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Newsboy Problem with Two Objectives, Fuzzy Costs and Total Discount Strategy

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Abstract: This study extends Newsboy problem with multi-product multi-constraint and two objectives. Also there is total discount on the purchasing prices. The constraints are the warehouse capacity and the batch forms of the order placements. The first objective of this problem is to find the order quantities such that the expected profit is maximized and the second objective is maximizing the service rate. Moreover, the decision variables are integer. A formulation to the problem is presented and shown to be an integer nonlinear fuzzy programming model. Finally, an efficient hybrid algorithm fuzzy simulation, goal programming and particle swarm optimization is provided to solve the model and the results are shown with a numerical example.

Key words: Newsboy, goal programming, particle swarm optimization

INTRODUCTION

The Newsboy problem is one of the stochastic inventory control problem in which the demand is considered uncertain variable. The buyer should order products only once and only at the beginning the cycle. Hence, on the one hand the buyer may incur a cost to dispose of them and on the other if he initially decides to buy smaller amounts of these commodities, shortages may occur, causing loss of revenue. The question becomes how to determine the quantity to be ordered to minimize (maximize) the costs (profit). The answer to this question is the main objective of the classical Newsboy model.

In real world situations, many products have a limited selling period, so the Newsboy model is often used to aid decision-making in fashion, sporting, service industries, etc. to manage capacity or evaluate advanced booking of orders (Taleizadeh *et al*., 2009).

In extension about multi-product Newsboy problem Abdel-Malek and Montanari (2005), Abdel-Malek and Areeratchakul (2007) and Vairaktarakis (2000) extended that with budget constraint. Matsuyama (2006) extended the Newsboy model in multi-period situation. Moreover, Alfares and Elmorra (2005) analyzed the Newsboy problem in both single-periodic and multi-periodic frames in which random yield and fixed order cost were considered.

Reyes (2005) used the Newsboy model in a supply chain in which both sides had incomplete information on the demand. Mostard and Teunter (2006) considered a single-period model in which a percentage of the sold products were returned. They assumed returned products could be returned in a specific range of time and could be sold again if not damaged. Keren and Pliskin (2006) presented the Newsboy as a risk-averse model and calculated the optimal order quantity by using the utility theory. Chen and Chuang (2006) analyzed the Newsboy problem along with the shortage level constraint. Abdel-Malek and Montanari (2005) presented the Newsboy problem with budget constraint and proposed different formulae to obtain the order quantity for three ranges of the budget quantity. Furthermore, Abdel-Malek and Areeratchakul (2007) used the quadratic programming approach in a multi-product Newsboy problem with budget, capacity and order constraints. Moreover, Taleizadeh *et al.* (2009) developed a multi-product Newsboy model with incremental discounts and batch orders in which the service level and warehouse space assumed constraints. Then, they proposed a GA to solve the obtained non-linear integer model.

Dutta *et al.* (2005) presented a single-period inventory problem in an imprecise and uncertain mixed environment. Dutta *et al.* (2005) introduce demand as a fuzzy random variable. Ji and Shao (2006) considered the model for the SPP problem with fuzzy demands and quantity discounts in hierarchical decision system by manufacturer and retailer. Shao and Ji (2006) considered the multi-product SPP problem with fuzzy demands under budget constraint. In this research we developed the SPP problem in the situation of multi-products multi-constraints considering uncertain demands.

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In this study, a multi-inventory Newsboy problem is considered in which the demand follows a Poisson distribution; batch orders and warehouse capacity are considered constraints and either the total discount policy is used to purchase the items. The holding and shortage cost are fuzzy variables. The overall goal is to establish the optimal order quantity for each product that serves the dual purpose of maximizing expected profits and service level.

PROBLEM DEFINITION

Consider a company that orders products to a supplier with the following rules in that order should take place only once and only at the start of a period. The customer demand for each product (j) follows a Poisson distribution with parameter λ_j. The order quantity of each product should only be integer multiples of packets each with n_j items. There is no enforced constraint by the supplier to supply an order. The entire capacity of the warehouse is assigned to the products. The company intends to maximize both the expected profit and service rate. Shortage is licensable and takes the lost sale condition. The shortage and holding costs are fuzzy and deployed at the end of the period and increase in quadratic fashion. The transportation cost is deployed to carry the products and has two components as fixed cost for each shipment and variable cost for each unit of the products. Discount for purchasing items is allowed and follows total discount rule. Since, the transportation and the order-processing times are relatively very small to the cycle length, we assume that the lead-time is equal to zero, which is the common practice in the Newsboy problems. The goal is to determine the order quantity of each product such that the constraints are satisfied and both the expected profit and service rate are maximized.

PROBLEM MODELING

Since the orders are placed only once and at the beginning of each period, one may take advantage of the classical Newsboy problem and develop a mathematical model for the problem at hand. To do this, the variables and the parameters of the model are first defined. Then, the costs and the constraints are determined. Finally, the model is presented.

