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A Model of Non-Cooperative Dynamic Game to Conflict Resolution Among Common Natural Resource Operators

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Abstract: In this study, non-cooperative dynamic game to resolve conflict among common natural resources operators is represented. Bidestan aquifer, where two municipal and agricultural operators are simultaneously pumping common aquifer, is chosen as case study. Based on the cooperation among aquifer operators, aquifer operation is modeled by 3 scenarios (1) non-cooperative static game, (2) non-cooperative dynamic game and (3) cooperative game. Results show the benefits of cooperative model are more than non-cooperative models. Employing proposed dynamic game has led to 25% more extraction than static game as well.

Key words: Aquifer extraction, Nash equilibrium, tragedy of common, decision maker

INTRODUCTION

In common natural resources operation generally and in groundwater extraction particularly, there are so many evidences that have proved possibility of tragedy of commons (Clarke *et al.*, 1997). As the optimization models maximize one or more object to satisfy just one Decision Maker (DM), practical results arising from applying optimization models in competitive situations, have not met the expected success.

Game is called to the situation in which there is more than one DM and decisions of each DM affect the rest of DMs' payoff. Modern game theory may be said to have begun with the work of and Von Neumann and Morgenstern (1944). Next major development was John Nash's modification of the Von Neumann and Morgenstern's (1994) approach. Nash (1950) formally defined equilibrium of a non-cooperative game to be a profile of strategies, one for each player in the game.

Many researchers have applied non-cooperative game to conflict resolution among common natural operators. Transboundary fishery management (Cave, 1987; Fisher and Mirman, 1992), air pollution and environment protection (Breton *et al.*, 2006). Nakao *et al.* (2002), analyzed potential gains from cooperation in the withdrawal of water from the Hueco Bolson aquifer. Salazar *et al.* (2007) applied game theory to a multi-objective conflict problem for an aquifer in Mexico, where economic benefits from agricultural production should be balanced with associated negative environmental impacts. Rubio and Casino (2001) distinguished between cost and externalities by analyzing two equilibrium concepts, open

loop Nash and stationary Markov feedback in nonlinear strategies to characterize private extraction. Msangi (2006) studied asymmetric external effects on aquifer operators' behaviors. Coppola and Szidarovszky (2004) analyzed a two-person conflict, where a water company and a community are the players and water supply and health risk constitute the payoff functions.

When players interact by playing a static game in finite stages, the game is called a stage dynamic game. In this study, a non-cooperative dynamic game was developed to imply in conflict resolution among aquifer operators. In most of the mentioned study in groundwater extraction to reduce calculation content, it has been assumed that changes in the water level are transmitted instantaneously to all users. Most of the researches on conflict resolution among common natural resources operators have applied static game theory, but it seems that dynamic games are more compatible with the fact. In this study to simulate the exact effects of DMs' decisions on groundwater table fluctuation, the well-known Tiess Well equation was employed.

The proposed model is developed based on the earlier studies of Mangasarian and Stone (1964) and Bellman (1957). Solution to static games was represented by Mangasarian and Stone (1964). Bellman (1957) represented dynamic programming for change complex equation with many variables to many equations with little variable (Denardo, 2003).

In this study, based on the amount of cooperation among the operators, common natural resources operation is modeled in 3 scenarios: (1) non-cooperative static games, (2) non-cooperative dynamic games and

(3) cooperative games. Proposed scenarios are applied to conflict resolution among common aquifer operators in Bidestan area in Iran. In this area, two municipal and agricultural operators are simultaneously pumping common aquifer to provide potable water and irrigate wheat fields, respectively.

NON-COOPERATIVE STATIC GAME

Game theory is a mathematical method for analyzing strategic interaction. This theory would be useful when two or more DMs with conflicting objects try to decide on a common goal.

Here, we briefly review the equilibrium solution to the non-cooperative static game. Let I and II denote two players and $M_I = \{1...m_I\}$ and $M_{II} = \{1...m_{II}\}$ be the sets of all pure strategies available, $S_I = \{x \in R_+^{m_I} | e^T x = 1\}$ and $S_{II} = \{y \in R_+^{m_{II}} | e^T y = 1\}$ are strategy spaces and x and y are mixed strategies of players I and II, respectively, e is the unit vector and T denotes the transposition of the vector. When player I chooses a pure strategy $i \in M_I$ and player II chooses a pure strategy $j \in M_{II}$, the payoffs of players I and II are p^I and p^{II} , respectively. The bi-matrix game is defined by $BG = \{S_I, S_{II}, A, B\}$, where, A and B are the players I and II's payoff matrices, respectively.

