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H_∞ Controller for Consensus of Swarm Agents with Complete and Incomplete Communication Graphs

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Abstract: This study mainly focuses on design method for H_∞ controllers achieving consensus among the autonomous agents within swarm. A robust controller is designed for the entire closed-loop system to achieve consensus among the agents, while the non-consensus part is Lyapunov stable based on the robust disturbance rejection. Two cases are considered. The first one is that the agents in the swarm can interact with every other agent when each agent can receive information from every other. The other one is that the topology structure composed by the agents is fixed when each agent in the swarm can only exchange information with some agents but not all other agents. Simulation results demonstrate that designed controller for the system can make the closed-loop system reach consensus with non-consensus part be Lyapunov stable meeting the H_∞ performance for disturbances attenuation.

Key words: Swarms, consensus theory, robust control, state feedback, distributed control

INTRODUCTION

Recent years have seen the emergence of consensus control of swarm agents as a topic of significant interest to the controls community. Such swarm systems have appeared in many applications including mobile vehicles, formation flight of Unmanned Air Vehicles (UAVs), clusters of satellites and automated highway systems.

Consensus problem has attracted research in physics, mathematics and computers. In fact, the consensus phenomena in nature including schooling fish, flocking birds and herds, have motivated the researchers (Hanspeter and Charlotte, 2003; Simon *et al*., 2004; Couzin *et al*., 2002; Cucker and Smale, 2007; Okubo, 1986; Couzin and Franks, 2003; Inada, 2001). The earliest computer model of flocks is set up (Reynold, 1987). Reynolds (1987) proposes the famous boid model. Individuals in the swarm interact with each other based on local information and follow three rules: collision avoidance, velocity matching and flock centering. A special version of the model introduced by Reynolds (1987) is the Vicsek model proposed by Vicsek *et al*. (1995). Some very interesting simulation results by Vicsek *et al*. (1995) show that all agents eventually move in the same direction based on local information without any central control or leaders. Flocking behaviors have been analyzed in detail by Jadbabaie *et al*. (2003), Saber and Murray (2003, 2004), Moreau (2005), Ren and Beard (2005) and Saber *et al*. (2007). A theoretical explanation for Vicsek model is presented by Jadbabaie *et al*. (2003). Moreover, convergence results for case of leader following are also provided. Consensus problems for networks of dynamic, autonomous agents with fixed and switching topologies are discussed by Saber and Murray (2003, 2004). A theoretical framework for design and analysis of distributed flocking algorithms is presented by Saber (2000) in the view of control engineering. Stability analysis of swarm agents are considered mainly by Moreau (2005), Saber (2006) and Fax and Murray (2004).

Some consensus algorithms with noises are analyzed based on stochastic matrices and graph theory to verify that under some act of control the consensus will be achieved (Castro and Paganini, 2004). A decentralized state feedback control law that guarantees consensus for the closed-loop system is designed by Castro and Paganini (2004). Castro and Paganini (2004) propose a new way, that is think globally and act locally, to analyze swarm system by linear matrix inequality theory. The convex synthesis

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of controllers for consensus is developed and the conditions for the existence of a controller which make the closed-loop system achieves consensus with optimal H_2 performance on the non-consensus part of the system.

It is very clear that there is always disturbances and noises in swarm systems, especially in practical engineering systems. With the disturbances the system will be not stable any more. So, the controller for the system in case of no noise will do not work again. It is very important and necessary to design a robust controller to make the states of swarm agents reach consensus with some performance under various of external and internal disturbances. This study mainly focuses on designing a controller for the system so as to make the closed-loop system reach consensus with non-consensus part be Lyapunov stable meeting the H_∞ performance for disturbances attenuation. Two cases are considered. The first one is that the agents in the swarm can interact with every other agent when each agent can receive information from every other. The other one is that the topology structure composed by the agents is fixed when each agent in the swarm can only exchange information with some agents but not all other agents. Since, the structure of communication among the swarm are usually restricted, it is very necessary to design H_∞ controller for the closed-loop system with certain fixed topology structure.