The problem parameters and variables: For i = 1,2,...,n and j = 1,2,...,T the parameters and the variables of the model are (Taleizadeh *et al.*, 2009):

T	:	The number of products
X_j	:	The stochastic demand of the jth product
λ_j	:	The expected demand of the jth product
$f_{xj}(x_j)$	:	The probability mass function of the jth product demand
V_j	:	The number of items in the packets of the jth product
$\hat{H}_j(x)$	:	The fuzzy holding cost function of the jth product at the end of a period
$\hat{h}_{1j}$	:	The fuzzy linear coefficient of the quadratic holding cost function of the jth product
$\hat{h}_{2j}$	:	The fuzzy quadratic coefficient of the quadratic holding cost function of the jth product
$\hat{\pi}_j(x)$	:	The shortage cost function of the jth product at the end of a period
$\hat{\pi}_{1j}$	:	The fuzzy linear coefficient of the quadratic shortage cost function of the jth product
$\hat{\pi}_{2j}$	:	The fuzzy quadratic coefficient of the quadratic shortage cost function of the jth product
A	:	The fixed transportation cost for each shipment
K_j	:	The variable transportation cost for each unit of the jth product
m	:	No. of shipments
Q_j	:	A decision variable representing the order quantity of the jth product
B_j	:	A decision variable representing the number of packets that have been ordered for the jth product
f_j	:	The space required for each packet of the jth product
$\hat{f}$	:	The capacity of a shipment
α_j	:	The minimum service level of the jth product
q_{ij}	:	The ith discount break point of the jth product
C_{ij}	:	The purchase cost of the ith product in the jth break point
F	:	The total available warehouse space
C_{H_j}	:	The expected holding cost of the jth product at the end of a period
C_{B_j}	:	The expected shortage cost of the jth product at the end of a period
C_{P_j}	:	The expected purchasing cost of the jth product
C_T	:	The transportation cost of the product(s)
P_j	:	The selling price of the jth product
R_j	:	The expected revenue of the jth product
Z_P	:	The expected profit
Z_{RS}	:	The expected service rate

Modeling the first objective (profit): In order to formulate the problem with a single-product problem, first the revenue and then the costs including holding, shortage, transportation and purchase are modeled. Finally, the problem in multi-product frame will be modeled.

Revenue: If the total demand quantity of jth product is more than the order quantity, then the sold quantity is Q_j. Otherwise, it is X_j. Since, the density function of the demand for product j is $f_{X_j}(x_j)$, the expected sold quantity of the jth product at the end of the period is determined as:

$$\sum_{X_j=0}^{Q_j-1} X_j.f_{X_j}(x_j) + \sum_{X_j=Q_j}^{+\infty} Q_j f_{X_j}(x_j) \quad (1)$$

So, the expected revenue will be:

$$R_j = \sum_{X_j=0}^{Q_j-1} P_j.X_j.f_{X_j}(x_j) + \sum_{X_j=Q_j}^{+\infty} P_j.Q_j.f_{X_j}(x_j) \quad (2)$$

Costs: The costs of the problem are holding, shortage, transportation and purchasing and are determined as follows.

Holding cost: To determine the expected holding cost we need to determine the expected inventory. If the total demand quantity is more than the order quantity i.e., $X_j \geq Q_j$, then the inventory quantity at the end of the period is zero. However, if the total demand quantity is less than the order quantity, then the inventory quantity at the end of the period is Q_j-X_j. Because the density function of the demand for product j is $f_{X_j}(x_j)$, the expected inventory at the end of the period will be:

$$\sum_{X_j=0}^{Q_j-1} (Q_j - X_j) f_{X_j}(x_j) \quad (3)$$

Finally, considering the quadratic increase of the holding cost (Taleizadeh *et al*., 2009), the expected holding cost at the end of the period which is calculated at the start of the period is:

$$C_{H_j} = \sum_{X_j=0}^{Q_j-1} \left(\hat{h}_{1_j}(Q_j - X_j) + \hat{h}_{2_j}(Q_j - X_j)^2\right) f_{X_j}(x_j) \quad (4)$$

Shortage cost: To determine the expected shortage cost we need to determine the expected shortage quantity at the end of the period. In this case, if the total demand quantity is more than the ordered quantity, i.e., $X_j>Q_j$, then at the end of the period the shortage quantity will be X_j-Q_j. However, if the total demand quantity at the end of the period is less than the order quantity, the shortage quantity at the end of the period will be zero. So, the expected shortage at the end of the period is:

$$\sum_{X_j=Q_j+1}^{+\infty} (X_j - Q_j) f_{X_j}(x_j) \quad (5)$$

Taking into account the quadratic shortage cost function by Taleizadeh *et al.* (2009), we have:

$$C_{B_j} = \sum_{X_j=Q_j+1}^{+\infty} \left(\hat{\pi}_{1_j}(X_j - Q_j) + \hat{\pi}_{2_j}(X_j - Q_j)^2\right) f_{X_j}(x_j) \quad (6)$$

Transportation cost: The transportation cost is calculated based on Eq. 7, in which f_jB_j is the required space to ship the order from the supplier.

$$C_T = \begin{cases} A + K_jQ_j; & 0 < f_jB_j \leq \hat{f} \\ 2A + K_jQ_j; & \hat{f} < f_jB_j \leq 2\hat{f} \\ \vdots & \vdots \\ mA + K_jQ_j; & (m-1)\hat{f} < f_jB_j \leq m\hat{f} \end{cases} \quad (7)$$

