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# New Method for Determination of Depth-Averaged Velocity for Estimation of Longitudinal Dispersion in Natural Rivers

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**Abstract:** A number of equations have been proposed to determine dispersion coefficient of stream analytically since 1950. However, it is realized that it is difficult to use these equations for predicting the dispersion coefficients because they require the detailed information on velocity profile and cross-sectional geometry. To solve the problem, researchers tried to introduce empirical equations. This research presents an attempt to get back to the fundamental analytical equations, because it is believed that a simple model proposed by Maghrebi can provide a good prediction of the normalized isovel contours in the cross-section of open or closed channels that are irregular in roughness as well as geometry. The input data for the model are the bed profile and its shear and roughness distributions. Having obtained the isovel contours, the depth-averaged velocity profile at a cross-section can be extracted. In order to extract the parameters that characterize the longitudinal dispersion coefficients, applications of the model to the cross-sections of the Sacramento Delta in the Northern California and Clinch River near Speers Ferry in the US are presented. The evaluated parameters are the velocity deviation intensity ratio r, a dimensionless parameter I and the dispersion coefficient K, which is derived from the depth-averaged velocity profile by triple integration suggested by Fischer. The results of this study show that the longitudinal dispersion coefficient predicted by the use of isovel contours is close to the coefficients produced by measurement as well as theoretical methods.

**Key words:** Dispersion coefficient, depth-averaged velocity, pollution management, river flow

## INTRODUCTION

Vertical and transverse velocity distributions are fundamental for the understanding of the state of flow in stream, which is required in a wide range of applications in various disciplines such as pollution management and modeling, ecological studies, emergency spill management and the scientific study of water quality processes. Usually, the velocity in a cross-section, varies from point to point, which results from water surface effects and shear stress at the bed. The velocity distribution in open channels is three-dimensional and complex, so, evaluation of the global equation to calculate the velocity in open channels is not easy. Considering the significance of the velocity distribution for the estimation of a number of hydraulic characteristics it would be an advantage and use a general, accurate and user-friendly method for the estimation of longitudinal dispersion coefficient.

The intensity of longitudinal dispersion is measured by the longitudinal dispersion coefficient (Tayfur and Singh, 2005). Hence, the transport process and the consequent fate of pollutants are described by the longitudinal dispersion coefficient. That is why the dispersion coefficient has been extensively investigated by Elder (1959), Sooky (1969), Fischer *et al*. (1979), Deng *et al*. (2002) and Seo and Baek (2004).

Taylor (1954) proposed a theoretical method to predict the longitudinal dispersion coefficient. Elder (1959) developed the method and derived an equation to compute the longitudinal dispersion coefficient for uniform flow in an infinitely wide-open channel, assuming a logarithmic velocity profile in the vertical direction. Elder's equation is simple and has a sound theoretical background, so, it has been widely used. But Elder's equation does not describe the longitudinal dispersion in natural stream (Fischer *et al*., 1979). According to Fischer's (1979) earlier study, Elder's equation was found to significantly underestimate the natural dispersion in real stream because it does not consider the transverse variation of the velocity profile across the stream. Fischer (1975, 1979) postulated that in most natural stream the transverse velocity profile is far more important in producing the longitudinal dispersion than the vertical profile and using the lateral velocity profile instead of the vertical velocity profile, obtained an

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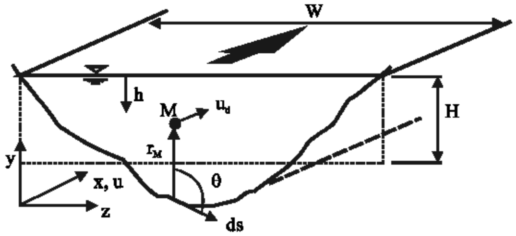


Fig. 1: Definition of symbols and coordinate system

integral relation for the dispersion coefficient in natural stream having large width-to-depth ratio as:

$$K=-\frac{1}{A}\int_0^W hu'\left(\int_0^z \frac{1}{\varepsilon_t h}\left(\int_0^z hu'dz\right)dz\right)dz \tag{1}$$

where, K is the longitudinal dispersion coefficient, A is the cross-sectional area of the stream, W is the stream width, h is the local depth of flow, u' is the deviation of the depth-averaged velocity $u_d$, from the cross-sectional mean velocity U, i.e.,u' = $u_d$- U, $\varepsilon_t$ is the transverse mixing coefficient and z is the lateral coordinate measured from the left bank of the stream (Fig. 1).

