005).

Nonlinear dynamic time history analysis offers the most realistic description of the response of a structure to earthquake excitation. However, the limitations of such a methodology are many. The constitutive relations that represent the physical properties of the structure elements for this method are very complex. Furthermore, the analysis procedure is computationally intensive and

requires experienced engineers. Finally, the validity of the available data used as input in the analysis is generally lower than the accuracy of the computed response. For these reasons parametric analyses are expensive, thus excluding the possibility of using nonlinear dynamic analysis for design purposes (Taucer and Negro, 2002).

Another promising method that has been incorporated in modern seismic codes is the Incremental Dynamic Analysis (IDA) (FEMA, 2000; Vamvatsikos and Cornell, 2002). This method can predict capacity and demand of structures in regions ranging from elasticity to global dynamic instability. This prediction is done by using a series of nonlinear dynamic analyses under suitably multiple-scaled ground motion records. This method needs large number of nonlinear dynamic analyses that makes it expensive. Also extraordinary variability from record to record of the IDA curves for a single building and legitimacy of scaling records are other drawbacks of this method. Still, professional practice favors simplified methods, mostly using SDOF systems that approximate MDOF system's behaviour. These methods could be developed to reach far into the nonlinear range and approximate the results of IDA (Vamvatsikos and Cornell, 2005).

Endurance Time (ET) method is a new nominee in the framework of PBEE. In this method, structures are subjected to intensifying acceleration functions (This term is used instead of accelerograms to prevent confusion with ground motions and simulated accelerograms that are usually compatible with ground motions). The performance of the structures is assessed based on the length of the time interval that they can satisfy required performance objectives. Endurance Time method is analogue of the exercise test applied by cardiologists in order to assess cardiovascular system condition of patients (Estekanchi *et al.*, 2004). In exercise test, the patient is asked to run on a treadmill while speed and slope of the treadmill increase. The test stops when the patient is exhausted or abnormal vital conditions are observed. Results of this relatively simple experiment are compared with certain standards. Finally the cardiologist can assess the overall fitness of the patient and possibly prescribe medication or surgery in severe cases. In ET method, a similar concept is applied to assess fitness of structures for enduring dynamic forces imposed in probable earthquakes. This relatively simple test procedure can be very useful in preliminary design stage of various structures where the performance parameters are hard to estimate with a reasonable accuracy. The procedure can also be very useful when comparing the relative performance of two different structures or design alternatives for the same structure. Endurance Time

method thus provides a handy method for the design modification by trial and error. Final verification can be carried out using more accurate and expensive procedures such as IDA.

Application of ET method in linear seismic analysis of structures has been studied before (Estekanchi *et al.*, 2007). In this research compliance and level of accuracy of this method in nonlinear seismic analysis of SDOF structures is investigated. Initially, the concept of ET method is explained and prospective procedures to implement it are discussed. A numerical optimization procedure for design of intensifying acceleration functions, compatible with ground motions, is developed. Results of ET analysis for inelastic SDOF systems are compared with ground motions analysis results for different strength ratios, ductilities and damping ratios. The accuracy of ET method in predicting the response of systems with stiffness degradation and strength deterioration is verified. Results of ET analysis are verified with different sets of ground motions. Finally, application of ET method in PBEE is described by a simple example of a SDOF system. The results show that the procedure can be used with reasonable validity for different SDOF systems in nonlinear range.

CONCEPT OF ET METHOD

Concept of ET method can be explained by an imaginary shaking table experiment. It is assumed that three different structures with unknown structural properties are to be ranked according to their performance against dynamic excitations. All of the structures are fixed on a shaking table and the test begins by subjecting the structures to a gradually intensifying acceleration function. Because of the increasing demand of the acceleration function, structures gradually go through elastic to yielding and inelastic phases. Eventually the structures collapse in different time. If the acceleration function somehow correspond with earthquake excitations, the structures can be ranked based on the test results. In this case, the structure which failed first is ranked as the worst and the structure which endured last is ranked as the best.

The increasing trend of an acceleration function gives a special meaning to time variable in ET method. It can be said that time is a representative of Intensity Measure (IM). In Fig. 1, a sample ET analysis result showing the increasing trend of IM for these structures through time is demonstrated. It should be noted that ET analysis results are usually presented by special increasing curves. In these curves the y coordinate at each time value t , corresponds to the maximum absolute value of the required parameter in the time interval $(0, t)$ as in Eq. 1.

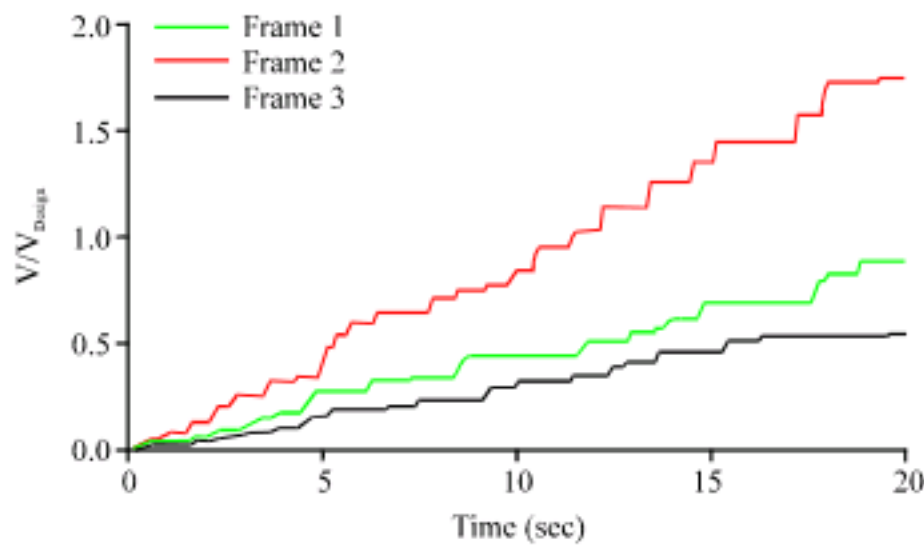


Fig. 1: Time vs. ratio of base shear to design base shear (V/V_{Design}) for three different frames

$$\Omega(f(t)) \equiv \text{Max}(\text{Abs}(f(\tau)); \tau \in [0, t]) \quad (1)$$

where, Ω is the Max-Abs operator as defined above and $f(t)$ is the response history such as base shear, inter-story drift, damage index or other parameters of interest.

ET method can be implemented into an overall performance-based design procedure with multiple performance objectives. These objectives can be described as various parameters obtained under different intensity of ground motions. The range of response versus the range of potential levels of IM can be estimated with ET method. Also the changes in the nature of the structural response like changes in peak deformation patterns with height, onset of stiffness degradation and strength deterioration and onset of dynamic instability can be predicted. Additionally, limit-states can be easily defined in this method (e.g., $\theta_{max} = 2\%$ for IO level).

Obviously, existence of appropriate intensifying acceleration functions is a vital prerequisite for successful implementation of ET method (Riahi and Estekanchi, 2006). Design of intensifying acceleration functions should be in a way that different structures with different fundamental periods can be evaluated with ET method. Therefore using a single harmonic function would certainly be unsuitable for designing acceleration functions. Consideration of the significance of the fundamental period of vibration in dynamic response of structures, leads one to the concept of response spectra. Accordingly, it must be a good idea to design ET acceleration functions so that their response spectra match those of certain earthquakes response spectra. Since, ET acceleration functions are supposed to be intensifying with time, their response spectra should increase through time. Therefore, a suitable response spectrum and an appropriate function for increasing this spectrum should be selected for design of ET acceleration

functions. At this stage, a linear function is assumed, i.e., if the response of ET acceleration function at time $t = t_1$ is to be S_{a1} , at $t = \alpha t_1$ it should be αS_{a1} , etc. Mathematically speaking, this requirement can be expressed as in Eq. 2.

$$S_{aT}(T, t) \equiv S_{aT}(T, t_{Target}) \times \frac{t}{t_{Target}} \quad (2)$$

where, t_{Target} is the time when the proportionality factor equals 1.0 i.e., when ET acceleration response directly equals the proposed response spectra function. In this way, definition of required acceleration function can be formulated as in Eq. 3.

$$\text{Find } a_g(t) | \forall T \in [0, \infty), t \in [0, \infty) \rightarrow \Omega(\ddot{u}(t)) = S_{aT}(T, t) \quad (3)$$

where, $\ddot{u}(t)$ is the acceleration response of a SDOF system with period T and damping ratio ξ , which is considered to be 5% in this research, subjected to ground acceleration defined by $a(t)$.

Analytical approaches to the solution of the problem stated in Eq. 3 are formidably complicated and the problem does not seem to have an exact solution. However, from the practical point of view, the important question is how closely an acceleration function $a(t)$ can be found to match the requirement stated in Eq. 3. For this purpose, a numerical optimization approach can be adopted. Considering that in practical applications only a limited range of parameters T and t are actually needed, Eq. 3 can be formulated as the following optimization problem:

$$\text{Minimize } F(a_g(t)) = \int_0^{T_{max}} \int_0^{t_{max}} \text{Abs}\{[S_a(T, t) - S_{aT}(T, t)]\} dt dT \quad (4)$$

where, T_{max} and t_{max} are maximum practical values for period and time considered approximately equal to 50 and 20 sec, respectively in this research.

For numerical calculation, Eq. 4 should put into matrix form as in Eq. 5. In Eq. 5, the differences between values with power of 2 are used instead of their absolute function for better numerical performance.

$$\begin{aligned} \text{Minimize } F(a_g) &= \sum_{i=1}^n \sum_{j=1}^m [S_{i,j}^a - S_{i,j}^{aT}]^2 \\ S_{i,j}^a &= S_a(T_i, t_j) \quad ; S_{i,j}^{aT} = S_{aT}(T_i, t_j) \\ t_i &= (i-1) * \Delta t \quad ; i \in [0, 2048] ; \Delta t = 0.01 \end{aligned} \quad (5)$$

where, T_i is a vector containing the target periods.