PRELIMINARIES AND BACKGROUND

Model description: We consider the swarm system composed of N interconnected agents, where each agent has the following dynamics:

$$\dot{x}_i = A_{ii}x_i + \sum_{j\neq i} A_{ij}x_j + B_{1i}u_i + B_{2i}w_i, i \in \{1, 2, \cdots, N\}$$

where, x_i represents the ith individual's state variable in the swarm, which is assumed to be p dimensions, that is, $x_i \in R^p$. And x_j is the state of the jth individual, which is the neighbor of the ith individual. That the individual j is the neighbor of individual i means that i can sense i and communicate with it. $\mu_i \in R^q$ is control input of the individual i. $\omega_i \in R^r$ represents the disturbances or noises into I. $A_{ii} \in R^{p\times p}$, $A_{ij} \in R^{p\times p}$, $B_{1i} \in R^{p\times q}$, $B_{2i} \in R^{p\times r}$ are the corresponding coefficient matrices and A_{ij} shows the relationship between i and j, meaning there exists information exchange between neighbors. If A_{ij} is equal to zero, it means that i has no contact with j.

Let's consider this swarm system globally, we can get the state space equation:

where,

$$\begin{aligned}
&\dot{x} = Ax + B_1u + B_2\omega \\
&x = [x_1 \;\; x_2 \;\; \ldots \;\; x_N]^T \in R^{Np} \\
&u = [u_1 \;\; u_2 \;\; \ldots \;\; u_N]^T \in R^{Np} \\
&B_1 = \begin{bmatrix} B_{11} & & \\ & \ddots & \\ & & B_{1N} \end{bmatrix} \in R^{Np\times Nq} \\
&B_{21} = \begin{bmatrix} B_{21} & & \\ & \ddots & \\ & & B_{2N} \end{bmatrix} \in R^{Np\times Nr} \\
&A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix} \in R^{Np\times Np}
\end{aligned} \quad (1)$$

Consensus: Now we give the explanation and definition to consensus or agreement of dynamic systems. We consider the dynamic system composed of N agents whose dynamics are same as:

$$\dot{x}_i = A_{ii}x_i + \sum_{j\neq i} A_{ij}x_j, \qquad i \in \{1, 2, \cdots, N\} \quad (2)$$

The Eq. 2 can be rewritten as global equation:

$$\dot{x} = Ax \quad (3)$$

Definition 1: Let β_i, i = 1,..., Np be an orthonormal basis in R^{Np}. The Eq. 3 achieves consensus to the subspace:

$$S = \mathrm{span}\{\beta_i, i = 1,\ldots,Np\}$$

If, S is a minimal set such that for any initial condition, the state x(t) converges to a point in S. The definition implies that every point α' in S is marginally stable equilibrium point of the system Eq. 3, i.e., $A\alpha' = 0$. α' can be normalized as α.

Now an example is used to explain Definition 1. Let us consider the dynamic system made up of Vicsek model, in which each agent moves at the same constant velocity but has different movement direction. Finally all of N agents in the system have the same direction, that is, the states of the system reach consensus. Then from (Jadbabaie *et al*., 2003) we can get that states of the system converges to the space S. Let α' be one point in S, α' can be normalized as:

$$\alpha = \frac{1}{\sqrt{N}}[1 \;\; 1 \;\; \cdots \;\; 1]^T$$

In order to analyze the problem for its convenience, we decompose the system states into two parts. One is consensus part and the other is non-consensus part. The following analysis shows that the non-consensus part of states eventually converges to 0 and the states converge to consensus. We assume that Eq. 3 converge to the space S. Let $\alpha_\perp$ be orthonormal complement of α, $\alpha_\perp^T$ be the transpose of α. Then we can get:

$$\alpha_\perp^T,\ \alpha_\perp = I,\ \alpha_\perp^T\alpha = 0$$

We decompose the system states as follows :

$$x = \begin{bmatrix}\alpha_\perp & \alpha\end{bmatrix}\begin{bmatrix}\theta \\ \delta\end{bmatrix}$$

where, $\delta \epsilon R^n$, $\theta \epsilon R^{Np-n}$, represent consensus and non-consensus part of Eq. 3, respectively, we rewrite Eq. 3 as follows:

$$\dot{x} = [\alpha_\perp \quad \alpha]\begin{bmatrix}\dot{\theta} \\ \dot{\delta}\end{bmatrix} = A[\alpha_\perp \quad \alpha]\begin{bmatrix}\theta \\ \delta\end{bmatrix}$$