By introducing the binary variables Y_k; k = 1,2,...,m, the transportation cost can be incorporated with the mathematical model of the problem as:

$$\begin{aligned} & C_T = K_jQ_j + \sum_{k=1}^{m} kAY_k \\ & 0 < f_jB_j \leq \hat{f}Y_1 \\ & \hat{f}Y_2 < f_jB_j \leq 2\hat{f}Y_2 \\ & \quad \vdots \\ & (m-1)\hat{f}Y_m < f_jB_j \leq m\hat{f}Y_m \\ & Y_1 + Y_2 + \cdots + Y_m = 1 \\ & Y_k = 0,1 \end{aligned} \quad (8)$$

Purchasing cost under total discount: Since, the purchasing cost of the company for the jth product at the beginning of a period calculated according to total discount policy we have:

$$C_{P_j} = \begin{cases} C_{1_j}Q_j; & 0 < Q_j \leq q_{1_j} \\ C_{2_j}Q_j; & q_{1_j} < Q_j \leq q_{2_j} \\ \vdots & \\ C_{n_j}Q_j; & Q_j \geq q_{n_j} \end{cases} \quad (9)$$

Equation 9 can change in linear form to Eq. 10 as below:

$$\begin{aligned} & C_{P_j} = C_{1_j}\lambda_{1_j}W_{1_j} + C_{2_j}\lambda_{2_j}W_{2_j} + \cdots + C_{nj}\lambda_{nj}W_{nj} \\ & 0 \leq W_{1_j} \leq q_{1_j}\lambda_{1_j} \\ & q_{1_j}\lambda_{2_j} \leq W_{2_j} \leq q_{2_j}\lambda_{2_j} \\ & \quad \vdots \\ & q_{n-1,j}\lambda_{n-1,j} < W_{n-1,j} \leq q_{nj}\lambda_{n-1,j} \\ & q_{nj}\lambda_{nj} < W_{nj} \\ & \lambda_{1j} + \lambda_{2j} + \cdots + \lambda_{nj} = 1, \quad \lambda_{ij} = 0,1; \quad i = 1,2,...,n \end{aligned} \quad (10)$$

Modeling the second objective (service rate): Let the service rate of the jth product (Z_{SR}) be the ratio of the expected customers demands that are satisfied in a period to the average costumers demand. Since, the expected satisfied demand at the end of a period is determined by:

$$\sum_{X_j=0}^{Q_j-1} (Q_j - X_j) f_{X_j}(x_j) \quad (11)$$

Then, the second objective is obtained as:

$$Z_{SR} = \frac{\sum_{X_j=0}^{Q_j-1} (Q_j - X_j) f_{X_j}(x_j)}{\lambda_j} \tag{12}$$

Constraints: Warehouse space and ordering in batch form are the constraints of the model. The warehouse space constraint will be:

$$f_j B_j \leq F \tag{13}$$

In which, the space required for each packet of the jth product is f_j square meters, the number of packets that have been ordered for the jth product is B_j and the total available warehouse space is F square meters. However, we need the orders to be placed in packets of size V_j. In this case, we have:

$$Q_j = V_j B_j \tag{14}$$

Finally, the multi-product model with total discount and fuzzy costs (according to some definitions in fuzzy environment in appendix will be:

$$\begin{aligned}
\text{Max}: Z_P = & \sum_{j=1}^{T}\sum_{X_j=0}^{Q_j-1} P_j.X_j.f_{X_j}(x_j) + \sum_{j=1}^{T}\sum_{X_j=Q_j}^{+\infty} P_j.Q_j.f_{X_j}(x_j) \\
& - \sum_{j=1}^{T}\sum_{X_j=0}^{Q_{j-1}} \left(\hat{h}_{1j}(Q_j - X_j) + \hat{h}_{2j}(Q_j - X_j)^2\right)\frac{e^{-\lambda_j}\lambda_j^{X_j}}{X_j!} \\
& - \sum_{j=1}^{T}\sum_{X_j=Q_j+1}^{\infty} \left(\hat{\pi}_{1j}(X_j - Q_j) + \hat{\pi}_{2j}(X_j - Q_j)^2\right)\frac{e^{-\lambda_j}\lambda_j^{X_j}}{X_j!} \\
& - \sum_{j=1}^{T} K_j Q_j - \sum_{k=1}^{m} kAY_k - \sum_{j=1}^{T}\sum_{i=1}^{n} C_{ij}\lambda_{ij}W_{ij} \\
\text{Max}: Z_{SR} = & \frac{1}{T}\sum_{j=1}^{T}\sum_{X_j=0}^{Q_j-1} \frac{(Q_j - X_j) f_{X_j}(x_j)}{\lambda_j} \\
\text{s.t}: & \\
& \sum_{j=1}^{T} f_j B_j \leq F \\
& Q_j = V_j B_j \quad \forall j, \quad j = 1,2,\ldots,T \\
& 0 \leq W_{1_j} \leq q_{1_j}\lambda_{1_j} \\
& q_{1_j}\lambda_{2_j} \leq W_{2_j} \leq q_{2_j}\lambda_{2_j} \quad \forall j, \quad j = 1,2,\ldots,T \\
& \vdots \\
& q_{n-1,j}\lambda_{n-1,j} < W_{n-1,j} \leq q_{nj}\lambda_{n-1,j} \quad \forall j, \quad j = 1,2,\ldots,T \\
& q_{nj}\lambda_{nj} < W_{nj} \quad \forall j, \quad j = 1,2,\ldots,T \\
& 0 < \sum_{j=1}^{T} f_j B_j \leq \hat{f}Y_1 \\
& (k-1)\hat{f}Y_i < \sum_{j=1}^{T} f_j B_j \leq k\hat{f}Y_k \quad \forall k = 2,3,\cdots,m \\
& \sum_{k=1}^{m} Y_k = 1 \quad , \quad Y_k = 0,1 \;;\; \forall k = 1,2,\ldots,m \\
& \sum_{i=1}^{n} \lambda_{ij} = 1 \;;\; \lambda_{i_j} = 0,1 \quad \forall j, \quad j = 1,2,\cdots T \text{ and } \forall i, \quad i = 1,2,\cdots,n \\
& B_j \geq 0 \text{ and integer} \quad \forall j, \quad j = 1,2,\cdots,T
\end{aligned} \tag{15}$$

