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## A Novel Discrete Multi-Tone Constellation Algorithm

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**Abstract:** In this study, a novel constellation algorithm for application in Discrete Multi Tone (DMT) modulation is presented. As the signal space diagram obtained is similar to snail, we have called the resulting modulation scheme as Snail Shift Keying (SSK), whose major property is its non-symmetric configuration where demands of subscribers can be prioritized conveniently. Quadrature Amplitude Modulation (QAM) and SSK, with the same power, have been applied to the DMT system with AWGN channels. Compared to a well-known QAM modulation scheme, based on the simulation results, the constellation algorithm presented has an improved Bit Error Rate (BER) performance with the same complexity in implementation.

**Key words:** Snail shift keying, QAM, ADSL, modulation

## INTRODUCTION

Asymmetric Digital Subscriber Line (ADSL) provides a high bit-rate downstream channel and a lower bit-rate upstream channel over twisted pair copper wires. The transmission method chosen for ADSL is based on DMT modulation (Acker et al., 2003). The available bandwidth of DMT is divided into parallel sub-channels or tones (Ysebeart et al., 2003). The incoming serial bitstream is divided into parallel streams, which are used to QAM modulation (Cioffi et al., 1999). After modulation with an Inverse Fourier Transform (IFFT), a cyclic prefix is added to each symbol (Kim and Lee, 2007). Rectangular QAM is used in the conventional DMT (Ghazi-Maghrebi et al., 2007). Rectangular QAM constellations are, in general, sub-optimal in the sense that they do not maximally space the constellation points for a given energy and less complexity in implementation than non-rectangular QAM (Panigrahi et al., 2006). In this research, a new nonrectangular QAM constellation algorithm with better performance than the rectangular QAM with the same complexity is introduced. In this pattern the points of M-ary constellation, with of, are selected based on the equation:

$$S(t) = A_i$$
,  $p(t) cos(2\pi f_i t + \theta_i) i = 0, 1, 2, ..., M-1 (1)$ 

where, p(t) is a unit energy pulse shaping function and  $f_c$  is the carrier frequency.  $A_i$  and  $\theta_i$  are, respectively, amplitude and phase of ith point of constellation algorithm, which are related by:

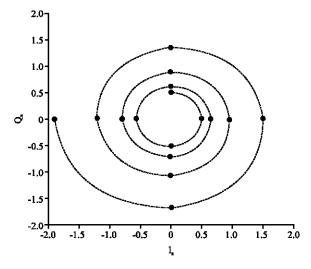


Fig. 1: The snail constellation points for M = 16

$$A_i = A_0 e^{\alpha \theta_i}$$
 and  $\theta_i = i \frac{\pi}{2}$  (2)

where,  $A_0$ = 0.054 and  $\alpha$  = 0.165 have been determined to achieve a better snail-like constellation. All the constellation points reside either on the horizontal or vertical axes. This means that the constellation points differ from each other by  $\pi/2$  in phase, but their amplitudes vary exponentially as shown in Fig. 1 in which  $I_n$  and  $Q_n$  are the amplitudes of in-phase and quadrature-phase of n-th symbol period, respectively. It means that in the receiver, we search only four phases and  $M=2^N$  amplitudes like QPSK and PAM, respectively, in which N

is the number of bits for each point and M is the number points of the constellation algorithm.

The mean,  $m_s$  and variance,  $\sigma_s^2$ , of the modulated signal, S(t), are respectively

$$m_{s} = E \left\{ s_{i}(t) \right\} = A_{0} E \{ e^{(j+a)\frac{\pi}{2}i} \} = \frac{A_{0}}{M} \cdot \frac{1 - e^{(j+a)\frac{\pi}{2}M}}{1 - e^{(j+a)\frac{\pi}{2}}} = \frac{A_{0}}{M} \cdot \frac{1 - e^{\frac{a^{\frac{\pi}{2}M}}{2}M} e^{\frac{j^{\frac{\pi}{2}M}}{2}}}{1 - je^{\frac{a^{\frac{\pi}{2}M}}{2}}}$$

$$\tag{3}$$

and

$$\sigma_{s}^{2} = E\{s_{i}^{2}\} - m_{s}^{2} = \frac{A_{0}^{2}}{M} \cdot \frac{1 - e^{(j+a)M\pi}}{1 - e^{(j+a)\pi}} - \frac{A_{0}^{2}}{M^{2}} \cdot \left(\frac{1 - e^{(j+a)M\pi/2}}{1 - e^{(j+a)\pi/2}}\right)^{2}$$
(4)