Because $[\alpha_\perp \quad \alpha]$ is unitary matrix, Eq. 3 is equivalent to this equation:

$$\begin{bmatrix}\dot{\theta} \\ \dot{\delta}\end{bmatrix} = [\alpha_\perp \quad \alpha]^{-1} A[\alpha_\perp \quad \alpha]\begin{bmatrix}\theta \\ \delta\end{bmatrix}$$
$$= \begin{bmatrix}\alpha_\perp^T \\ \alpha^T\end{bmatrix} A[\alpha_\perp \quad \alpha]\begin{bmatrix}\theta \\ \delta\end{bmatrix} = \begin{bmatrix}\alpha_\perp^T A\alpha_\perp\theta + \alpha_\perp^T A\alpha\delta \\ \alpha^T A\alpha_\perp\theta + \alpha^T A\alpha\delta\end{bmatrix}$$

When the states reach consensus, i.e., $A\alpha = 0$:

$$\begin{bmatrix}\dot{\theta} \\ \dot{\delta}\end{bmatrix} = \begin{bmatrix}\alpha_\perp^T A\alpha_\perp\theta \\ \alpha^T A\alpha_\perp\theta\end{bmatrix} = \begin{bmatrix}\alpha_\perp^T A\alpha_\perp & 0 \\ \alpha^T A\alpha_\perp & 0\end{bmatrix}\begin{bmatrix}\theta \\ \delta\end{bmatrix} \quad (4)$$

eigenvalues of Eq. 4 are composed of 0 and eigenvalues of matrix $\alpha_\perp^T A\alpha_\perp$, then that states of Eq. 3 achieve consensus is equivalent to the following conditions:

- $\alpha_\perp^T A\alpha_\perp$ is negative definite matrix
- $A\alpha = 0$

Then Theorem 1 is shown (Castro and Paganini, 2004).

Theorem 1: The states of Eq. 3 converges to consensus in S by Definition 1, if and only if:

- $A\alpha = 0$ (5)
- There exists X>0 such that:

$$\alpha_\perp^T A\alpha_\perp X + X(\alpha_\perp^T A\alpha_\perp)^T < 0 \quad (6)$$

State feedback H_∞ control: It is assumed that states of Eq. 1 can be measured directly, the state feedback controller can be:

$$u = Kx \quad (7)$$

With the controller the closed-loop system is:

$$\dot{x} = (A + B_1K)x + B_2\omega$$
$$z = (C_1 + D_{12}K)x$$

If this closed-loop is internal stable and closed-loop transfer function $T_{wz}(s)$ satisfies:

$$\|T_{wz}(S)\|_\infty = \|(C_1+D_{12}K)[sI-(A+B_2K)]^{-1}B_1\|_\infty < \gamma$$

then, it is said that the control law of Eq. 7 is a state-feedback γ-optimal H_∞ controller of Eq. 1, where, $z = (C_1 = D_{12}K)x + D_{11}\omega$ is regulated output.

Theorem 2: For Eq. 1, given a $\gamma>0$, there exists a state-feedback controller u = Kx satisfying system performance specific $\|T_{wz}(s)\|_\infty<\gamma$, if and only if there exists positive definite matrices X, P and matrix W, such that:

$$\begin{bmatrix} AX + B_1W + (AX + B_1W)^T & B_2 & (C_1X + D_{12}W)^T \\ B_2^T & -I & D_{11}^T \\ C_1X + D_{12}W & D_{11} & -\gamma^2 I \end{bmatrix} < 0$$

If, this inequalities has feasible solutions X*, W*, then $u = W^*(X^*)^{-1}x$ is a state-feedback γ-optimal H_∞ control law of Eq. 1. Theorem 2 is extracted from Dullerud and Paganini (2000).

MAIN RESULTS

Here, some conditions are given to design H_∞ controller for consensus of swarm agents with complete and incomplete communication graphs.