A SOLUTION ALGORITHM

Since, the model in Eq. 15 is mixed-integer-nonlinear in nature, reaching an analytical solution (if any) to the problem is difficult (Gen and Cheng, 1997). As a result, here a stochastic search algorithm is used to solve the model. However, since the models have two objectives, a goal programming framework is first applied to formulate them and then a particle swarm optimization is employed to solve them.

Goal programming modeling: The scope of this research is limited to the application of Goal Programming (GP) approach to real life manufacturing situations in a multi-objective environment. For a rigorous mathematical analysis of multi-objective programming approach, the reader is referred to Steuer (1985).

The multi-objective models in the context of manufacturing were formulated and solved in the recent past (Kalpic *et al*., 1995; Nagarur *et al*., 1997) to provide information on the trade-off among multi-objectives. However, although it represents a viable approach to production planning, Multi-Objective Goal Programming (MOGP) is not as widespread among manufacturing companies as desired.

The GP appears to be an appropriate, powerful and flexible technique for decision analysis of the troubled modern decision maker who is burdened with achieving multiple conflicting objectives under complex environmental constraints. The extensive surveys of the GP by Tamiz *et al.* (1998) and Aouni and Kettani (2001) have reflected this. The modeling approach of GP does not attempt to maximize or minimize the objective function directly as in the case of conventional linear programming. Instead it seeks to minimize the deviations between the desired goals and the actual results to be obtained according to the assigned priorities. A commonly used generalized model for goal programming is as follows (Kwak *et al*., 1991):

$$\begin{aligned}
& \text{Min}: z = \sum w_i p_i (d_i^+ + d_i^-) \\
& \text{S.t}: \\
& \quad \sum_j a_{ij} x_{ij} + d_i^- - d_i^+ = b_i \quad \forall i, \; i = 1,2,\ldots,m \\
& \quad x_{ij}, d_i^-, d_i^+ \geq 0 \\
& \quad d_i^-.d_i^+ = 0, \quad (i = 1,2,\ldots,m; j = 1,2,\ldots n)
\end{aligned} \tag{16}$$

where, p_i is the preemptive factor/priority level assigned to each relevant goal in rank order ($p_1 \geq p_2 \geq \ldots \geq p_n$) and w_i are non-negative constants representing the relative weights assigned within a priority level to the deviational variables, d_i^+, d_i^- for each ith corresponding goal, b_i. The x_{ij} represents the decision variables and a_{ij} represents the decision variable coefficients.

Based upon the generalized GP model in Eq. 16, here, the model in Eq. 15 is changed in goal programming form. To determine the goal (b_1, b_2), this model is first solved as single objective ones. However, $b_2 = 1$ is considered by default. The new model for total discount policy will become:

$$
\begin{aligned}
&\text{Min}: Z = p_1W_1d_1^- + p_2W_2d_2^- \\
&\text{s.t}: \\
&\sum_{j=1}^{T}\sum_{X_j=0}^{Q_j-1} P_j.X_j.f_{X_j}(x_j) + \sum_{j=1}^{T}\sum_{X_j=Q_j}^{+\infty} P_j.Q_j.f_{X_j}(x_j) \\
&-\sum_{j=1}^{T}\sum_{X_j=0}^{Q_{j-1}}\left(\hat{h}_{1j}(Q_j-X_j)+\hat{h}_{2j}(Q_j-X_j)^2\right)\frac{e^{-\lambda_j}\lambda_j^{X_j}}{X_j!} \\
&-\sum_{j=1}^{T}\sum_{X_j=Q_j+1}^{\infty}\left(\hat{\pi}_{1j}(X_j-Q_j)+\hat{\pi}_{2j}(X_j-Q_j)^2\right)\frac{e^{-\lambda_j}\lambda_j^{X_j}}{X_j!} \\
&-\sum_{j=1}^{T}K_jQ_j - \sum_{k=1}^{m}kAY_k - \sum_{j=1}^{T}\sum_{i=1}^{n}C_{ij}\lambda_{ij}W_{ij} + d_1^- - d_1^+ = b_1 \\
&\quad \frac{1}{T}\sum_{j=1}^{T}\sum_{X_j=0}^{Q_j-1}\frac{(Q_j-X_j)f_{X_j}(x_j)}{\lambda_j} + d_2^- - d_2^+ = b_2 \\
&\quad \sum_{j=1}^{T} f_jB_j \le F \\
&\quad Q_j = V_jB_j \quad \forall j, \quad j=1,2,\ldots,T \\
&\quad 0 \le W_{1_j} \le q_{1_j}\lambda_{1_j} \\
&\quad q_{1_j}\lambda_{2_j} \le W_{2_j} \le q_{2_j}\lambda_{2_j} \quad \forall j, \quad j=1,2,\ldots,T \\
&\quad \vdots \\
&\quad q_{n-1,j}\lambda_{n-1,j} < W_{n-1,j} \le q_{nj}\lambda_{n-1,j} \quad \forall j, \quad j=1,2,\ldots,T \\
&\quad q_{nj}\lambda_{nj} < W_{nj} \quad \forall j, \quad j=1,2,\ldots,T \\
&\quad 0 < \sum_{j=1}^{T} f_jB_j \le \hat{f}Y_1 \\
&\quad (k-1)\hat{f}Y_i < \sum_{j=1}^{T} f_jB_j \le k\hat{f}Y_k \quad \forall k=2,3,\cdots,m \\
&\quad \sum_{k=1}^{m} Y_k = 1, \; Y_k = 0,1 \;;\; \forall k=1,2,\ldots,m \\
&\quad \sum_{i=1}^{n} \lambda_{ij} = 1, \; \lambda_{i_j} = 0,1 \;;\; \forall j, \quad j=1,2,\cdots T \text{ and } \forall i, \quad i=1,2,\cdots,n \\
&\quad d_2^-.d_2^+ = 0, \\
&\quad d_1^-.d_1^+ = 0 \\
&\quad d_1^-, d_1^+, d_2^-, d_2^+ \ge 0,\; B_j \ge 0 \text{ and integer} \quad \forall j, \quad j=1,2,\cdots,T
\end{aligned}
\tag{17}
$$

Particle swarm optimization: Particle Swarm Optimization (PSO) was invented by Kennedy and Eberhart (2001) in the mid 1990s while attempting to simulate the choreographed, graceful motion of swarms of birds as part of a socio-cognitive study investigating the notion of collective intelligence in biological populations. In PSO, a set of randomly generated solutions (initial swarm) propagates in the design space towards the optimal solution over a number of iterations (moves) based on large amount of information about the design space that is assimilated and shared by all members of the swarm. PSO is inspired by the ability of flocks of birds, schools of fish and herds of animals to adapt to their environment, find rich sources of food and avoid predators by implementing an information sharing approaches (Kennedy and Eberhart, 2001). Examples of researches on PSO can be found by Sha and Hsu (2008), Lin and Tsai (2006), Yapicioglua *et al*. (2007), Guo *et al*. (2008), Rahimi-Vahed *et al*. (2007) and Taleizadeh *et al*. (2009).

Basic PSO algorithm: The basic PSO algorithm consists of three steps; generating particle's positions and velocities, velocity update and position update.

Initializing particles' positions and velocities: A particle refers to a point in the designed space that changes its position from one move (iteration) to another, based on velocity updates. Using the upper and the lower bounds on the design variables values, X_{min} and X_{max}, the positions, X_k^i and velocities, V_k^i, of the initial swarm of particles can be first generated randomly. As PSO is a population-based optimization algorithm, in which each particle is an individual and the swarm is composed of particles. The relationship between swarm and particles in PSO is similar to the relationship between population and chromosomes in Genetic Algorithm (GA) and we will denote the swarm size by N. The positions and velocities are given in a vector format where the superscript and subscript denoting the ith particle at time k. Rand is a uniformly distributed random variable that can take any value between 0 and 1. This initialization process allows the swarm particles to be generated randomly across the design space. Equations 18 and 19 are used to initialize particles, in which Δt is the constant time increment.

$$X_0^i = X_{min} + \text{Rand}(X_{max} - X_{min}) \tag{18}$$

$$V_0^i = \frac{X_{min} + \text{Rand}(X_{max} - X_{min})}{\Delta t} = \frac{\text{Position}}{\text{Time}} \tag{19}$$

The initialization is a very important process for the PSO algorithm to become convergent. Two common methodologies to generate the initial solution are: (1) generating feasible solutions, or (2) random generation. In this study to generate the initial solution we used the first methodology. In other words, since a solution vector, because of its velocity, may dissatisfy a constraint, we check the feasibility of a generated solution in each step. As a result, if a constraint is not satisfied by a solution vector, the related vector solution will be punished by a big penalty.

Updating the velocities: By using the fitness values which are functions of the particles current positions in the design space at time k, the velocities of all particles at time k + 1 are updated. The fitness function value of a particle not only determines which particle has the best global value in the current swarm, P_k^g, but also determines the best position of each particle over time, P^i, in the current and all previous moves. The velocity update formula uses these two pieces of information for each particle in the swarm along with the effect of current motion, V_k^i, to provide a search direction, V_{k+1}^i for the next iteration and to ensure good coverage of the design space and avoid entrapment in local optima.