Fischer showed that the agreement between measurements and the prediction obtained through Eq. 1 is within a factor of 4 in non-uniform streams and within an error of 30% in uniform streams (Seo and Baek, 2004). Eq. 1 is rather difficult to use since detailed transverse profiles of both the depth-averaged velocity and the cross-sectional geometry are required. So, Fischer (1975) developed a simpler equation by introducing dimensionless quantities in the following form:

$$K=\frac{IrU^2W_1^2}{E_t} \tag{2}$$

where, $W_1$ is the characteristic length associated with shear stress due to the transverse velocity distribution, $E_t$ is the cross-sectional mean value of the transverse mixing coefficient, r and I are velocity deviation intensity ratio and dimensionless integral, respectively, which are given by:

$$r=\frac{\overline{u'^2}}{U^2} \tag{3}$$

$$I=-\int_0^1 h'u''\left(\int_0^{z'} \frac{1}{h'\varepsilon'_t}\left(\int_0^{z'} h'u''dz'\right)dz'\right)dz' \tag{4}$$

In Eq. 4, the dimensionless variables are defined as:

$$h'=\frac{h}{H};z'=\frac{z}{W};u''=\frac{u'}{\sqrt{\overline{u'^2}}};\varepsilon'_t=\frac{\varepsilon_t}{E_t} \tag{5}$$

where, H is the cross-sectional average depth of channel and $\sqrt{\overline{u'^2}}$ is the intensity of velocity deviation in the streamwise direction, which is a measure of the deviation of the turbulent averaged velocity from its cross-sectional mean throughout the cross-section. Fischer (1975) selected $W_1$ = 0.7 W as a reasonable choice for a real stream with some degree of asymmetry.

Deng *et al.* (2002) performed a sensitivity and error analysis and showed that the velocity U, channel width W, mean depth of flow H and shear velocity $u_*$ take the weighted factors of 5, 2.5, 1.25 and 1 in importance in estimating the dispersion coefficient K, respectively. Therefore, it is concluded that in order to obtain a reliable prediction of dispersion coefficients, more attention should be paid to the proper estimation of U and $u_d$. To estimate the longitudinal dispersion coefficient from the velocity distribution, the transverse dispersion coefficient must be known. The transverse dispersion coefficient is very difficult to predict, especially in meandering channels. This places severe limitations on our ability to estimate the longitudinal dispersion coefficient from velocity measurement.

**Normalized depth-averaged velocity:** Bogle (1997) worked on the transverse depth-average velocity profile and its application on the estimation of the longitudinal dispersion coefficient in natural stream. He used the Sacramento Delta data and suggested an empirical equation for the transverse distribution of velocity in a stream channel of width W, by a quartic function as follows:

$$\frac{u_d}{U}=A_q+B_q\bar{z}^2+C_q\bar{z}^4 \tag{6}$$

where, $\bar{z}$ = (2z/W)-1, $B_q$ = 5$A_q$ -7.5, $C_q$= -7$A_q$+7.5 , $A_q$ is the regression coefficient and z is the lateral coordinate (Fig. 2).

Bogle's equation, suggested for prediction of depth-average velocity in channel stream, is plotted in Fig. 2. Equation 6 predicts negative velocities in the vicinities of both banks that are obviously unrealistic. By increasing $A_q$, the affected area under negative velocity is decreased. Bogle (1997) in his researches on the Sacramento Delta and in his suggested equation (Eq. 6) has implied that the quartic is a convenient choice because by specifying $A_q$ one can define a profile that can be more or less sharply peaked ($A_q$ = 1 gives a parabola, $A_q$ = 1.5 gives a profile that is flat over 2/3 of the width).