In this research, target periods vector has two parts. The first part consists of 200 period values distributed in the range of 0 to 5 sec. A semi logarithmic distribution is

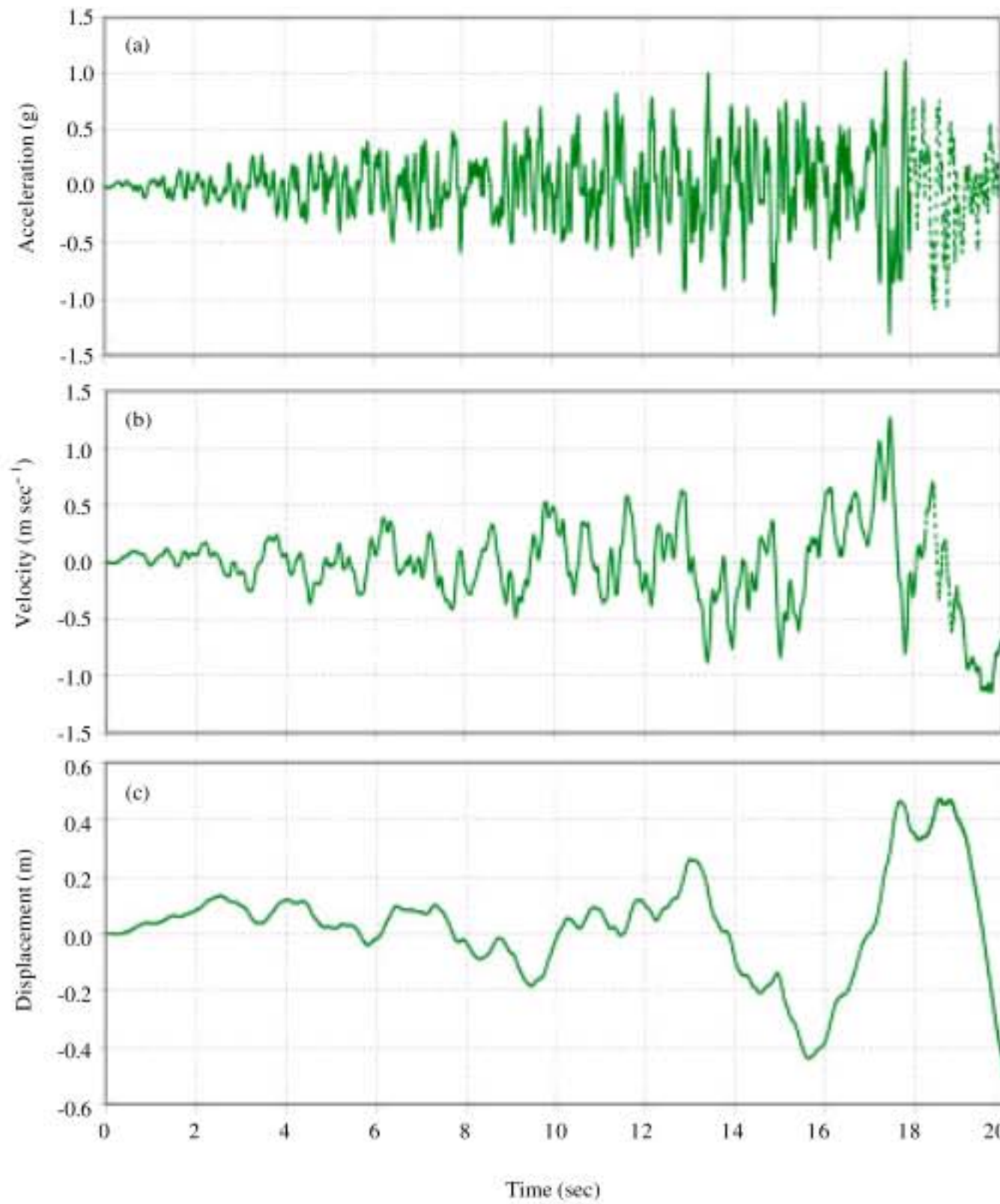


Fig. 2: (a) ETA20f01 acceleration function, (b) ETA20f01 velocity function and (c) ETA20f01 displacement function

used for this part to have more points in the short period range (Estekanchi *et al.*, 2007). The second part consists of 20 period values uniformly spaced in the range of 6 to 50 sec. The next minor improvement is to directly include displacement response in the optimization formula. Displacement and acceleration can be correlated for linear systems with low damping ratios. However, direct inclusion of the displacement is believed to result in better convergence for entire range of periods. Therefore, generation of ET acceleration function has been formulated as in Eq. 6.

$$\text{Minimize } F(a_g) = \sum_{i=1}^n \sum_{j=1}^m [S_{i,j}^a - S_{i,j}^{at}]^2 + \alpha [S_{i,j}^d - S_{i,j}^{dt}]^2 \quad (6)$$

where, α is an optimization weighing factor considered equal to 1.0 in this research. Higher α values causes better convergence in the long period range and lower α results in better convergence in the low period range.

Optimization was run considering the 2^{11} (i.e., 2048) acceleration data points at 0.01 sec time steps as optimization variables. A general non-constrained numerical optimization procedure was applied. The optimization problem defined thus is a computationally intensive one. Each acceleration function typically takes about 80 h on 3 GHz Pentium IV personal computer to converge with 200 cycles. It should be noted that the generated acceleration functions can be reused in any analysis for which the original response spectrum is valid. A sample acceleration function generated in this way is shown in Fig. 2.

Maximum degree of convergence that can be achieved using improved optimization problem imposes a challenging problem. However, the acceleration functions obtained using somewhat rough procedure explained above seem to be good enough for some practical applications.

PROPERTIES OF ET ACCELERATION FUNCTIONS IN NONLINEAR RANGE

In the development of PBEE, displacement rather than force has been recognized as the most suitable and direct performance or damage indicator (Moehle, 1992). If ET acceleration functions are to be used successfully as a tool for relative performance measurement, they should be capable of predicting displacements of nonlinear systems with reasonable accuracy and consistency. In order to estimate nonlinear displacement of the structures, different methods have been proposed. Gulkan and Sozen (1974) were the first researchers that developed the concept of substitute structure to estimate nonlinear structural response through an equivalent elastic model. This idea has been adopted by many investigators since then (ATC, 1996; Priestley and Calvi, 1997; Kowalsky *et al.*, 1994; Fajfar, 1999). Building an inelastic response spectrum is another way to account the nonlinear inelastic behaviour of a structural system (Krawinkler and Nassar, 1992; Miranda and Bertero, 1994; Vidic *et al.*, 1994; Ruiz-García and Miranda, 2003).

If ET acceleration functions coincide well with ground motions this method can be used for estimation of deformation of the inelastic SDOF systems for different levels of IM. Also the procedure can be implemented to MDOF systems adopting the idea of equivalent SDOF systems (Priestley and Calvi, 1997; Gulkan and Sozen, 1974). To evaluate the results of ET acceleration functions for nonlinear SDOF systems, an elastic perfectly plastic model is used. This model has been used widely in previous investigations and therefore it represents a benchmark to study the effect of hysteretic behaviour. Nonlinearity is introduced to the model by the means of normalized lateral strength. In this study the lateral strength is normalized by the strength ratio R , which is defined by Eq. 7.

$$R = \frac{mS_a}{F_y} \quad (7)$$

where, m is the mass of the SDOF oscillator and S_a is the spectral acceleration ordinate corresponding to the initial period of the system. F_y represents the lateral strength required to maintain the system elasticity, which sometimes is also referred to as the elastic strength demand. By using Eq. 7 nonlinear response spectrum of acceleration functions can be calculated for different R values.

The target response spectrum of ET acceleration functions used in earlier study (Estekanchi *et al.*, 2007) is the response spectrum of the Iranian National Building

Code (INBC) (BHRC, 2005) standard 2800 for soil type II. These acceleration functions are built using response spectrum values at 200 period points distributed in the range of 0 to 5 sec. In addition, 3 accelerograms are simulated by SIMQKE software. These accelerograms are compatible with the response spectrum of the INBC standard 2800 to compare the results in nonlinear range (Gasparini and Vanmarcke, 1976). For the comparison of ET acceleration functions results with others, response spectra of ET acceleration functions are obtained for the target time, here 10 sec. The compatibility of the ET acceleration functions spectra with target spectrum in linear range is very good. However in nonlinear range, results of ET acceleration functions are not attuned with target spectrum and the results of simulated accelerograms especially in long periods. The main problem of these ET acceleration functions is their target spectrum. The response spectrum of the INBC standard 2800 forces the displacement to increase with the period even for very long periods. In recent seismic codes this problem is somehow removed. Thus the relation of elastic acceleration and the period is formulated by a quadratic equation to consider the constant displacement range (ASCE 2005; FEMA, 2003). The reasonable feature of the elastic displacement design spectra is still under investigation (Bommer and Elnashai, 1999). A precise way to overcome the problem of ET acceleration functions in long periods is to consider the constant displacement range of the target response spectrum in the generation of acceleration functions. This time optimization process is extended in long periods for 20 points from $T = 6$ to $T = 50$ sec.