Definition (equilibrium solution): $(x^*, y^*) \in S_I \times S_{II}$ is said to be a Nash equilibrium strategy of bi-matrix game BG if $x^T A y^* < x^{*T} A y^*, \forall x \in S_I$ and $x^{*T} B y < x^{*T} B y^*, \forall y \in S_{II}$.

All the mixed strategies of player i that satisfy the relation above are called the best response of player i to the opponent. Therefore, the mixed strategy equilibrium point is a vector in which each player takes action the best response to his/her opponents. No player intends to change the strategy in case that all players play on this point and the vector above would be the equilibrium point. Based on the Nash Existence Theorem every bi-matrix game has at least one equilibrium solution (Owen, 1995), which will be found by following theorem:

Theorem (Mangasarian and Stone, 1964): A necessary and sufficient condition that (x^*, y^*) be an equilibrium solution of BG is that it is a solution of the following quadratic programming problem:

$$\begin{aligned} \max \quad & x^T (A+B)y - \alpha - \beta \\ \text{s.t.} \quad & Ay \leq \alpha e, \quad B^T x \leq \beta e, \quad x \in S^m, y \in S^n, \quad \alpha, \beta \in R \end{aligned} \tag{1}$$

Further, if $(x^*, y^*, \alpha^*, \beta^*)$ is a solution to the problem above, thus:

$$\beta^* = x^{*T} B y^* \quad \text{and} \quad x^{*T} (A+B)y^* - \alpha^* - \beta^* = 0 \tag{2}$$

DYNAMIC GAME AND SOLUTION

When players interact by playing a static game in finite stages, the game is called dynamic game. In every stage of such a game, players simultaneously move knowing the moves in the earlier stages.

Development of dynamic game: Let us consider non-cooperative dynamic game for two players. The decisions of players I and II at stage t are expressed as $D_t^I \in M_I$ and $D_t^{II} \in M_{II}$. Assume that the initial system state at stage t be R_t and the maximum and minimum possible values of it be R^{max} and R^{min} , respectively. The players' payoffs at stage t, denoted by p_t^I and p_t^{II} for players I and II, respectively, depend on the current system state R_t , player I and player II's decisions, so that: $p_t^I = f(R_t, D_t^I, D_t^{II})$ and $p_t^{II} = g(R_t, D_t^I, D_t^{II})$ where f and g are desired utility functions for players I and II, respectively. Each player chooses optimal policies to maximize his/her utilities all over the stages with regard to his/her opponent's probable action. Players' optimal decision at stage t will be a decision that leads to the maximum payoff to the end of the planning horizon for him/her. In other words, at each stage both Eq. 3 and 4, should be satisfied simultaneously:

$$U_t^I (R_t, D_t^I, D_t^{II}) = \max_{D_t^I} \{ p_t^I + U_{t+1}^I (R_{t+1}, D_{t+1}^I, D_{t+1}^{II}) \} \tag{3}$$

$$U_t^{II} (R_t, D_t^I, D_t^{II}) = \max_{D_t^{II}} \{ p_t^{II} + U_{t+1}^{II} (R_{t+1}, D_{t+1}^I, D_{t+1}^{II}) \} \tag{4}$$

where, U_t^I and U_t^{II} are the maximum expected cumulative payoffs for players I and II from stage t to the end of the planning horizon, respectively. D_t^I and D_t^{II} are the optimal decisions of players I and II, respectively at stage t as well. Attention to the right hand side of equations above shows that players' payoffs consist of two components: the first one is the current payoff and the second is the future payoff.

Solution to the dynamic game: To solve the stage dynamic game, the combination of dynamic programming and solution to the static games are employed. Dynamic programming is a theory extensively adopted by Bellman (1957). Programming starts with the final stage $t = T_f$. For all players' decisions, $D_{T_f}^i \in M_i, i = I \text{ and } II$, the players' payoff matrices are created. By solving each of static game by Eq. 1, the optimal decision of players I and II, $D_{T_f}^I$ and $D_{T_f}^{II}$, respectively, are obtained for each possible system state. At stage $t = T_f - 1$ for all players' decisions,

the current players' payoffs are obtained. Since, the system state at the end of stage $t = T_f - 1$ is equal to the beginning of the next stage $t = T_f$ and players' optimal decisions at stage $t = T_f$ have been defined previously, so, by adding the current and future payoffs, the cumulative payoffs are created at stage $t = T_f - 1$. By solving this game, the players make a decision which leads to the maximum payoff from stage $t = T_f - 1$ to the final stage. The mentioned process continues till it reaches the first stage $t = 1$.