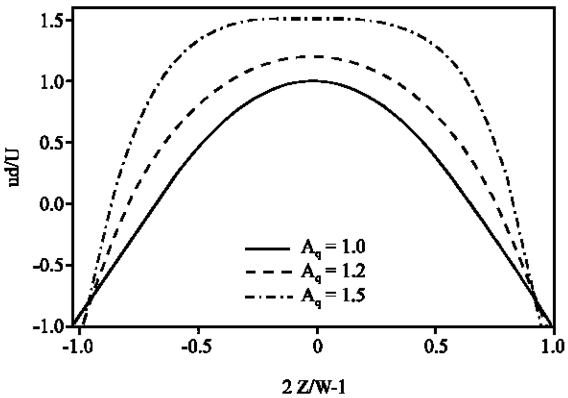


Fig. 2: Normalized depth-average velocity profiles by using the Bogle's equation (Seo and Baek, 2004)

Seo and Baek (2004) used the beta probability density function to describe the complicated properties of the transverse velocity distribution. They presented the dimensionless transverse velocity distribution as:

$$\frac{u_d}{U}=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(z')^{\alpha-1}(1-z')^{\beta-1},\quad 0<z'<1 \tag{7}$$

where, $\alpha$ and $\beta$ are the parameters that affect the velocity profile, $\Gamma$ is the gamma function and z' is the normalized lateral coordinate.

Unlike other empirical functions proposed by Sooky (1969), Deng *et al.* (2002), Bogle (1997) and Seo and Cheong (1998), this function can represent a complete spectrum of the properties of the transverse velocity distribution for natural stream e.g., this function can produce velocity profiles with symmetrical and skewed distributions with can be both flattened and sharp-peaked. The velocity profile produced by the beta function also shows good compatibility with the actual velocity profile (Seo and Baek, 2004). The reason of this good compatibility is that $\alpha$ and $\beta$ are calculated by using the measured depth-averaged velocity data. If there are no field data, it is impossible to use the beta function, unless $\alpha$ and $\beta$ of similar river be used (Seo and Baek, 2004).

Maghrebi (2003, 2006) has proposed a simple method, which is able to predict the normalized isovel contours at the cross-sections of straight ducts and irregular open channels both in the roughness and in geometry. It is assumed that each element of boundary influences the velocity at an arbitrary point (M) in the cross-section (Fig. 1). Then, the total effect of the boundary can be obtained by the use of integration along the wetted perimeter. Accordingly, he suggested that:

$$u=\int_{\text{boundary}} c_1 f(r_M)\times ds \tag{8}$$

where, u is the streamwise velocity vector at a point on the channel section, $f(r_M)$ is a velocity function which is similar to the dominant velocity profile over a flat plate with infinity large width, ds is the vector notation along the wetted perimeter and $c_1$ is a constant related to the boundary roughness.

The vector direction of velocity on the left hand side of Eq. 8 is the same as the vector product of $f(r_M)\times ds$ on the right hand side, which is a normal to flow section towards downstream. $f(r_M)$ is replaced by a power law relationship that is commonly used to fit velocity profiles in closed conduits and open channels, so, Eq. 8 may be written as (Maghrebi and Ball, 2006):

$$u(z,y)=\int_{\text{boundary}} c_1 c_2 \sin\theta\, u_* r_M^{1/m}\, ds \tag{9}$$

where, $r_M$ is the distance from a point in the river section to the boundary element, m is a constant, $\theta$ is the angle between the positional vector and the boundary elemental vector, $u_*$ is shear velocity, $c_2$ is a constant related to the nature of flow and u(z, y) is a local point velocity at an arbitrary position in the channel section.