DEVELOPMENT OF ET ACCELERATION FUNCTIONS COMPATIBLE WITH EARTHQUAKES

To investigate the potential capability of ET method in estimating nonlinear response of ground motions, two sets of ET acceleration functions (ETA20e and ETA20f) are generated using the average response spectrum of ground motions. To reach this goal, 20 accelerograms that are recorded on site class C, as defined by the NEHRP, and used in FEMA 440 are selected. From these ground motions, 7 records that their response spectra shape were more compatible with the response spectrum of soil type II of INBC standard 2800 are selected (Table 1) (Riahi and Estekanchi, 2007). It should be noted that the characteristics of site class C of NEHRP are the same as soil type II of the INBC standard 2800. These 7 accelerograms are scaled to produce response spectrum that is compatible with INBC standard 2800 spectrum. In this way, the produced template response spectrum

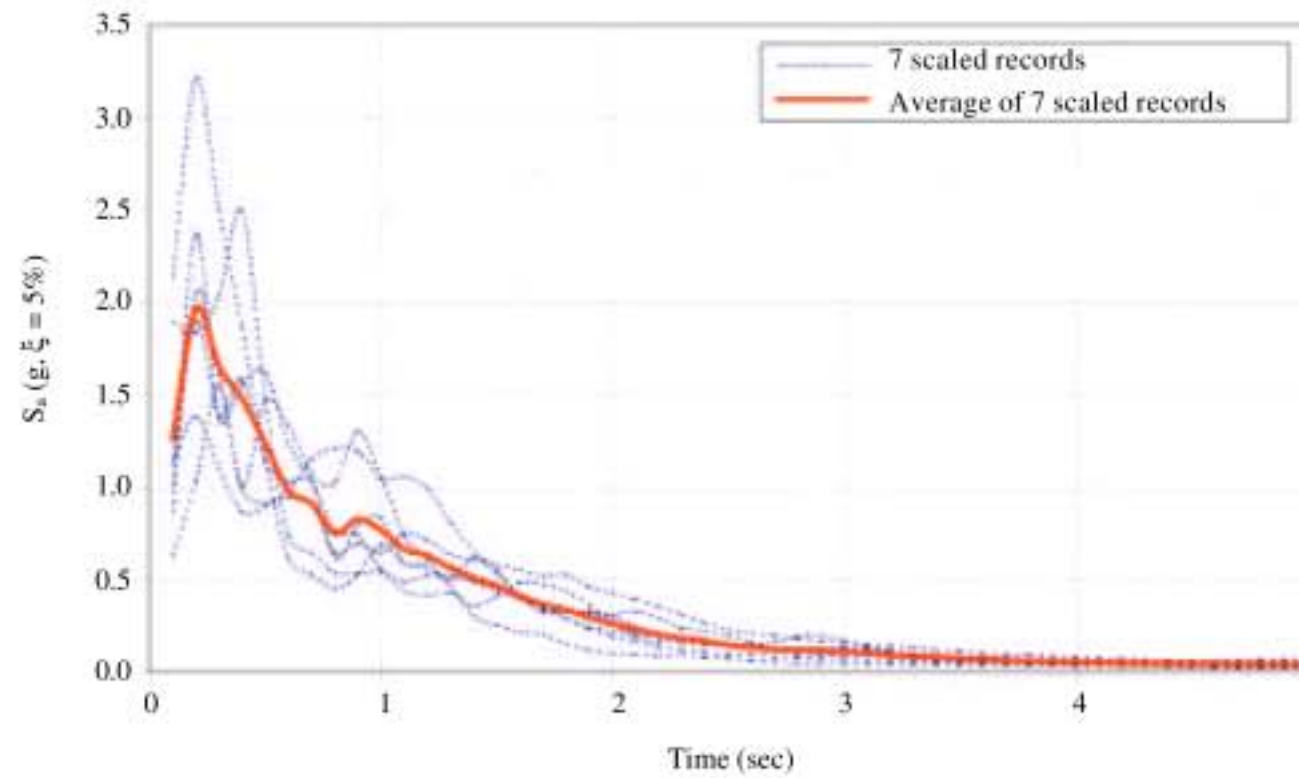


Fig. 3: Total acceleration response spectra of 7 scaled accelerograms and their average spectrum

Table 1: Description of 7 ground motions used in this study

Date	Earthquake name	Magnitude (Ms)	Station No.	Component (deg)	PGA (cm s ⁻²)	Abbreviation
06/28/92	Landers	7.5	12149	0	167.8	LADSP000
10/17/89	Loma Prieta	7.1	58065	0	494.5	LPSTG000
10/17/89	Loma Prieta	7.1	47006	67	349.1	LPGIL067
10/17/89	Loma Prieta	7.1	58135	360	433.1	LPLOB000
10/17/89	Loma Prieta	7.1	1652	270	239.4	LPAND270
04/24/84	Morgan Hill	6.1	57383	90	280.4	MHG06090
01/17/94	Northridge	6.8	24278	360	504.2	NRORR360

retains a degree of significance from the seismic design viewpoint i.e., it induces seismic demands that are comparable to codified values at target time. The range of periods in this scaling procedure is from 0 to 5 sec. Finally, average of pseudo acceleration spectrum of these scaled accelerograms is obtained and smoothed. The smoothed spectrum is used as the target spectrum in generating new ET acceleration functions. In Fig. 3, total acceleration spectra of 7 scaled accelerograms and their average spectrum are plotted. The response spectra of ETA20f set (3 ET acceleration functions) are shown in Fig. 4. Optimization process is done for 200 period points from T=0 to 5 sec and 20 points from T=6 to T=50 sec to account the effects of long periods. Difference between two sets of acceleration functions is the initial values of the functions used in the optimization process. Here, the results of new ET acceleration functions analysis are compared with the results of the analysis of 7 scaled accelerograms.

In Fig. 5, average of total acceleration spectra for 7 scaled accelerograms are compared with the average of total acceleration spectra for ETA20f series for different R values. Results demonstrate the reasonable compatibility between the new acceleration functions and 7 scaled accelerograms. In Fig. 6, the displacement response of ETA20f series and ground motions are compared.

There are some differences between the results of ET acceleration functions and records but a distinct trend cannot be observed. For example for R = 2 (Fig. 6a), displacements of earthquakes in long periods are more than ET method estimation but for R = 5 (Fig. 6b) this conclusion is not valid.

In this study, the results computed with nonlinear response-history analyses of the selected ground motions assumed to be the benchmark maximum displacements, $(\Delta_i)_{ex}$. The maximum displacements estimated with ET method analysis are supposed to be the approximate maximum displacements, $(\Delta_i)_{app}$. It should be noted that the nonlinear response-history results of the ground motions are just a convenient benchmark and there is no exact result for the responses. By these definitions a mean error measure can be defined for each period and strength ratio as follows:

$$\bar{E}_{T,R} = \frac{\bar{\Delta}_{ex} - \bar{\Delta}_{app}}{\bar{\Delta}_{ex}} \quad (8)$$

where, $\bar{\Delta}_{ex}$ and $\bar{\Delta}_{app}$ are average of maximum displacements of 7 scaled accelerograms and average of maximum displacements of three acceleration functions. Values of this error measure are shown in Table 2. Mean error measure is usually greater for larger R values but there is

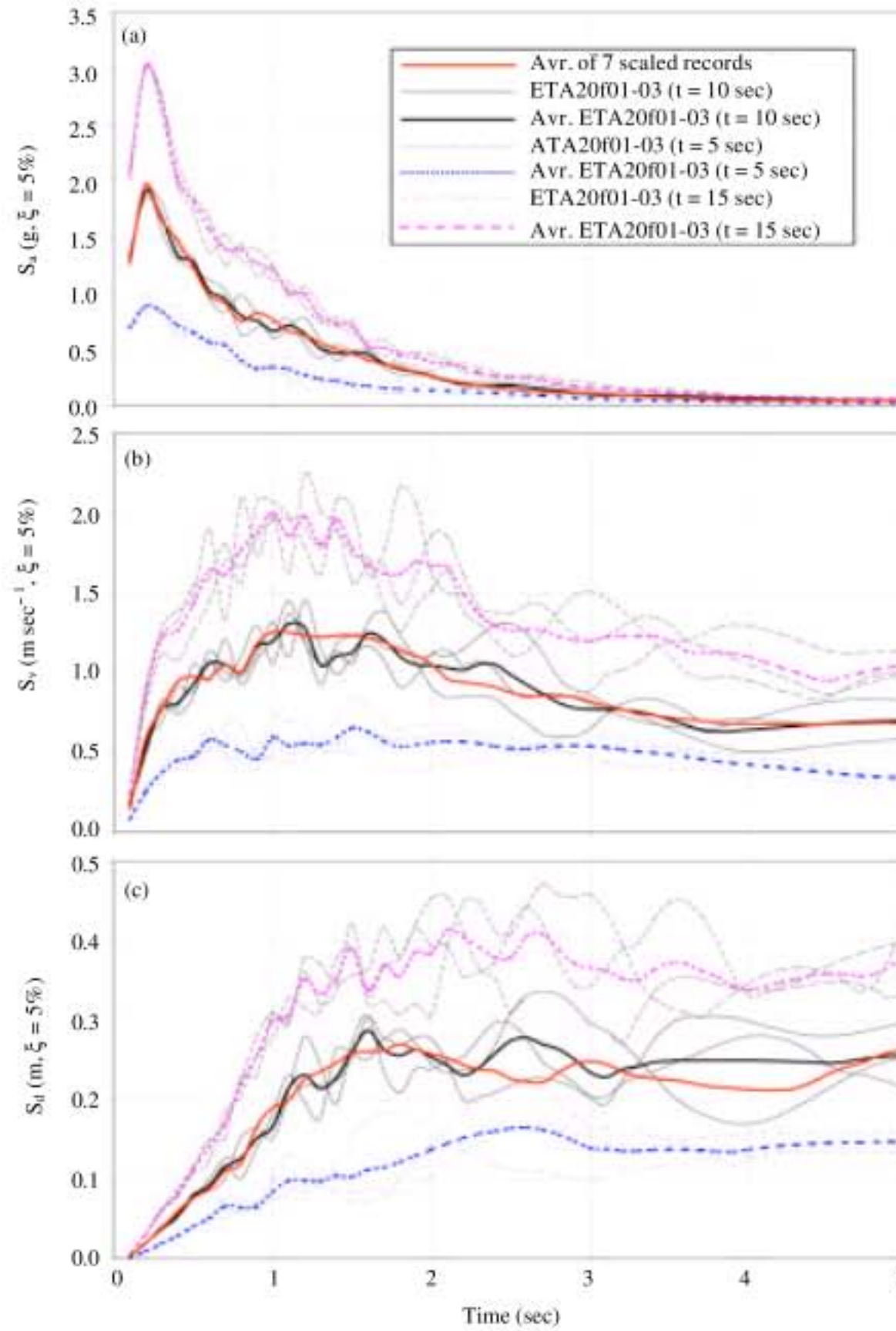


Fig. 4: Response spectra of ETA20f series acceleration functions for $\xi = 5\%$ at different time, (a) total acceleration, (b) velocity response and (c) displacement response