SCENARIOS OF AQUIFER OPERATION

Scenario 1: Non-cooperative static game: In this scenario, it is assumed that the players monthly decision is defined by myopic policy. In other words players are not long sighted and their monthly decision is just to reach monthly compromise. Therefore during the operation stages, players should solve T_f independent static games. If P_t^i denotes the payoff of player i at stage t , Eq. 5 shows the interaction of operators with each other:

$$\begin{aligned}
 & \text{for } t = 1: T_f \\
 & \quad \text{for } i = I:II \\
 & \quad \quad \max_{D_t^i \in M_t} p_t^i \\
 & \quad \text{end} \\
 & \text{end}
 \end{aligned} \tag{5}$$

Scenario 2: Non-cooperative dynamic game: In this scenario the goal of each operator is getting the maximum and possible benefit at the whole stages of operation and at the same time watching the probable moves of the opponent.

Scenario 3: Cooperative game: To better evaluation of before conflict resolution scenarios, another scenario with the aim of optimization of aquifer operation was prepared. The objective function of this model is maximizing the total extraction of players during the whole stages of the game. In other word, it is supposed that instead of two DMs, one DM is the owner of the wells and try to maximize the whole extraction of them:

$$\max_{(D_t^I \in M_t) \text{ and } (D_t^{II} \in M_t)} \left(\sum_{t=1}^{T_f} (P_t^I + P_t^{II}) \right) \tag{6}$$

CASE STUDY AND DECLINE EQUATIONS

Figure 1 indicates the position of Bidestan and the wheat field around it. This city is located at 150 km to the North-west of Tehran, the capital of Iran. Some of the potable water of this city, with 10000 inhabitants, is provided with M1 and M2 wells. The water for wheat fields is drawn from pumping A1 to A7 wells. To make the calculation easier, one municipal equivalent well (ME) instead of M1 and M2 and one equivalent agricultural well (AE) instead of A1 to A7 are used. The equivalent of the municipal pumping rate is $Q_{ME} = 1500 \text{ m}^3 \text{ day}^{-1}$ and the equivalent of the agricultural pumping rate is $Q_A = 2000 \text{ m}^3 \text{ day}^{-1}$.

First and second rows of Table 1 shows monthly value of municipal and agricultural operators. Third row indicates the monthly increasing level of groundwater table fed by precipitation.

The planning period is based on a one-year scale. The set of pure strategies for operator i is a discrete one which including ten elements, $M_i = \{0, 10\%, \dots, 100\%\}$ $i \in \{I, II\}$, where symbols I and II stand for municipal and agricultural operators, respectively. Groundwater level at the beginning of each month determines the system state at the beginning of each stage.

Operators payoff at stage t , depends on the current system state, operators' decisions and the amount of monthly demand, so that: $p_t^i = \min(D_t^i \times Q_i, Demand_t^i)$ Q_i is the pumping rate for player i , $Demand_t^i$ is the demand of operator i at month t . To avoid the aquifer overdraft if $h_{t+1} < 0$ thus punishment is enforced on the operator i . The amount of drawdown in the groundwater table at the end of each month is calculated by Tiess Well equation (Maidment, 1993):

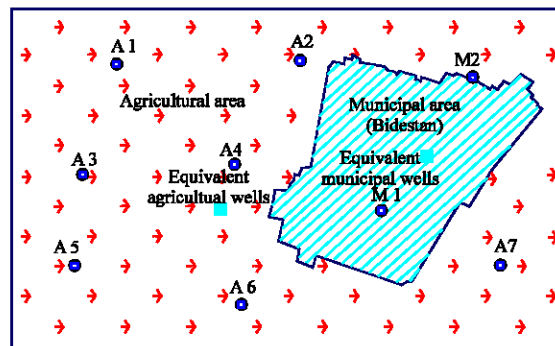


Fig. 1: Position of municipal and agricultural wells

Table 1: Monthly demand of operators ($\text{m}^3 \text{ day}^{-1}$) and groundwater table incensement at Bidestan aquifer (m)