The exponent m usually ranges between 4 and 12 depending on the intensity of turbulence (Yen, 2002). However, the sixth root of power law profile has been found to be equal to Manning's formula, which can be well applied to natural streams i.e., m = 6 (Chen, 1991). By using the average velocity, the normalized point velocity, U(z, y), is given by:

$$\tilde{U}(z,y)=\frac{u(z,y)}{U}=\frac{\int_{\text{boundary}} c_1 c_2 \sin\theta\, u_* r^{1/m}\, ds}{\frac{1}{A}\int_A\left(\int_{\text{boundary}} c_1 c_2 \sin\theta\, u_* r^{1/m}\, ds\right)dA} \tag{10}$$

Equation 10 provides the normalized velocity at a point as a simple function of the boundary geometry and relative roughness. Having obtained the isovels, it is easy to extract the depth-average velocity profiles. An advantage of the Maghrebi's model is that it allows the consideration of the hydraulic characteristics of the boundary and their influences on the flow.

**Field data:** Six detailed measurements of cross-sectional velocity distributions and dispersion coefficients are available from the US Geological Survey (USGS) field program in the Sacramento Delta and Godfrey and Frederick's field study (Seo and Baek, 2004). Two of the sites are on the Sacramento River near the Delta Cross Channel-WGA is just above the diversion point and WGB is just below it and the third, OLD, is on the OLD River

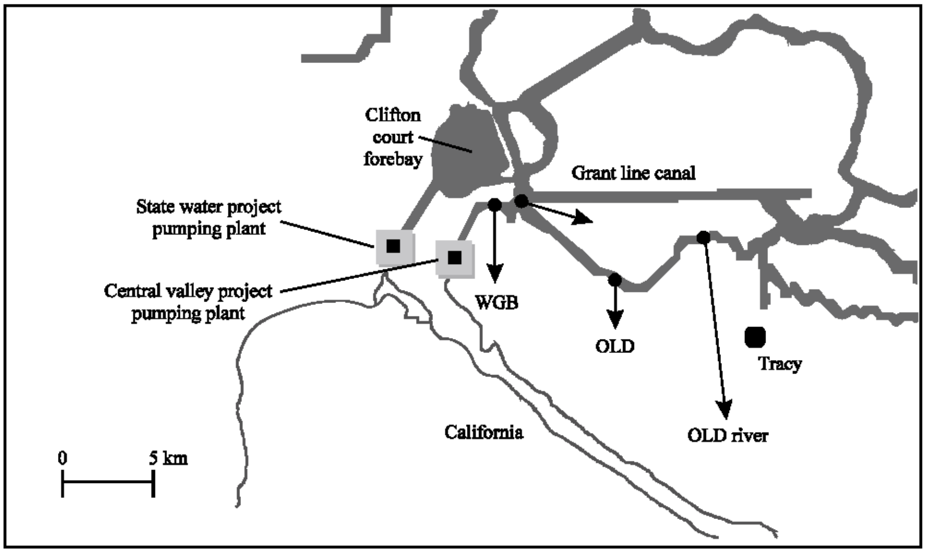


Fig. 3: Site of Sacramento Delta

Table 1: Channel dimensions, flow characteristics, correlation coefficient and average errors derived from Seo and Baek's equation, Maghrebi's model and Bogle's equation

| | | | | | Seo and Baek's Eq. | | Maghrebi's model | | Bogle's Eq. | |
|---|---|---|---|---|---|---|---|---|---|---|
| Case | W (m) | H (m) | U (m $sec^{-1}$) | Q ($m^3$ $sec^{-1}$) | R (%) | Average errors (%) | R (%) | Average errors (%) | R (%) | Average errors (%) |
| WGA41 | 127.0 | 8.30 | 1.07 | 1136.0 | 88.3 | -7.6 | 86.8 | -5.4 | $S_f<S_r$ | 41.6 |
| WGA60 | 111.0 | 7.10 | 0.38 | 298.0 | 88.7 | -16.9 | 77.6 | -23.2 | $S_f<S_r$ | 25.8 |
| WGB38 | 120.0 | 7.00 | 0.91 | 758.0 | 81.3 | -16.0 | 80.2 | -11.0 | $S_f<S_r$ | 20.8 |
| WGB57 | 108.0 | 5.10 | 0.16 | -85.0 | 78.4 | -4.4 | 73.9 | -1.0 | $S_f<S_r$ | 16.2 |
| OLD | 187.0 | 6.30 | 0.37 | 435.0 | 91.2 | -10.8 | 78.3 | -12.6 | $S_f<S_r$ | 70.7 |
| Clinch | 44.2 | 0.55 | 0.40 | 9.7 | 78.6 | -2.5 | 82.0 | 0.85 | $S_f<S_r$ | 2.1 |