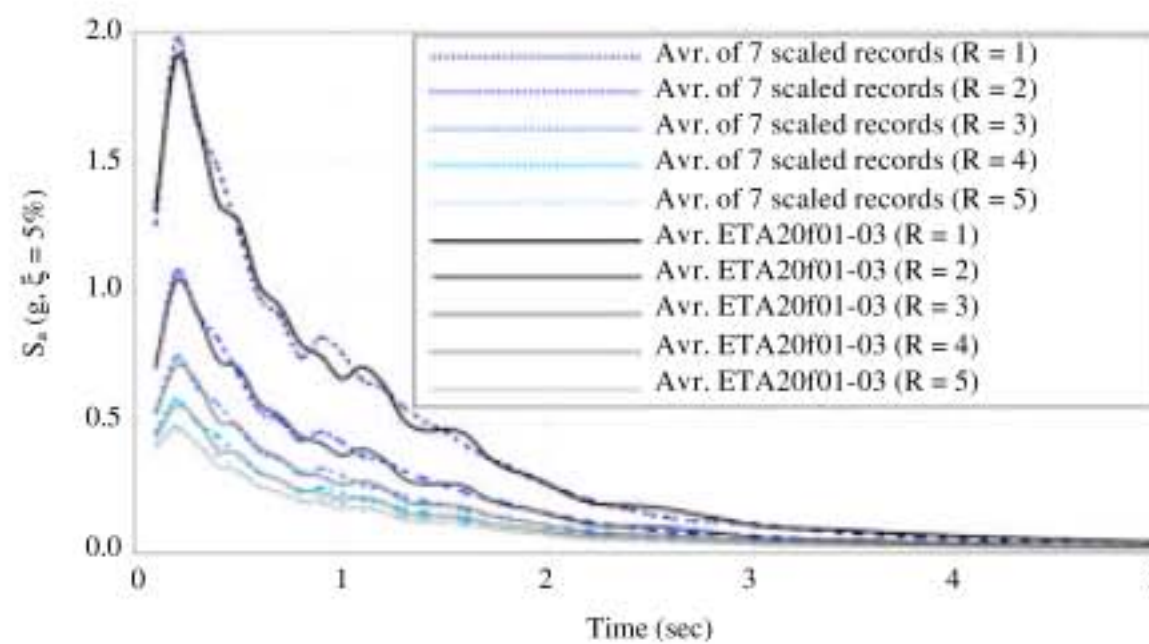


Fig. 5: Comparison of average of total acceleration spectra for 7 scaled accelerograms and ETA20f series for different R-values

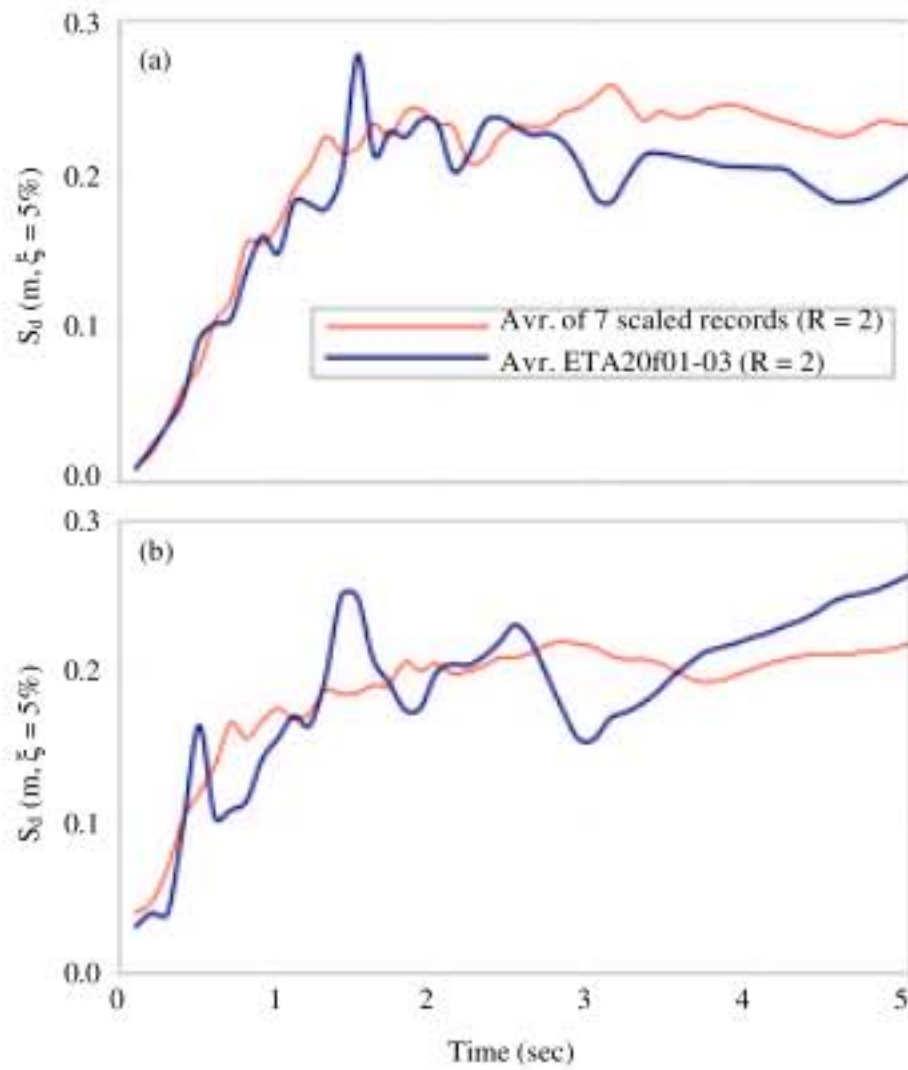


Fig. 6: Comparison of average of displacement spectra for 7 scaled accelerograms and ETA20f series, (a) R = 2 and (b) R = 5

Table 2: Maximum and minimum of mean error measure of acceleration functions for different R values

Acceleration function		Percentage of mean error measure				
		R = 1	R = 2	R = 3	R = 4	R = 5
ETA20f	Overestimate	13	29	39	35	43
	Underestimate	-25	-28	-41	-41	-38
ETA20e	Overestimate	18	29	55	52	49
	Underestimate	-20	-21	-37	-9	-23
ETA20f and ETA20e	Overestimate	12	24	44	35	37
	Underestimate	-18	-20	-25	-18	-18

not a logical relation for the changes of estimation of different acceleration functions. Most of these values occur in short periods between 0 to 1 sec but there are some exceptions as well. The third row in Table 2. examines the use of 6 acceleration functions together. As it is clear the results are better and the worst value of mean error measure is 44%. As can be shown in Table 2, by using the average of three acceleration functions, reasonable accuracy can be achieved for these sets of acceleration functions.

INVESTIGATION OF DUCTILITY DEMANDS

Another important criterion for comparison of the results of nonlinear systems is ductility. For an inelastic SDOF system with a bilinear force deformation relationship, ductility and strength ratio are related by:

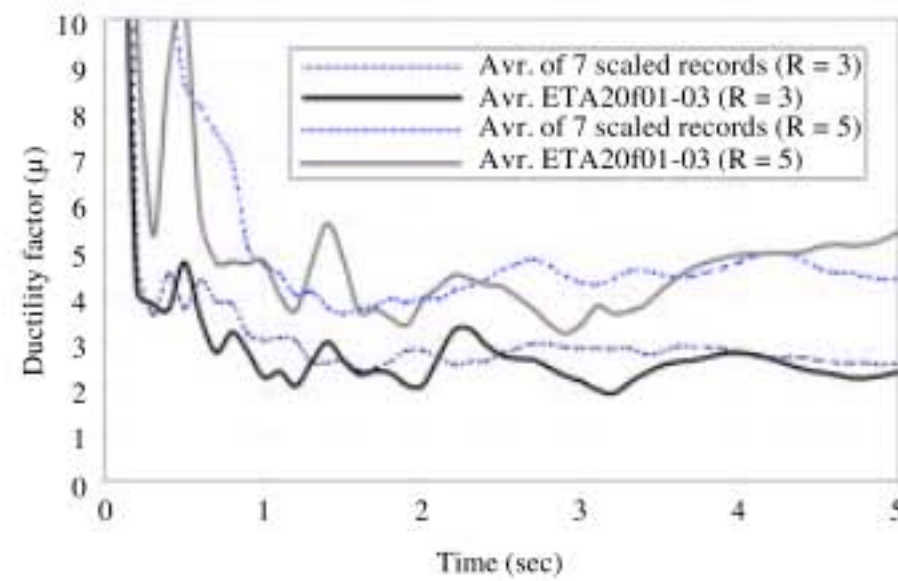


Fig. 7: Comparison of ductility factor spectra for 7 scaled accelerograms and ETA20f series for R = 3 and 5

$$\frac{u_m}{u_0} = \frac{\mu}{R} \tag{9}$$

where, μ is the ductility factor defined as the ratio between the maximum inelastic displacement (u_m) and the yield displacement (u_y) and u_0 is the maximum displacement of the elastic system. Many investigators define different R- μ relations for acceleration, velocity, and displacement sensitive spectral regions. By using these relations, the inelastic design spectrum can be constructed (Krawinkler and Nassar, 1992; Miranda and Bertero, 1994; Vidic *et al.*, 1994; Veletsos and Newmark, 1960; Newmark and Hall, 1982; Riddell *et al.*, 1989).