The velocity update formula includes some random parameters, rand, represented by the uniformly distributed variables. The three values that affect the new search direction, namely, current motion, particle own memory and swarm influence, are incorporated via a summation approach as shown in Eq. 16 with three weight factors, namely, inertia factor, w, self confidence factor, C_1 and swarm confidence factor, C_2, respectively. The constraint in Eq. 21 formed by V_{max}, a maximum velocity, is specified to clamp the excessive accelerations.

$$\underbrace{V_{k+1}^i}_{\substack{\text{Velocity of particle}\\ \text{i at time k+1}}} = \overset{[0.4,1.4]}{w}\ \underbrace{V_k^i}_{\substack{\text{Current}\\ \text{motion}}} + \underbrace{\overset{[1.5,2]}{C_1}\ \text{Rand}\frac{(P^i - X_k^i)}{\Delta t}}_{\text{Particle memory influence}} + \underbrace{\overset{[2,2.5]}{C_2}\ \text{Rand}\frac{(P_k^g - X_k^i)}{\Delta t}}_{\text{Swarm influence}} \tag{20}$$

$$\begin{aligned} &\text{if} \quad (V_{k+1}^i > V_{max}); \quad V_{k+1}^i = V_{max} \\ &\text{if} \quad (V_{k+1}^i < -V_{max}); \quad V_{k+1}^i = -V_{max} \end{aligned} \tag{21}$$

The inertia weight w controls how much of the previous velocity should be retained from the earlier step. A larger inertia weight facilitates a global search, while a smaller inertia weight facilitates a local search. A balance can be achieved between global and local exploration to speed up search results using a dynamically adjustable inertia weight formulation. Introducing a linearly decreasing inertia weight into the original PSO significantly improves its performance through the parameter study of inertia weight (Shi and Eberhart, 1999; Naka *et al*., 2001). The linear distribution of the inertia weight is expressed as follows (Naka *et al*., 2001):

$$w = w_{max} - \frac{w_{max} - w_{min}}{k_{max}} \times k \tag{22}$$

where, w_{max} and w_{min} are the initial and final values of weighting coefficient, respectively and k_{max} and k are the maximum iteration number and iteration counter, respectively. The related results of the two parameters w_{max} = 0.9 and w_{min} = 0.4 were investigated by Shi and Eberhart (1999) and Naka *et al*. (2001).

Salman *et al*. (2003) used the values of 0.9, 2 and 2 for w, C_1 and C_2, respectively and suggested upper and lower bounds on these values shown in Eq. 16. Other combinations of the parameter values usually lead to much slower convergence or sometimes non-convergence at all. The tuning of the PSO algorithm weight factors is a topic that warrants proper investigation but is outside the scope of this study. Meanwhile, according to some suggestions in the literature (Shi and Eberhart, 1999; Naka *et al*., 2001) in this research we considered V_{max} = 10 and Eq. 18 for w. Also different values for C_1, C_2, and N are considered and all possible combinations of them are examined.

Updating the position: Position update is the final step involved in iteration and is done by using the current particle position and its own updated velocity vector shown in the Eq. 23.

$$X_{K+1}^i = X_K^i + V_{K+1}^i \Delta t \tag{23}$$

Fuzzy simulation: In order to estimate the uncertain costs of the fuzzy model, we employ a simulation technique. Denoting $\hat{h}_{ij}$ by $\hat{h}_{ij} = (\hat{h}_{i1}, \hat{h}_{i2}, \ldots, \hat{h}_{iT})$, μ as the membership function of $\hat{h}$ and μ_{ij} are the membership functions of $\hat{h}_{ij}$, we randomly generate $\hat{h}_{ijk}$ from the α-level sets of fuzzy variables $\hat{h}_{ij}$, i = 1,2,...,n, j = 1,2,..., as T and k = 1,2,...,K as $h_k = (h_{i1k}, h_{i2k}, \ldots, h_{iTk})$ and $\mu(h_{ijk}) = \mu_1(h_{i1k}) \wedge \mu_2(h_{i2k}) \wedge, \ldots, \mu_t(h_{iTk})$, where, α is a sufficiently small positive number. Also we do this denoting for $\hat{\pi}_j$ as same as $\hat{h}_{ij}$.

Based on the definition in appendix Eq. 3, the expected value of the fuzzy variable is:

$$E\left[Z(\hat{h},\hat{\pi},Q)\right] = \int_0^{+\infty} Cr\left\{Z(\hat{h},\hat{\pi},Q) \geq r\right\} dr - \int_{-\infty}^0 Cr\left\{Z(\hat{h},\hat{\pi},Q) \leq r\right\} dr \tag{24}$$

Then, provided N is sufficiently large, for any number $r \geq 0$, $Cr\{Z(\xi,Q) \geq r\}$ can be estimated by:

$$Cr\left\{Z(\hat{h},\hat{\pi},Q) \geq r\right\} = \frac{1}{2}\left(\underset{k=1,2,\ldots,N}{Max}\left\{\mu_k \middle| Z(\hat{h},\hat{\pi},Q) \geq r\right\} + 1 - \underset{k=1,2,\ldots,N}{Max}\left\{\mu_k \middle| Z(\hat{h},\hat{\pi},Q) < r\right\}\right) \tag{25}$$