near Rock Slough (Fig. 3). Godfrey and Frederick's field study is located on the Clinch River near Speers Ferry in the US. The channel dimensions and flow characteristics of the mentioned cross-sections are shown in Table 1. In Table 1, W is the stream width, H is the mean depth of flow, U is the mean velocity and Q is the discharge (Fig. 1 for the symbols). WGB57 corresponds to discharges of -85 $m^3$ $sec^{-1}$ which is a reverse flow case, induced by exports at the Central Valley Project and the State Water Project pumping plants on the Southern edge of the Delta. The channel depth profile and the measured depth-averaged transverse velocity profile for six cross-sections are plotted in Fig. 4a-c and 5a-c. At the Sacramento River sites, the channel has a near-rectangular cross-section, which may not be representative of the Delta as a whole.

The normalized depth-averaged velocity profiles based on the field measurement, Bogle's equation, Seo and Baek's equation and Maghrebi's model as well as the isovel contours based on Maghrebi's model are shown in Fig. 4 and 5. In Fig. 4 and 5, the profiles produced by Seo and Baek's equation are close to the ones produced by Maghrebi's model. However, Bogle's profiles are not close to the measured data. For the case of river sections with irregular shapes, such as OLD, larger deviations between the Bogle's profiles and others can be observed (Fig. 5b).

In calculating the transverse normalized depth-average velocity by using Maghrebi's model, the shear velocity distribution on the wetted perimeter of the river cross-section is considered to be constant. It is obvious that if a better estimation of the shear velocity $u_*$ and roughness $c_1$, along the wetted perimeter are available, the results obtained by Maghrebi's model will be closer to the measured feild data.

Unlike the Bogle and the Seo and Baek's equations, which are able to predict the dimensionless transverse velocity distribution only through the calibration of the parameters, the proposed model by Maghrebi is able to predict the normalized transverse velocity distribution from the geometry of the cross-section and the nature of flow.

In Table 1, the correlation coefficient (R) and average error between the field and the predicted data by different models are shown. The correlation coefficient is calculated by the following equation:

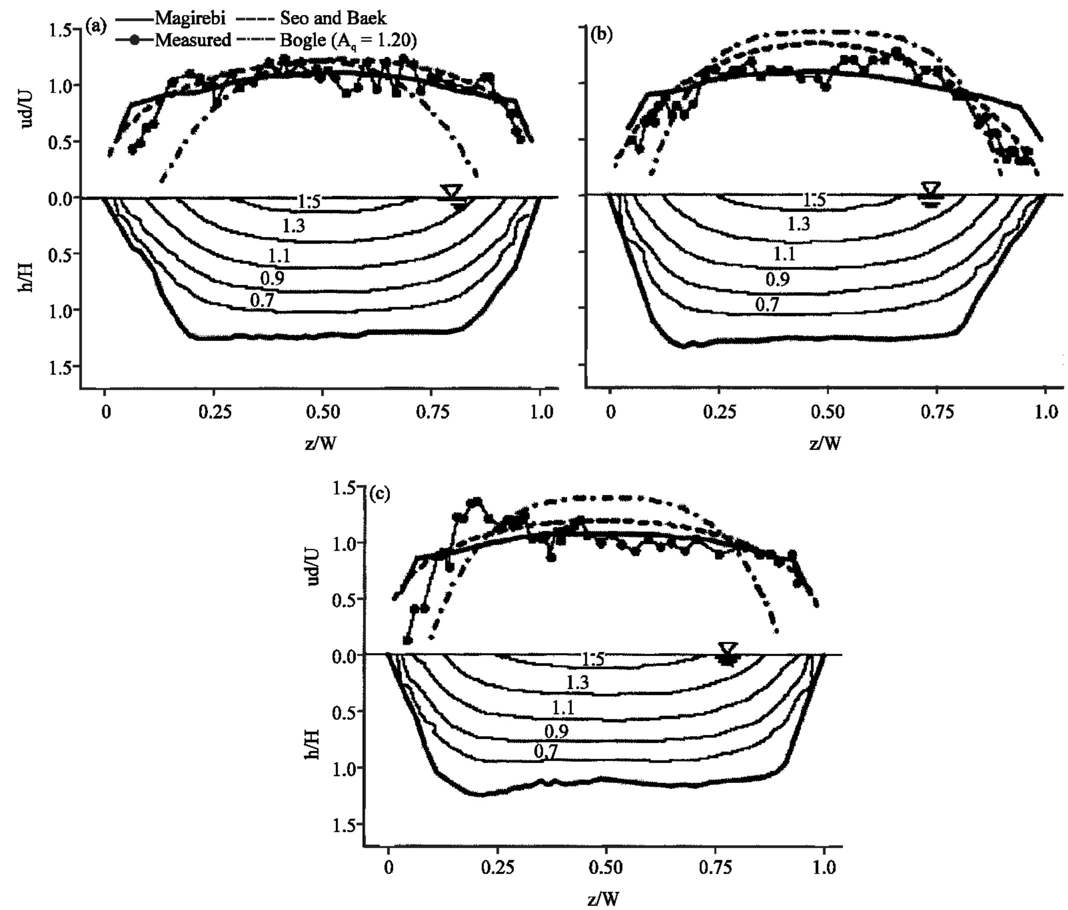


Fig. 4: Isovel contours predicted by Maghrebi's model and comparison of the measured data with depth-averaged velocity profiles predicted by different methods (a) WGA41 (b) WGA60 and (c) WGB38

$$R = \sqrt{\frac{(S_t - S_r)}{S_t}} \qquad (11)$$

where, $S_t$ is the sum of the squares of the residuals between the data points and the mean and $S_r$ is the sum of squares of the residuals between the data points and the data predicted by the model.

As mentioned previously, Seo and Baek (2004) used the field data to obtain the coefficients and as a consequence the correlation coefficient in Seo and Baek's equation is better than in Maghrebi's model. However, the difference of the R magnitudes in the two models is small. Investigation of Bogle's quartic equation reveals that this equation can only produce symmetrical depth-averaged velocity profiles. Negative velocities near the banks predicted by the quartic equation lead to $S_t<S_r$ (Table 1).

As shown in Fig. 4 and 5, in the middle of rivers the normalized depth-averaged velocities predicted by Seo and Baek's model and Maghrebi's model compare very well with the field data. The observed depth-average velocity shows some fluctuations. Neither model nor equation can model large fluctuations in the flow. The main reason for larger average errors, which are shown in Table 1, is the difference between the observed and predicted values in the bank regions. When the predicted results of Bogle's equation are compared with the observed data, due to unrealistic negative velocity near the banks, a higher average error will be obtained (Table 1).

According to Eq. 2, the deviation intensity ratio r and dimensionless integral I contribute to the evaluation of K. Application of the transverse velocity profiles in Eq. 3 and 4, obtained by the use of artificial velocity profiles such as beta density function (Eq. 7), Maghrebi's model and the quartic equation (Eq. 6), provide different results, as shown in Table 2. The calculated values of r according to the three methods are more or less comparable to each other. However, the calculated values of I based on the beta function and isovel contours are quite close to each other and very different from the one obtained by quartic function.