Figure 7 shows the relation between ductility factor and strength ratio for R = 3 and 5. Results of 7 scaled accelerograms and 3 acceleration functions demonstrate that ductility factor and strength ratio tend to be equal for long periods. This fact is similar to R- μ relations that are established by different researchers. The main difference of ET acceleration functions with ground motions in long periods is the scattering property of them. But in short periods the results are more interesting. For R = 3, the compatibility of the results of these methods is extraordinary but the results are so different for R = 5. Although that different responses of acceleration functions in short periods is a major problem for them but the general trend of them is compatible with ground motions. For example, the value of ductility factor is larger than the strength ratio in short periods for both acceleration functions and ground motions.

EFFECTS OF DAMPING

As it was described before, a fixed viscous damping ratio ($\xi = 5\%$) is used in generating acceleration functions in this research. The response spectrum of a ground motion record is typically highly depending on the

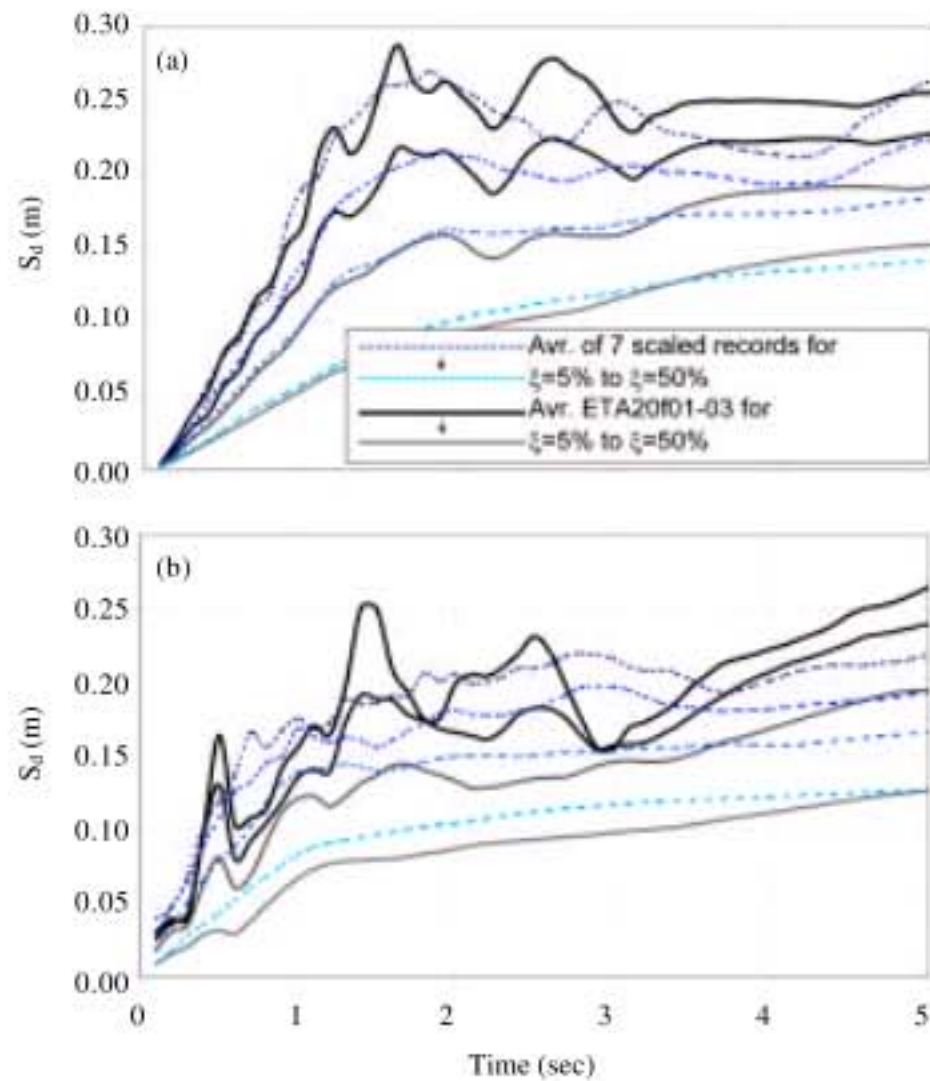


Fig. 8: Comparison of average of displacement spectra for 7 scaled accelerograms and ETA20f series for different damping ratios ($\xi=5\%$, 10% , 20% , 50%), (a) $R = 1$ and (b) $R = 5$

assumed level of damping. The assumed damping ratio, often 5%, represents a level that might be expected for a typical structure responding in the elastic range.

By applying 4 different levels for damping ratios, the consistency of ET method in estimating the response of SDOF systems with different damping ratios is investigated. Damping ratios are assumed to be 5, 10, 20 and 50%. As it can be shown in Fig. 8a, the results of 7 scaled accelerograms and 3 acceleration functions are compatible in linear range for different damping ratios. In linear range displacement values are decreased by increasing damping ratios. However by increasing the nonlinearity of the systems (Fig. 8b), ET acceleration functions underestimate the response in almost every point of periods for larger damping ratios. Again the dispersion of the results of ET acceleration functions is larger than the ground motions.

EFFECT OF STIFFNESS DEGRADATION AND STRENGTH DETERIORATION

Commonly, the nonlinear hysteretic characteristics of most buildings include both stiffness degradation and strength deterioration to some extent. The presence of stiffness degradation, pinching and strength deterioration

may influence peak displacement estimates. Because of the many parameters involved, it is difficult to capture the effects of all possible types of cyclic degradation by simple methods.

The general consensus appears to be that moderate levels of stiffness degradation and pinching will cause peak displacements of short period systems (below 0.3 to 0.6 sec) to increase slightly above those determined for EPP systems (Otani *et al.*, 2000; FEMA, 2005; Vidic *et al.*, 1994; Gupta and Kunnath, 1998). For periods of vibration larger than about 0.6 sec, the maximum displacements of stiffness degrading systems are to some extent smaller than that of the EPP systems (FEMA, 2005; Krawinkler and Nassar, 1992; Gupta and Kunnath, 1998; Clough, 1966; Chopra and Kan, 1973; Mahin and Bertero, 1981; Foutch and Shi, 1998).

To evaluate the accuracy of ET analysis results for systems with stiffness degradation and strength deterioration three models are used (Fig. 9):

- The Stiffness-Degrading (SD) system matches to the modified Clough (1966) model by Mahin and Lin (1983). This model was originally proposed as representative of a structure with a modest level of inelastic energy dissipation and stiffness degradation due to cyclic damage. The post-elastic stiffness in any cycle is always equal to zero
- The strength and stiffness-degrading (SSD) system is intended to reproduce the hysteretic behavior of structures in which lateral stiffness and lateral strength decrease when subjected to cyclic reversals. In this system, the amount of strength deterioration and stiffness degradation is a function of dissipated hysteretic energy (Rahnama and Krawinkler, 1993). Parameters of the system are set to represent severe strength deterioration and stiffness degradation. Degradation of the system only includes cyclic degradation and therefore the post-elastic stiffness in any cycle is always equal to zero
- The concrete (CONC) system is proposed to simulate the hysteretic behavior of structures that have negative post-elastic stiffness, degradation of stiffness and strength deterioration. In this system, the strength diminishes in the current cycle of deformation. This system can be prone to dynamic instability. Negative values of post-yield stiffness arise either due to the load-deformation behavior of the component or the presence of P-Delta effects. The parameters of CONC system is adopted from the concrete model used by Ibarra and Krawinkler (2005)