And for any number $r < 0$, $Cr\{Z(\hat{h},\hat{\pi},Q) \leq r\}$ can be estimated by:

$$Cr\left\{Z(\hat{h},\hat{\pi},Q) \leq r\right\} = \frac{1}{2}\left(\underset{k=1,2,\ldots,N}{Max}\left\{\mu_k \middle| Z(\hat{h},\hat{\pi},Q) \leq r\right\} + 1 - \underset{k=1,2,\ldots,N}{Max}\left\{\mu_k \middle| Z(\hat{h},\hat{\pi},Q) > r\right\}\right) \tag{26}$$

Solution approach procedure: In summary, according to some definitions about PSO, GP and FS the procedure of hybrid solution approach will be:

Step 0 : Initializations.
Step 1 : Update the velocities according to Eq. 19

Step 2 : Update the positions according to Eq. 23

Step 3 : Set $E_{ij} = 0$, $G_j = 0$

Step 4 : Randomly generate h_{ijk} and π_{jk} from α-level sets of fuzzy variables $\hat{h}_{ij}, \hat{\pi}_j$ and set $h_k = (h_{i1k}, h_{i2k}, \cdots, h_{iTk}), \pi_k = (\pi_{1k}, \pi_{2k}, \cdots, \pi_{Tk})$

Step 5 : Set $a_{ij}^1 = h_{j1} \wedge h_{j2} \wedge \cdots \wedge h_{jk}$, $a_j^2 = \pi_{j1} \wedge \pi_{j2} \wedge \cdots \wedge \pi_{jk}$

$b_{ij}^1 = h_{j1} \vee h_{j2} \vee \cdots \vee h_{jk}$, $b_j^2 = \pi_{j1} \vee \pi_{j2} \vee \cdots \vee \pi_{jk}$

Step 6 : Randomly generate r_{j1}, r_{j2} from Uniform $[a_{j1}, b_{j2}]$, $[a_{j2}, b_{j2}]$, respectively

Step 7 : If $r_{j1} \geq 0$, then $E_{ij} \leftarrow E_{ij} + Cr\{\hat{h}_{ij} \geq r_{j1}\}$

Step 8 : If $r_{j1} < 0$, then $E_{ij} \leftarrow E_{ij} - Cr\{\hat{h}_{ij} \leq r_{j1}\}$

Step 9 : If $r_{j2} \geq 0$, then $G_j \leftarrow G_j + Cr\{\hat{\pi}_j \geq r_{j2}\}$

Step 10 : If $r_{j2} < 0$, then $G_j \leftarrow G_j - Cr\{\hat{\pi}_j \leq r_{j2}\}$

Step 11 : Repeat the 6 to 9 steps for N times

Step 12 : $E[\hat{h}_{ij}] = a_{ij}^1 \vee 0 + b_{ij}^1 \wedge 0 + E_{ij} \times \frac{b_{ij}^1 - a_{ij}^1}{N_{FS}}$

Step 13 : $E[\hat{\pi}_j] = a_j^2 \vee 0 + b_j^2 \wedge 0 + G_j \times \frac{b_j^2 - a_j^2}{N_{FS}}$

Step 14 : Evaluate fitness value according to objective function

Step 15 : Update P_k^g and P^i if necessary

Step 16 : Until a sufficient good criterion (usually a desirable fitness or a maximum number of iterations) is met

A NUMERICAL EXAMPLE

Consider a multi-product Newsboy problem with fifteen products and deterministic and fuzzy general data given in Table 1 and 2, respectively. The total available warehouse space is 1750 m^2; the fixed cost for each shipment is 500, $K_j = 0.1C_j$ in total discount. Moreover, $W_1 = W_2 = 1$, $p_1 = 1$, $p_2 = 10000$, $b_1 = 10349$ and $b_2 = 1$. Table 3 shows different values of the PSO parameters used to obtain the solution. In this research all the possible combinations of the PSO parameters (C_2, C_1 and N) are employed and using the max(max) criterion the best combination of the parameters has been selected. Table 4 shows the best results of the algorithm. The best combinations of the parameters of PSO algorithm are $C_1 = C_2 = 2$, N = 100. Also we in fuzzy simulation approach we used $\alpha = 0.9$, $N_{FS} = 10$.