	1	2	3	4	5	6	7	8	9	10	11	12
Municipal operator	900	1000	1000	1200.0	1500.0	1500.0	1200.0	1000	1000	1000	900	1200
Agricultural operator	1500	1500	1000	200.0	200.0	500.0	1000.0	1500	1500	500	200	200
Inflow	2	2	1	0.5	0.5	0.5	0.5	1	2	2	2	3

$$\begin{aligned}
 \text{The well equation: } s_i^t &= -\frac{1}{4\pi T} \sum_{i=1}^2 De_i^t \times Q_i \times W_i(u_i^t) \\
 W(u_i^t) &= \int_{u_i^t}^{\infty} \frac{e^{-x}}{x} dx, \quad u_i^t = \frac{r_i^2}{4\alpha D}, \quad \alpha = \frac{T}{S_s}
 \end{aligned}
 \tag{7}$$

where, s_i^t is the amount of water decline in well i at month t , r_i is the distance from the point at which the decline is measured to well i is 1000 m, T is the aquifer transmissivity equal to $432 \text{ m}^2 \text{ day}^{-1}$, D is the duration of operation with the fixed pumping rate equal to 30 days and S_s is the specific storage coefficient equal to 4×10^{-4} .

RESULTS AND ANALYSIS OF EQUILIBRIUM POINTS

Here, we analyze the results arising from different scenarios. Table 2 and 3 explain the amount of extracted water, Fig. 2 and 3 show the equilibrium points and Fig. 4 and 5 shows the changes on ground water basin, respectively in the locations of municipal and agricultural operating wells.

Comparison and analysis of tables and figures above shows as the following:

- According to Table 2 and 3 the average of obtained water by players in scenario 3 is more than scenario 1 and 2. The reason of this matter is because of the cooperation between the two operators. As in scenario 3 extracting the maximum amount of water by two operators are assumed as the goal function, so the players cooperate with each other to achieve a collective aim. In this scenario the collective aims of operators is prior to individual aims
- In scenarios 1 and 2 to resolve the conflict among the operators, individual interests are considered. According to Table 2 and 3 the amount of obtained water arising from scenario 2 is more than scenario 1. In scenario 1 the operators on the basis of monthly needs and also the probable movements of the opponent choose a decision that leads into the

individuals benefits at the mentioned month. But in scenario 2 the players in addition to watching the probable moves of opponents analyze the effects of current decisions on acquired water on the oncoming months and take decisions that require more extraction at the sum of current stages and future stages

- Comparing Fig. 2-5 show that in scenario 1 municipal and agricultural operators without considering the effects of current decision on future payoff tries to extract water from aquifer. The process of aquifer extraction in scenario 1, emphasis on this fact that as long as possible the operators tried to extract water from the aquifer. In other word, in scenario 1, players obey myopic policy. In this scenario obtaining water at the first half of the year is more than the second half of the year. But in scenarios 2 and 3 water extraction is done gradually throughout the year with the monthly need of operators
- Generally, the results show that cooperative model has more profits for the group of players. But we should emphasize that the operation on the base of optimization is not applicable. Because this method

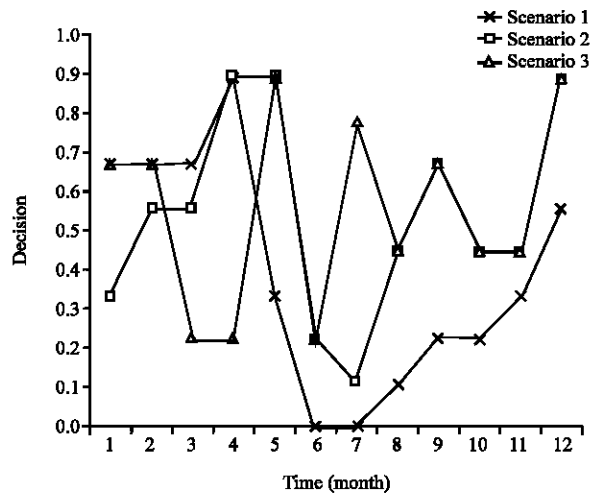


Fig. 2: Monthly decision of municipal operator in different scenarios

Table 2: The amount of extracted water by municipal operator in different scenarios

Scenarios	1	2	3	4	5	6	7	8	9	10	11	12	Average
1	900	1000	1000	1200	500	0	0	167	333	333	500	833	563.88
2	500	833	833	1200	1333	333	167	667	1000	667	667	1200	783.33
3	900	1000	333	333	1333	333	1167	667	1000	667	667	1200	800.00

Table 3: The amount of extracted water by agricultural operator in different scenarios

Scenarios	1	2	3	4	5	6	7	8	9	10	11	12	Average
1	1500	1500	1000	200	200	500	444	222	444	444	200	200	571.28
2	1111	1500	667	0	0	222	667	1333	1111	500	200	200	625.93
3	1111	1111	889	200	0	222	889	1333	1111	500	200	200	647.22