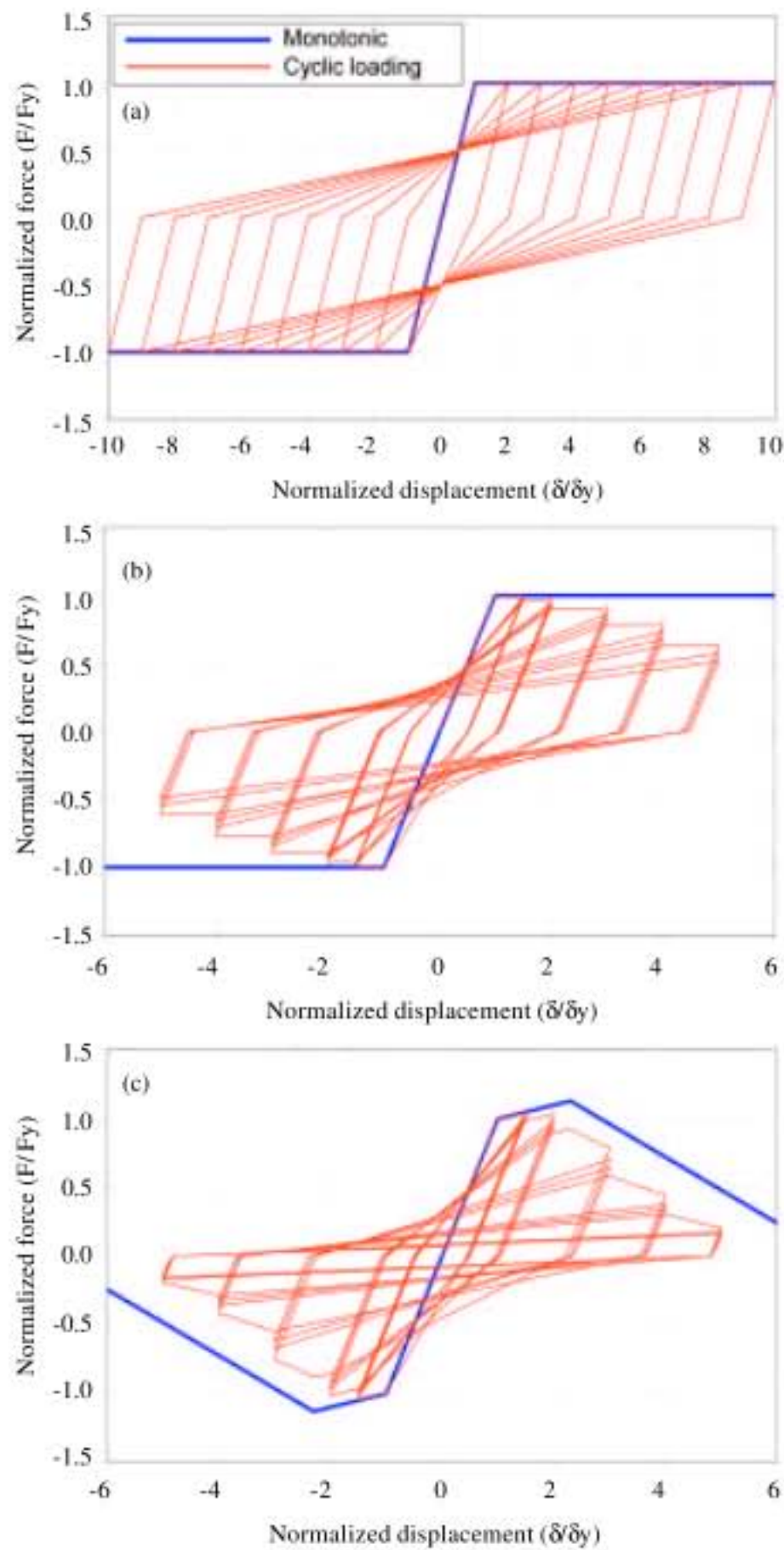


Fig. 9: Degradation hysteretic models used in this study. (a) stiffness-degrading (SD), (b) strength and stiffness-degrading (SSD) and (c) concrete (CONC)

Figure 10a-c show the ratio of the maximum displacement of SD system to EPP system for 7 scaled accelerograms, ETA20f series and ETA20e series. For ground motions the results are consistent with previous studies of different investigators. For SDOF systems with $T < 0.6$ sec, displacement responses of SD systems are larger than EPP systems and for longer periods the situation is vice versa. For ET acceleration functions, displacement responses of SD systems are larger than EPP systems for $T < 0.3$ sec. But for periods between 0.3 to 1.0 sec the displacement responses of SD systems are

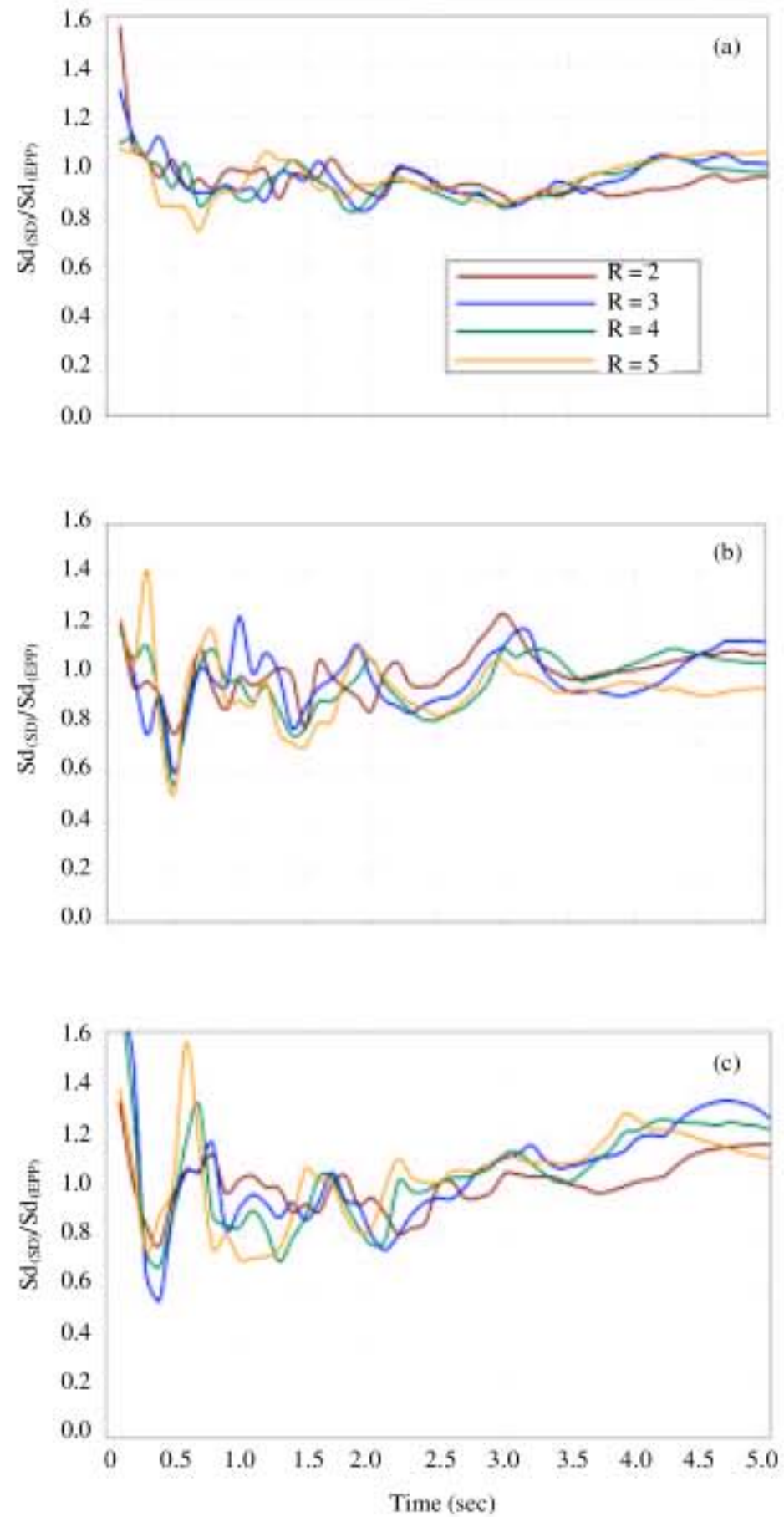


Fig. 10: Ratio of the maximum displacement of SD system to EPP system. (a) 7 scaled accelerograms, (b) ETA20f series and (c) ETA20e series

sometimes smaller and sometimes larger than the displacement responses of EPP systems. For longer periods displacement responses of acceleration functions for SD systems are smaller than that of the EPP systems, except in some local points, but the dispersion of the results are more than ground motions.

Figure 11 shows the ratio of the maximum displacement of SSD system to SD system for 7 scaled accelerograms, ETA20f series and ETA20e series for $R = 5$. Both ground motions and acceleration functions foresee the same trend for the displacement and the results of SSD systems are larger for $T < 0.6$ sec. For longer periods the responses are really close together.