Table 1: Deterministic general data

Products	1	2	3	4	5	6	7	8	9	10
V_j	5	10	5	3	10	6	6	3	1	1
λ_j	102	123	95	62	19	83	89	91	52	66
f_j	3	2	3	4	5	4	5	6	4	1
C_{1j}	18	30	20	10	35	25	30	28	100	100
C_{2j}	15	25	18	8	30	21	27	25	90	90
C_{3j}	12	23	15	7	28	18	24	22	80	80
C_{4j}	10	18	13	5	20	16	21	20	60	75
q_{1j}	30	40	30	15	40	15	40	40	20	30
q_{2j}	90	70	90	30	70	35	70	70	40	45
q_{3j}	100	110	100	50	140	70	100	100	60	70

Table 2: Fuzzy general data

Products	h_{1j}	h_{2j}	π_{1j}	π_{2j}
1	[0.5,1,1.5]	[1.5,2,2.5]	[5,7,9]	[10,12,14]
2	[3,4,5]	[4,5,6]	[25,30,35]	[30,35,40]
3	[2,4,6]	[3,5,7]	[28,30,32]	[40,45,50]
4	[1,2,3]	[2,3,4]	[25,30,35]	[30,35,40]
5	[4,5,6]	[5,6,7]	[40,45,50]	[45,50,55]
6	[1,2,3]	[2,3,4]	[20,21,22]	[24,26,28]
7	[4,5,6]	[5,6,7]	[32,34,36]	[35,40,45]
8	[3,5,7]	[4,6,8]	[10,15,20]	[15,20,25]
9	[1,3,5]	[3,4,5]	[8,10,12]	[13,15,17]
10	[1,2,3]	[2,3,4]	[40,45,50]	[50,52,54]

Table 3: The parameters of the PSO algorithm

C_2	C_1	N
1.5	1.50	10
2.0	1.75	50
2.5	2.00	100

Table 4: The best result

Product (j)	1	2	3	4	5	6	7	8	9	10
Q_j	90	70	90	57	30	90	72	81	40	59
Z					2635					
Z_{SR}					0.8126					

CONCLUSION AND RECOMMENDATIONS FOR FUTURE RESEARCH

In this study, a multi-product Newsboy model with total discount policy in which the batch orders and warehouse space are constraints, was developed. Then, the meta-heuristic solution algorithm of FS + GP + PSO has been proposed to solve the obtained non-linear integer model. At the end, a numerical example was given to demonstrate the application of the proposed method. Some of the future study of this research are:

- The holding and shortage costs may be considered to occur during a period
- Emergency order can be deployed to the model to refuse the shortage

APPENDIX

Some definitions in fuzzy and rough environments: At first let us to present some definitions in fuzzy environment that will use to modeling the problem and after that we will present the definitions needed in rough environment. We adopt the concepts of the credibility, possibility and necessity theory, as well as credibility of fuzzy event and the expected value of a fuzzy variable.

Definition 1: A Fuzzy number is of LR-type, if there exist reference functions L (for the left), R (for the right) and scalars $\alpha>0$, $\beta>0$ with:

$$\mu(\tilde{\xi}) = \begin{cases} 1 & \tilde{\xi} \in [m,n] \\ L(\frac{m-\tilde{\xi}}{\alpha}) & \tilde{\xi} \le m \\ R(\frac{\tilde{\xi}-n}{\beta}) & \tilde{\xi} \ge n \end{cases} \quad (1)$$

and $\tilde{\xi}$ is denoted by $\tilde{\xi} = (m,n,\alpha,\beta)_{L-R}$. The triangular and trapezoidal fuzzy variable are specific kind of LR-Type.

Definition 2: Let $\tilde{\xi}$ be a fuzzy number with the membership function $\mu(\tilde{\xi})$. Then the possibility, necessity and credibility measure of the fuzzy event $\tilde{\xi} \ge r$ can be represented, respectively, by:

$$\text{Pos}\{\tilde{\xi} > r\} = \sup_{\tilde{\xi} \ge r} \mu(\tilde{\xi}) \quad (2)$$

$$\text{Nec}\{\tilde{\xi} \ge r\} = 1 - \sup_{\tilde{\xi} < r} \mu(\tilde{\xi}) \quad (3)$$

$$\text{Cr}\{\tilde{\xi} \ge r\} = \frac{1}{2}\left[\text{Pos}\{\tilde{\xi} \ge r\} + \text{Nec}\{\tilde{\xi} \ge r\}\right] \quad (4)$$

Definition 3: The expected value of a fuzzy variable $\tilde{\xi}$ is defined as:

$$E\left[\tilde{\xi}\right] = \int_0^{\infty} \text{Cr}\{\tilde{\xi} \ge r\}dr - \int_{-\infty}^{0} \text{Cr}\{\tilde{\xi} \le r\}dr \quad (5)$$

the expected value of a triangular fuzzy variable $\tilde{\xi} = (\xi_1, \xi_2, \xi_3)$ is:

$$E[\tilde{\xi}] = \frac{1}{4}(\xi_1 + 2\xi_2 + \xi_3) \quad (6)$$

Definition 4: Let $\tilde{\xi}$ be a fuzzy variable. Then the optimistic function of α is defined as:

$$\tilde{\xi}_{\sup}(\alpha) = \sup\left[r \middle| \text{Cr}\{\tilde{\xi} \ge r\} \ge \alpha\right], \quad \alpha \in (0,1] \quad (7)$$

Definition 5: Assume $C_1C_2. . .C_k$, are real constants and $G_1(\tilde{\xi}), G_2(\tilde{\xi}), \cdots G_k(\tilde{\xi})$ are functions of fuzzy variable, then:

$$E\left[\sum_{k=1}^{K} C_k G_k(\tilde{\xi})\right] = \sum_{k=1}^{K} C_k E(G_k(\tilde{\xi})) \quad (8)$$

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