H_∞ controller for consensus without communication restriction: The closed-loop system is:

$$\dot{x} = (A + B_1K)x + B_2\omega$$
$$z = (C_1 + D_{12}K)x \quad (8)$$

When the states are decomposed as:

$$x = [\alpha_\perp \quad \alpha]\begin{bmatrix}\theta \\ \delta\end{bmatrix}$$

the closed-loop Eq. 8 is equivalent to:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} \alpha_\perp^T(A+B_1K)\alpha_\perp\theta \\ \alpha^T(A+B_1K)\alpha_\perp\theta \end{bmatrix} = \begin{bmatrix} \alpha_\perp^T \\ \alpha^T \end{bmatrix} B_2\omega$$

$$z = (C_1 + D_{12}K)[\alpha_\perp \quad \alpha]\begin{bmatrix} \theta \\ \delta \end{bmatrix}$$

Now let us analyze consensus for Eq. 8. According to Definition 1, when the Eq. 8 achieve consensus in S, the system is marginally stable and the meaningful definition for performance variable is with respect to the stable part of the state x, namely:

$$\dot{\theta} = \alpha_\perp^T(A+B_1K)\alpha_\perp\theta + \alpha_\perp^T B_2\omega$$

The meaningful definition for performance variable is:

$$z_\theta = (C_1 + D_{12}K)\alpha_\perp\theta$$

According to Theorem 1-3 can be concluded. Theorem 3 guarantees the existence of a robust controller that achieves consensus under persistent disturbances, provided a complete communication graph.

Theorem 3: Given $\gamma>0$, there exists a control law $u = Kx$, such that the states of the Eq. 8 reach consensus in S with the transfer function T_{zw} from the noises ω in the non-consensus part to performance variable $z_\theta = (C_1+D_{12}K)\alpha_\perp\theta$ satisfying $\|T_{wz}(s)\|_\infty < \gamma$, if and only if there exists X>0, W such that:

$$Ax\alpha + B_1W\alpha = 0 \tag{9}$$

$$\begin{bmatrix} \alpha_\perp^T(AX + B_1W + XA^T + W^TB_1^T)\alpha_\perp & \alpha_\perp^T B_2 & (C_1X\alpha_\perp + D_{12}W\alpha_\perp)^T \\ (\alpha_\perp^T B_2)^T & -I & 0 \\ C_1X\alpha_\perp + D_{12}W\alpha_\perp & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{10}$$

$$X = \alpha_\perp\alpha_\perp^T X\alpha_\perp\alpha_\perp + \alpha\alpha^T X\alpha\alpha^T \tag{11}$$

The proof of Theorem 3 is as follows:

Necessity: According to Theorem 1 and 2, if states of swarm system reach consensus in S, the following inequalities can be got:

$$(A+B_1K)\,\alpha = 0 \tag{12}$$

$$\alpha_\perp^T(A+B_1K)\alpha_\perp P + P(\alpha_\perp^T(A+B_1K)\alpha_\perp)^T < 0 \tag{13}$$

$$\begin{bmatrix} \alpha_\perp^T(A+B_1K)\alpha_\perp P + (\alpha_\perp^T(A+B_1K)\alpha_\perp P)^T & \alpha_\perp^T B_2 & ((C_1+D_{12}K)\alpha_\perp P)^T \\ (\alpha_\perp^T B_2)^T & -I & 0 \\ (C_1+D_{12}K)\alpha_\perp P & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{14}$$

For the Eq. 8, 12-14 can be got directly corresponding from Theorem 1 and 2. We note that Eq. 13 and 14 can be united into Eq. 14 Let:

$$\alpha_\perp P = X\alpha_\perp,\ W = KX$$

and satisfying X>0 with the equality:

$$X = \alpha_\perp\alpha_\perp^T X\alpha_\perp\alpha_\perp^T + \alpha\alpha^T X\alpha\alpha^T \tag{15}$$

Then, Eq. 13 can be rewritten:

$$\alpha_\perp^T(AX + B_1W + XA^T + W^TB_1^T)]\alpha_\perp < 0 \tag{16}$$

Because $K = WX^{-1}$, the left side of Eq. 12:

$$(A+B_1K)\alpha = (A+B_1WX^{-1})\alpha$$

Since, $X = \alpha_\perp\alpha_\perp^T X\alpha_\perp\alpha_\perp^T + \alpha\alpha^T X\alpha\alpha^T$
then $X^{-1} = \alpha_\perp(\alpha_\perp^T X\alpha_\perp)^{-1}\alpha_\perp^T + \alpha(\alpha^T X\alpha)^{-1}\alpha^T$, the left side of Eq. 12 can be written as:

$$(A + B_1WX^{-1})\alpha = A\alpha + B_1W\alpha(\alpha^T X\alpha)^{-1}$$

If, we multiply $\alpha^T X\alpha$ to the both sides of the equality, we can get:

$$Ax\alpha + B_1W\alpha = 0 \tag{17}$$

In Eq. 14:

$$\begin{aligned} &\alpha_\perp^T(A+B_1K)\alpha_\perp P + (\alpha_\perp^T(A+B_1K)\alpha_\perp P)^T \\ &= \alpha_\perp^T(AX + B_1W + XA^T + W^TB_1^T)]\alpha_\perp \\ &(C_1 + D_{12}K)\alpha_\perp P = C_1X\alpha_\perp + D_{12}W\alpha_\perp \end{aligned}$$

Then, Eq. 14 can be reformed as Eq. 18:

$$\begin{bmatrix} \alpha_\perp^T(AX + B_1W + XA^T + W^TB_1^T)]\alpha_\perp & \alpha_\perp^T B_2 & (C_1X\alpha_\perp + D_{12}W\alpha_\perp)^T \\ (\alpha_\perp^T B_2)^T & -I & 0 \\ C_1X\alpha_\perp + D_{12}W\alpha_\perp & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{18}$$

According to Eq. 15-18, we can conclude Eq. 9-11. Thus the proof of necessity of theorem is finished.

Sufficiency: If we have Eq. 12-14, then sufficiency of Theorem 3 can be proved based on Theorem 1 and 2. Because Eq. 12-14 satisfy:

$$\alpha_\perp P = X\alpha_\perp, W = KX \text{ and } X > 0$$
$$X = \alpha_\perp\alpha_\perp^T X\alpha_\perp\alpha_\perp^T + \alpha\alpha^T X\alpha\alpha^T$$

They are equivalent to Eq. 15-18, i.e., equivalent to Eq. 9-11. Thus the sufficiency is proved.

The proof of Theorem 3 is over.

The control law can be got from Eq. 9-11, if the feasible solutions are X*, W*, then:

$$u = W^* (X^*)^{-1} x$$

H_∞ controller for a given topology structure: In general, a solution obtained from Theorem 3 would have a full block state feedback matrix K, i.e., each agent can receive information from every other agent in the swarm. However, it is more practical to consider the swarm system with a certain communication structure. Now, we consider a network composed of three agents with the following structure. With the structure like Fig. 1, the first agent labeled one can receive information from itself and the third agent labeled three, the second agent labeled two can get information from the first agent and itself, the third agent can get information from itself and second agent.

According to Theorem 3, we can get the state feedback matrix K, if we consider the given structure like Fig. 1, an additional condition must be added, that is, K must satisfy the requirement:

$$K = \begin{bmatrix} K_{11} & 0 & K_{13} \\ K_{21} & K_{22} & 0 \\ 0 & K_{32} & K_{33} \end{bmatrix}, K_{ij} \in R^{q\times p} \qquad (19)$$

Now, to design the state feedback H_∞ consensus controller for given structure is to find a feasible solution for the conditions in the Theorem 3 such that K has the form Eq. 19. According to Theorem 3, $K = W^*(X^*)^{-1}$.

In order to simplify the problem, we consider matrix X with a special structure. Assuming that $X \in R^{3p\times 3p}$ be a block diagonal matrix diag (X_1, X_2, X_3), with each block $X_i \in R^{p\times p}$.

Then K will have the desired structure if and only if W has the desired structure for K. For the structure as Fig. 1 gives, if:

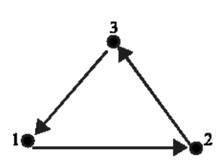

Fig. 1: Topology structure of 3 agents

$$W = \begin{bmatrix} W_{11} & 0 & W_{13} \\ W_{21} & W_{22} & 0 \\ 0 & W_{32} & W_{33} \end{bmatrix}$$

then,

$$K = WX^{-1} = \begin{bmatrix} W_{11} & 0 & W_{13} \\ W_{21} & W_{22} & 0 \\ 0 & W_{32} & W_{33} \end{bmatrix} \begin{bmatrix} X_1^{-1} & 0 & 0 \\ 0 & X_2^{-1} & 0 \\ 0 & 0 & X_3^{-1} \end{bmatrix}$$
$$= \begin{bmatrix} W_{11}X_1^{-1} & 0 & W_{13}X_3^{-1} \\ W_{21}X_1^{-1} & W_{22}X_2^{-1} & 0 \\ 0 & W_{32}X_2^{-1} & W_{33}X_3^{-1} \end{bmatrix}$$