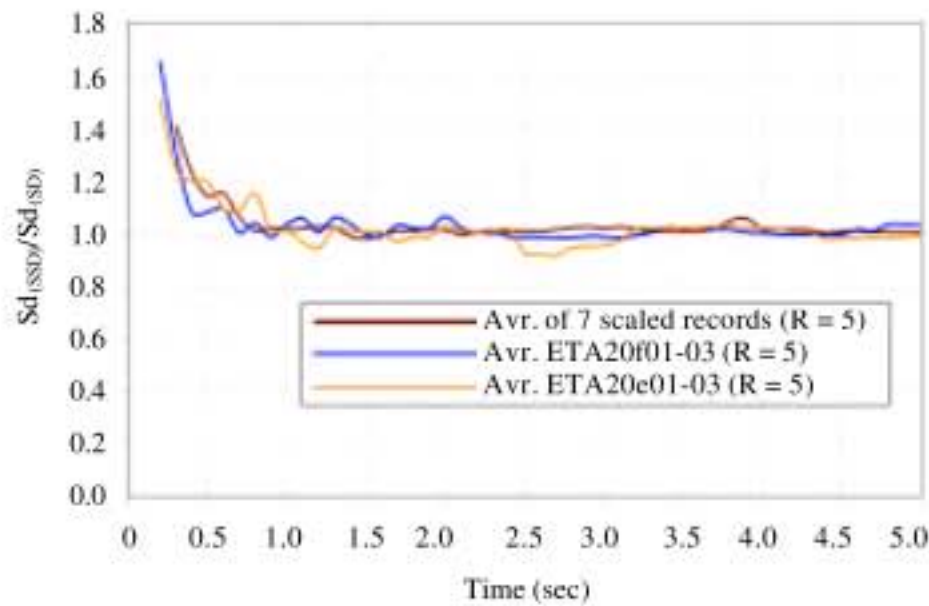


Fig. 11: Ratio of the maximum displacement of SSD system to SD system for R = 5

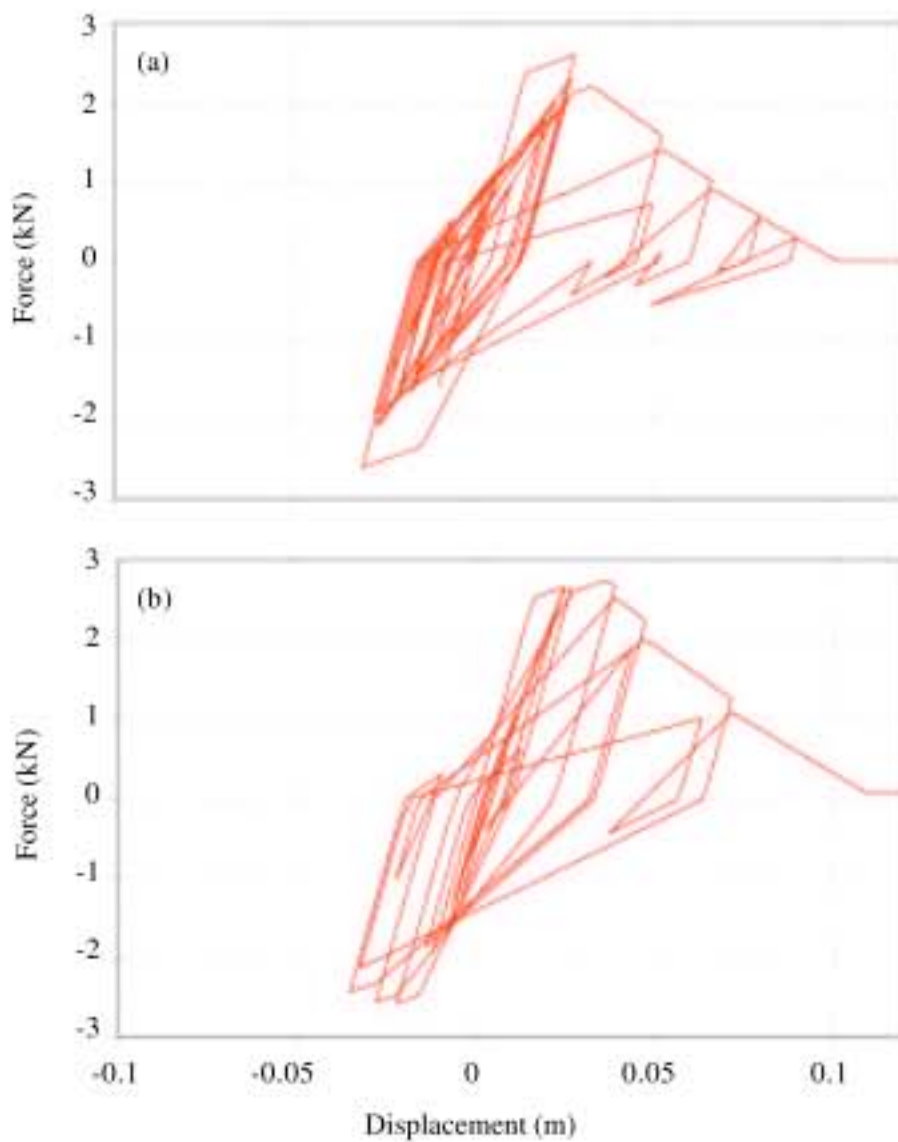


Fig. 12: Hysteretic behaviour of a SDOF system with T = 0.5 sec and CONC material model. (a) LPLOB000 accelerogram and (b) ETA20f03 acceleration function

As it was mentioned earlier, negative values of post-yield stiffness, arising due to the load-deformation behaviour of the system like CONC system can cause increases in peak displacement. In these systems dynamic instability may occur and the responses of the systems after that are not reliable. Displacement responses of 7 scaled accelerograms for CONC system indicate dynamic instability for SDOF systems with short periods.

The range of the periods which dynamic instability may occur in it increases by increasing R values. For example for R = 2 dynamic instability occurs for systems with $T < 0.5$ sec and for R = 5 it occurs for systems with $T < 1.5$ sec. In Fig. 12a the hysteretic behavior of a SDOF system with T = 0.5 sec and R = 5 for LPLOB000 ground motion is shown. For large values of R, dynamic instability occurs sometimes in long periods for 7 scaled accelerograms. Occurrence of dynamic instability is highly dependent on the ground motion.

Displacement responses of CONC system for ET acceleration functions are like ground motions with the difference that the range of period that dynamic instability occurs in it is shorter for acceleration functions. For example for R = 5, dynamic instability occurs for systems with $T < 1.0$ sec. Dynamic instability does not occur in long periods for acceleration functions even for large values of R. In Fig. 12b the hysteretic behavior of a SDOF system with T = 0.5 sec and R = 5 for ETA20f03 acceleration function is shown.

COMPARISON WITH ALTERNATE SET OF GROUND MOTIONS

To ensure the accuracy of the results of ET method analysis another set of ground motions is used. A Large Magnitude-Long Distance set, LMLR, ($6.5 < M_w < 7.0$, $30 \text{ km} < R < 60 \text{ km}$) of ground motion recorded on NEHRP site class D is selected (Medina and Krawinkler, 2004). This set consists of 20 records. This set is intentionally selected to be different from the ground motions used for the generation of ET acceleration functions. Therefore the capability of ET method in predicting the results even for the ground motions with different characteristics can be investigated. Six SDOF oscillators with EPP and SSD systems are used for this comparison. The oscillators have two yield strengths (R = 2, 4) and three different periods (T = 0.5, 1.0, 3.0). To compare the results, each ground motion is scaled so that the spectral ordinate at the period of the oscillator matches the spectral value of the ET acceleration function (with $t = t_{\text{target}}$) at the same period. Ground motions are eliminated selectively to avoid motions with unacceptably large scaling factors.

Figure 13 and 14 show the results of EPP and SSD systems for ET acceleration functions and LMLR set. As it can be seen the general trend of the results are similar. For SDOF system with T = 0.5 sec, displacement responses are increased by increasing R value and for SDOF system with T = 3.0 sec, displacement responses are decreased by increasing R value. The response of SDOF system with T = 1.0 sec does not change a lot by modifying R value. The change of the responses for EPP

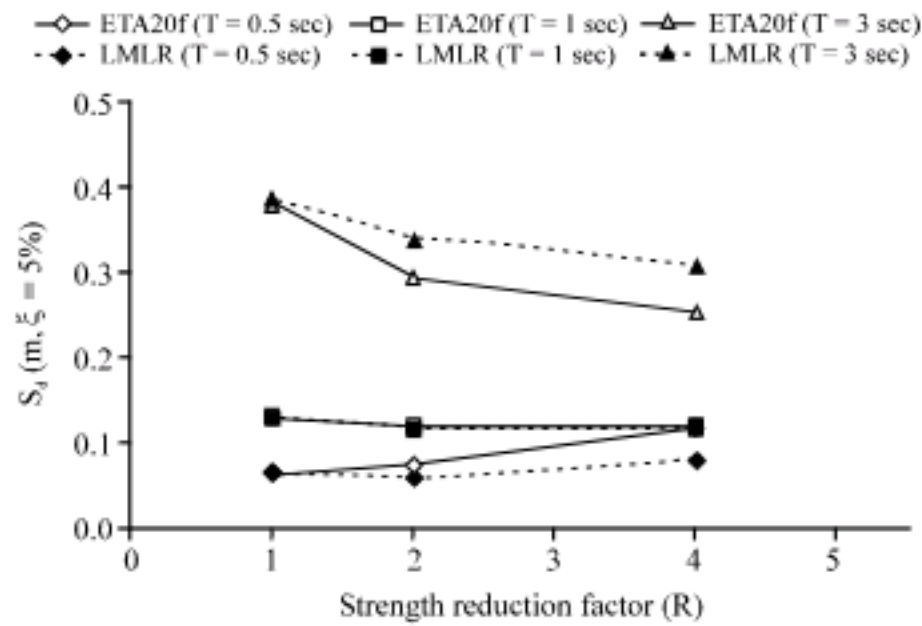


Fig. 13: Comparison of responses of oscillators for ETA20f and LMLR series for different R values (EPP system)

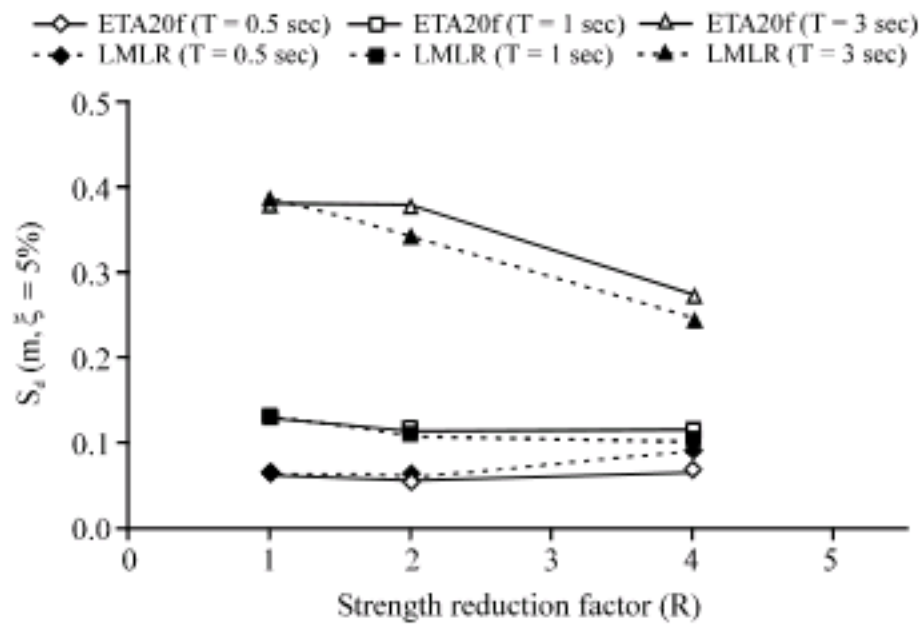


Fig. 14: Comparison of responses of oscillators for ETA20f and LMLR series for different R values (SSD system)

system estimated by ET acceleration functions is more than what is obtained by ground motions. Vice versa the change of the responses for SSD system estimated by ET acceleration functions is less than what is obtained by ground motions.