Also bandwidth of M-SSK is reduced by increasing the M as same as the total energy of signal increasing. To detect the received signal, an optimum receiver based on minimum distance can be utilized (Golden *et al.*, 2006). In this method if an unfiltered Gaussian noise with zero mean and power spectral density  $N_{\text{o}}/2$  is applied to the channel, then the average symbol error probability of  $p_{\text{s}}$  will be

$$p_{s} = \frac{2}{K} \sum_{k=0}^{K} Q \left( \sqrt{\frac{M \log_{2} M}{2}} \left( \frac{1 - e^{\alpha \pi}}{1 - e^{\alpha M \pi}} \right) \left( \frac{E_{b}}{N_{o}} \right) \Delta_{k_{i,j}}^{2} \right)$$
(5)

where,  $E_b$  is the energy per bit. K is the number of effective terms in computation of yhe distance between ith and jth points,  $\Delta^2_{k,j}$ . By selecting the constant  $\alpha$ , experiments show that for every  $E_b/N_0$ , the probability of error of SSK is lower than QAM. In other words for any specified BER, the required  $E_b/N_0$  in SSK is grater than the required  $E_b/N_0$  for QAM.

One of the major features of this modulation scheme, based on the Fig. 1, is that the distance between the constellation points is not constant, so that one can assign points to the data according to their preferred performance or BER. That is, the farthest points are assigned to the data with higher performance. Some of the main parameters of modulated signals are the average symbol power, peak-to-average power ratio (PAR) and the minimum Euclidean distance (Kou *et al.*, 2007). The average power is defined as:

$$P = \frac{1}{2^{M}} \sum_{n=1}^{2M} (I_n^2 + Q_n^2)$$
 (6)

The PAR, in dB, is calculated as a ratio of the peak and average powers given by:

$$PAR = 10\log_{10}\left(\frac{\max_{n}(P_{n})}{P}\right) + 3 \quad (dB)$$
 (7)

Table 1: Main parameters of two constellations

Parameters	QAM	SSK
P	1.11	1.11
PAR(dB)	5.557	8.097
η(dB)	-10.00	-25.67

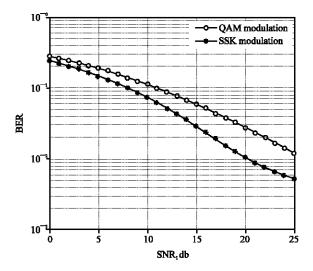


Fig. 2: Comparing SSK and QAM modulations in DMT system

where,  $P_n = I_n^2 + Q_n^2$  is the power in the *n*-th symbol. The Euclidian distance d(k, i) is the geometric distance between points k and i of the constellation diagram (Xiong, 2000). Taking minimum Euclidian distance 2-QAM,  $d_2$ , as a reference, the noise immunity of M-point constellations can be estimated in dB as:

$$\eta = 20\log_{10} \frac{d_{M}}{d_{2}} (dB)$$
(8)

where,  $d_M$  is minimum Euclidian distance of M-QAM. Equation 6-8 will be used to make comparison QAM and SSK. The QAM and SSK modulation schemes were applied to a DMT system. The simulation specifications are as shown in Table 1. As it is shown from the Table, their average powers are the same.

The channel has AWGN noise with zero mean and the channel impulse response length is equal to cyclic prefix length plus one.

Based on the results of simulation, the proposed SSK for different SNRs in an AWGN noisy channel has a BER better performance with respect to conventional QAM modulation in DMT system (Fig. 2). Furthermore, the proposed SSK modulation has a simple implementation and a low complexity as the same as QAM In other words we conclude that, SSK constellation like another non-rectangular QAM constellation is optimum and like rectangular QAM constellation can be easily, as the same complexity as, implemented.

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