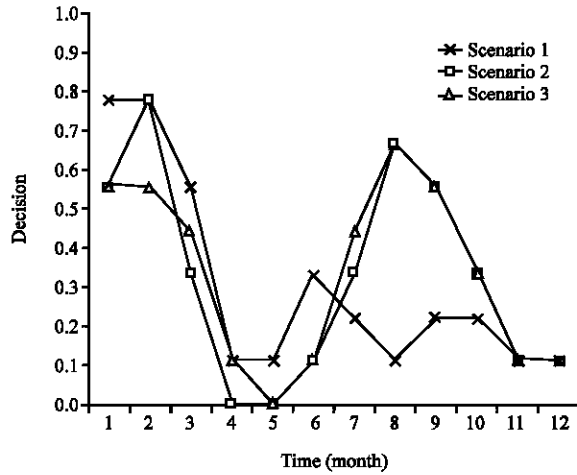


Fig. 3: Monthly decision of agricultural operator in different scenarios

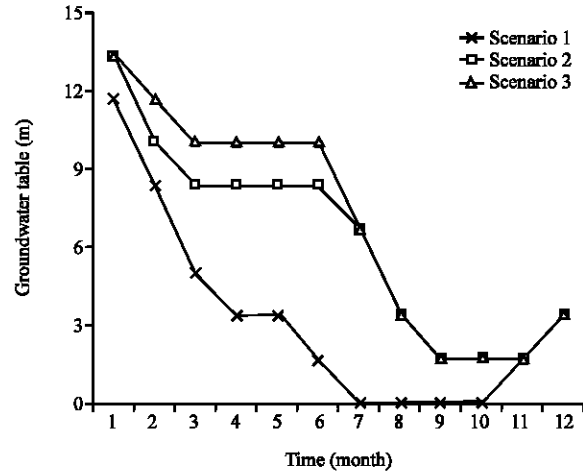


Fig. 5: Monthly fluctuation of groundwater table in agricultural well in different scenarios

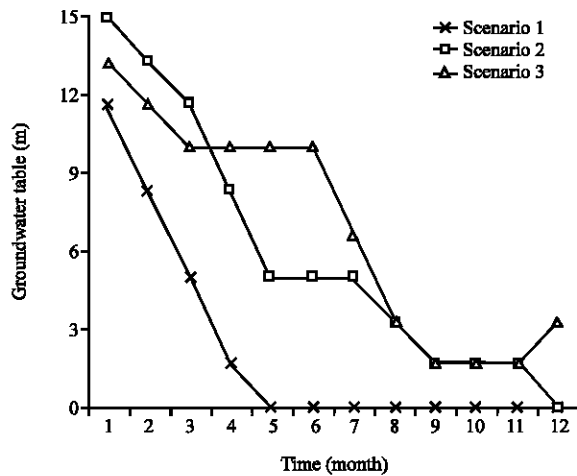


Fig. 4: Monthly fluctuation of groundwater table in municipal well in different scenarios

is assumed on the basis of cooperation among the operators, but it is possible that the operators don't keep their words on the agreement or by optimization model be spoiled the interest of one of the operators on behalf of other operators. As it was said in introduction, despite having less benefit for the players, using non-cooperative conflict resolution model is a necessity. There is no need to the agreement of operators with each other but logical operators follow the operation rules which arises from non-cooperative conflict resolution models. Results show that among the non-cooperative conflict resolution models, the results of the proposed model (scenario 2) on providence of

operators and in addition to conflict resolution among the operators, there is a little difference with the results of cooperative model in a way that the differences of suggested conflict resolution and cooperative model in the case study is 3%

CONCLUSION

In this research, the dynamic conflict resolution is presented on the basis of compilation of static games study and the dynamic programming. The theory of static games for conflict resolution among operators and dynamic programming for transferring players payoff from one stage to another stage is used. This model application is in long term and mid term programming of conflict resolution for common natural resources. The proposed model is applied for conflict resolution among municipal and agricultural operators from Bidestan aquifer, which is located in Iran. To analyze the effectiveness of the proposed model, the static non-cooperative and cooperative model on the Bidestan aquifer were applied also. Results showed that among the above scenarios, cooperative model has more benefit for the players but in practice the possibility of using cooperative model is less. The results show that in non-cooperative conflict resolution the results of the proposed model on the basis of operators' providence has a little difference with the results of cooperative model.

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