APPLICATION OF ET METHOD IN NONLINEAR RANGE

As explained in the outset of this study, the major purpose of ET method is not to provide accurate estimates of the response of structures due to earthquakes. Endurance Time (ET) method tries to provide a simple tool for screening performance differences among different systems and designs. Endurance Time (ET) method tries to judge about the performance of the structures based on the time interval during which they can meet predefined performance criteria. A simple SDOF

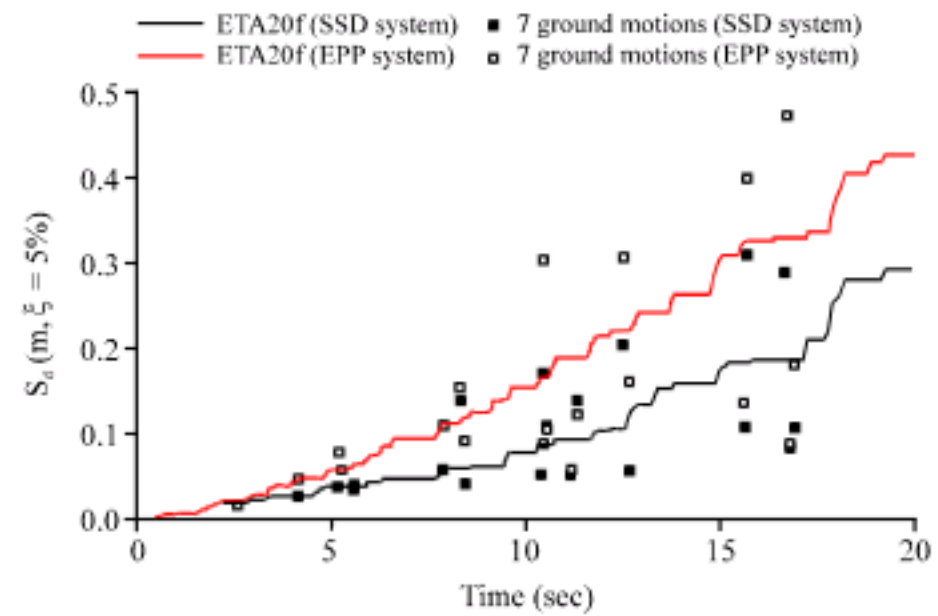


Fig. 15: Displacement response of SDOF oscillators with T = 0.5 sec with EPP and SSD system for ETA20f series and 7 ground motions with scale factors = 1, 2, 3, 4

example is provided in this section to clarify this subject. Suppose that two SDOF structures with T = 0.5 sec are given with EPP and SSD systems described before. The yield strength of EPP system and SSD system are 2000 and 4000 kN, respectively. Considering that the system with higher yield strength includes a degrading characteristic, it is not clear as which system is expected to perform better when subjected to earthquakes on stiff soil conditions. To understand the behavior of these systems in different intensity measures and find the structure with better overall performance, ET analysis is done for both of the systems. Also, in order to compare the results with earthquakes, 7 ground motions used in the generation of ET acceleration functions are scaled with four scale factors (SF = 1, 2, 3, 4) and their results are shown in Fig. 15. To obtain the equivalent time for ground motions analysis, maximum total acceleration of each ground motion obtained in linear analysis is compared with the target response spectrum of ET acceleration functions at T = 0.5 sec and the equivalent time is calculated.

Figure 15 shows that both ET acceleration functions and ground motions separate the systems for different levels of intensity measures. EPP systems with lower yield strength experiences more displacement than SSD system with higher yield strength. At low intensity measures both systems remain elastic but by increasing the time that is equal to IM, two systems separate from each other. At t = 10 sec that is the target time of ET acceleration function generation, the results are compatible with the results of mentioned earlier. Estimation of displacement by ET method is more than ground motions for EPP system and is less than ground motions for SSD systems. By increasing time, the

difference between the results for EPP system is increased but for SSD system the difference is decreased and even for $t = 20$ sec the estimation of displacement by ET method is more than ground motions. This can be attributed to the fact that by increasing time, the strong motion duration of the ET acceleration function is increased but it is essentially constant for ground motions in different levels of IM. Since larger magnitude earthquakes actually have longer durations, this characteristic of ET acceleration functions can make them somewhat closer to what is expected in the real situation. Certainly more research is required before passing any judgment about this characteristic of ET acceleration functions. It should be noted that ground motion (record to record) variability that is clear in Figure 15 cannot be represented in ET method.

RESULTS AND DISCUSSION

In this research, Endurance Time (ET) method has been introduced as a new time-history based dynamic analysis procedure. In this method, structures are subjected to a gradually intensifying acceleration function. The performance of the structures is assessed based on the length of the time interval that they can satisfy required performance objectives. A numerical optimization procedure for generating ET acceleration functions is explained. This procedure is time consuming, but when accomplished, ET acceleration functions can be reused as far as their design parameters can be considered appropriate.

Endurance time method can be used as a useful tool in the preliminary performance-based design stage when performance at different levels of dynamic excitation should be estimated. At first glance it can be said that by doing a simpler analysis like pushover analysis, this goal can be attained. But there are some major drawbacks for pushover analysis that different researchers try to solve them by proposing enhanced pushover procedures to account for higher modes effects, inelastic response spectrum and variant load pattern. These drawbacks limit the usage of pushover analysis for special complicated cases. IDA tries to solve this problem but this method is even much more complex than ET analysis. It can be said that ET analysis somehow tries to be a tool in PBEE between pushover analysis and IDA.

Of course at this time the body of work to relate the results of ET analysis to inelastic dynamic analysis results does not exist and lots of research should be done to examine its validity. This research and the following works are the first steps to validate ET method. As shown, displacement responses of SDOF systems from ET acceleration functions, optimized to be compatible with

ground motions, match well with responses from ground motions in linear and nonlinear ranges with reasonable accuracy. Highest differences between the results were observed in short period range ($T = 0$ to $T = 1$ sec).

Although, a fixed viscous damping ratio ($\xi = 5\%$) is used in generating acceleration functions in this research, accuracy of the response of SDOF systems in linear range for different damping ratios is reasonable. By increasing the nonlinearity of the systems, ET acceleration functions underestimate the response in almost all periods for larger damping ratios. The general trend of ET acceleration functions results are compatible with ground motions for SD and SSD systems with the exception that in ET acceleration functions the dispersion of the results are more than in ground motions.

Displacement responses of systems with negative values of post-yield stiffness for ET acceleration functions are similar to ground motions with the difference that the range of period that dynamic instability occurs is shorter for acceleration functions. The range of the periods where dynamic instability is imminent increases by increasing R values for these systems.

Sample application of ET method in case of two SDOF systems with different nonlinear material characteristics was explained. This example verifies the potential of ET method for screening different design alternatives based on their performance in a convenient manner. However, it should be noted that the numerical values from the results of ET method and the average of ground motions are not equal. This issue can be attributed to the strong motion duration of the ET acceleration function that is different from ground motions. Improved acceleration functions are required if results from ET procedure are to be used for estimating response parameters with reasonable validity in nonlinear range.

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NOMENCLATURE

- Abs : Absolute value function
- a_g : ET acceleration function in vector form
- $a_g(t)$: ET acceleration function
- $\bar{E}_{T,R}$: Mean error measure of ET acceleration functions for each period and R factor
- $F(a_g)$: Optimization target function

F_y	: Lateral yield strength of the SDOF system
m	: Mass of the SDOF system
Max	: Maximum of the values
n	: Count of vector or matrix elements
R	: Strength ratio
S_a	: Spectral acceleration
$S_a(T, t)$: Acceleration response for period T at time t
$S_aT(T, t)$: Target acceleration response for period T at time t
$S_u(T, t)$: Displacement response value for period T at time t
$S_uT(T, t)$: Target displacement response value for period T at time t
T	: Free vibration period
t	: Time
t_{Target}	: Target time
T_{max}	: Maximum free vibration period (sec) to be considered in the optimization
t_{max}	: Time corresponding to the end of acceleration function
u_0	: Maximum displacement of the elastic system
u_m	: Maximum inelastic displacement
u_y	: Yield displacement
$\ddot{u}(t)$: Acceleration response of a SDOF system
α	: Weighing factor in optimization target function
$\bar{\Delta}_{\text{app}}$: Average of maximum displacement estimated with ET acceleration functions
$\bar{\Delta}_{\text{ex}}$: Average of maximum displacement estimated with scaled ground motions
$(\Delta_i)_{\text{app}}$: Maximum displacement estimated with ET acceleration functions
$(\Delta_i)_{\text{ex}}$: Maximum displacement estimated with scaled ground motions
μ	: Ductility factor
θ_{max}	: Maximum drift ratio
Ω	: Max-Abs operator
ξ	: Viscous damping ratio
$ $: Such that
\forall	: For all values

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