K has the desired form, so we can get the required K by the feasible solutions W and X. For the general case, let $K = K_\chi$ denote that a matrix K belongs to the set of prescribed structure $\chi = \{K : K_{ij} = 0 \in R^{q\times p}, \quad \text{for}(i, j) \in I\}$. That is, some blocks indexed by some set I are zeros. Then, we can get Corollary 4 for the given topology structure. Corollary 4 guarantees the existence of a robust controller that achieves consensus under persistent disturbances and an incomplete communication graph.

Corollary 1 (Structured H_∞ control for consensus): Let $\chi = \{K : K_{ij} = 0 \in R^{q\times p}, \text{for } (i, j) \in I\}$ be a given structure. Given $\gamma>0$, there exists a state feedback law $u = K_\chi x$ that make Eq. 8 reach consensus in S with the transfer function T_{zw} from the noises ω in the non-consensus part to performance variable $z_\theta = (C_1 + D_{12} K)\alpha_\perp\theta$ satisfying $\|T_{zw}\|_\infty<\gamma$ if and only if there exists $W = W_\chi$ and $X_i>0$, $i = 1,\ldots,N$, such that:

$$Ax\alpha + B_1 W_\chi \alpha = 0 \qquad (20)$$

$$\begin{bmatrix} \alpha_\perp^T(AX + B_1W_\chi + XA^T + W_\chi{}^T B_1{}^T)]\alpha_\perp & \alpha_\perp^T B_2 & (C_1X\alpha_\perp + D_{12}W_\chi\alpha_\perp)^T \\ (\alpha_\perp^T B_2)^T & -I & 0 \\ C_1X\alpha_\perp + D_{12}W_\chi\alpha_\perp & 0 & -\gamma^2 I \end{bmatrix} < 0 \qquad (21)$$

$$\begin{aligned} \operatorname{diag}(X_1,\cdots,X_N) = {} & \alpha_\perp\alpha_\perp^T \operatorname{diag}(X_1,\cdots,X_N)\alpha_\perp\alpha_\perp^T \\ & + \alpha\alpha^T \operatorname{diag}(X_1,\cdots,X_N)\alpha\alpha^T \end{aligned} \qquad (22)$$

The control law can be got by:

$$K_\chi = W_\chi \operatorname{diag}(X_1^{-1},\cdots,X_N^{-1})$$

Proof: The proof is directly extracted by substituting the matrix W, X with W_χ and $\operatorname{diag}(X_1,\ldots,X_N)$ for $\chi = \{K : K_{ij} = 0 \in R^{q\times p}\}$, for $(i, j) \in I\}$ in the original conditions in Theorem 3.

NUMERICAL SIMULATIONS

Let's consider the swarm systems composed of 3 integrators each agent having the same dynamics:

$$\dot{x}_i = A_{ii}x_i + \sum_{j\neq i} A_{ij}x_j + B_{1i}u_i + B_{2i}\omega_i, i \in \{1,2,3\}$$

where, $A_{ii} = 0$, $B_{1i} = 1$, $B_{2i} = 1$, $p = 1$, $N = 3$, then

$$\dot{x} = Ax + B_1u + B_2\omega$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} + \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix}$$

So, $A = 0$, $B_1 = I$, $B_2 = I$.

The performance variable:

$$Z_\theta = (C_1 + D_{12}K)\alpha_\perp\theta$$

represents one part of the output when the limited-power noises are input into the system. The other part of the output is not considered here.

Since, $\alpha = (\sqrt{3})^{-1}\begin{bmatrix}1\\1\\1\end{bmatrix}$, then $\alpha_\perp = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}}\\ 0 & \frac{2}{\sqrt{6}}\\ -\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}}\end{bmatrix}$

Let $C_1 = \begin{bmatrix}\alpha_\perp^T\\ 0\end{bmatrix}$, $D_{12} = \begin{bmatrix}0\\ I\end{bmatrix}$, $D_{11} = 0$

When the agents can exchange information with every other agent, that is, the graph composed of the agents is complete connected Eq. 21. From Theorem 3, we solve the linear matrix inequalities Eq. 9-11, we can get the control law $u = W^*(X^*)^{-1}x$,

$$u = W^*(X^*)^{-1}x = \begin{bmatrix}-2.0451 & 1.0225 & 1.0226\\ 1.0225 & -2.0451 & 1.0225\\ 1.0226 & 1.0225 & -2.0451\end{bmatrix}x$$

$$\|T_{zw}\|_\infty < 1$$

Given that the initial states of the system is $x(0) = [1\ \ 3\ \ 2]^T$, the states are convergent to the same value which is depicted in Fig. 2 when the infinite norm of transfer function from disturbances to regulated output is $\|T_{zw}\|_\infty = 0.326<1$.

From Fig. 2, we can obviously see that the states of system achieve consensus under the control law designed based on Theorem 3. When the communication topology structure is given as Fig. 1, from Corollary 4, we solve the linear matrix inequalities Eq. 20-22, we can get the control law:

$$u = W_\chi^{**}(X^*)^{-1}x = \begin{bmatrix}-1.9379 & 0 & 1.9379\\ 1.9379 & -1.9379 & 0\\ 0 & 1.9379 & -1.9379\end{bmatrix}x$$

when $\|T_{zw}\|_\infty < \gamma = 1$. Given that the initial states of the system is $x(0) = [1\ \ 3\ 2]^T$, the states are convergent to the common value which is depicted in Fig. 3 when the infinite norm of transfer function from disturbances to regulated output $\|T_{zw}\|_\infty = 0.3010<1$.

Figure 3 has clearly shown that the states of system achieve consensus under the control law designed based on Corollary 1.

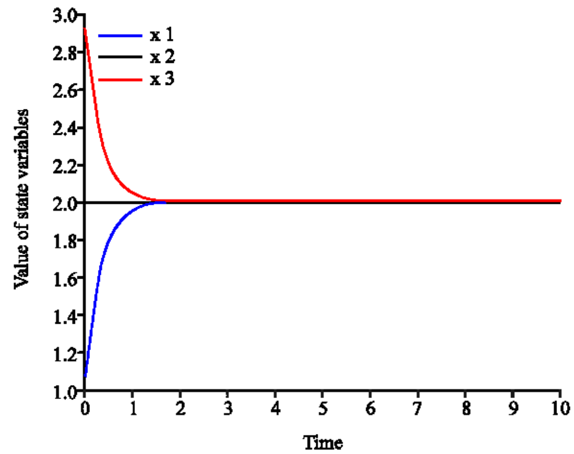

Fig. 2: The states of swarm agents for no restricted communication structure under the act of H_∞ controller

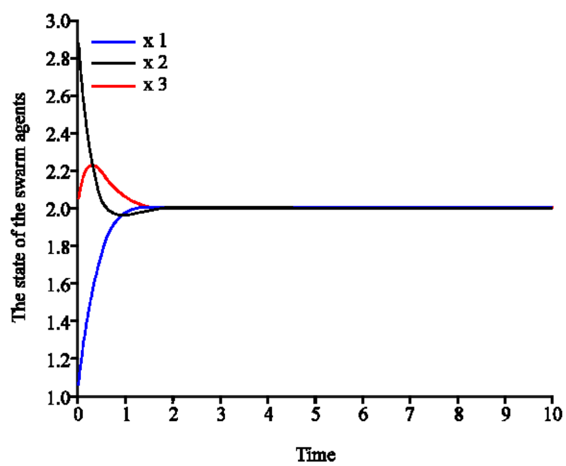

Fig. 3: The states of swarm agents for a given topology structure under the act of H_∞ controller

CONCLUSIONS

This study provides a theoretical analysis for design the H_∞ controller of consensus for two cases of no-restriction communication structure and some fixed topology structure. Theorem 3 and Corollary 1 are concluded. The controller is designed based on linear matrix inequalities theory. By the controller the closed-loop system can achieve consensus satisfying robust performance. We give a specific example to show the validity of our theory, the results show that the control law can make the states of the system reach consensus with the non-consensus part be Lyapunov stable meeting the H_∞ performance disturbances attenuation.

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