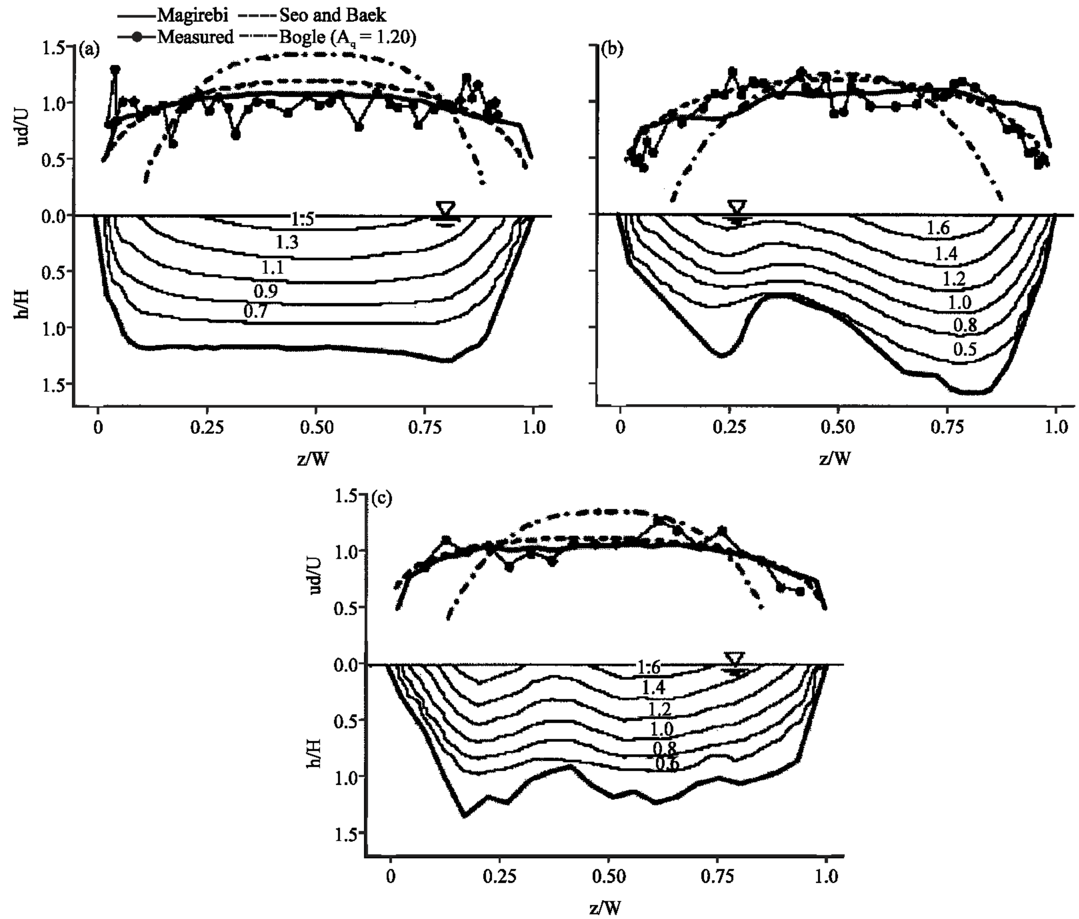


Fig. 5: Isovel contours predicted by Maghrebi's model and comparison of the measured data with depth-averaged velocity profiles predicted by different methods (a) WGB57 (b) OLD and (c) Clinch

Table 2: Calculation values of r and I at six different river sections

| | Seo and Baek's Eq. | | Maghrebi's model | | Bogle's Eq. | |
|---|---|---|---|---|---|---|
| Case | r | I | r | I | r | I |
| WGA41 | 0.0521 | 0.0161 | 0.0375 | 0.0139 | 0.694 | -0.0236 |
| WGA60 | 0.1153 | 0.0185 | 0.0362 | 0.0140 | 0.633 | -0.0124 |
| WGB38 | 0.0478 | 0.0177 | 0.0442 | 0.0175 | 0.625 | 0.0059 |
| WGB57 | 0.0142 | 0.0157 | 0.0245 | 0.0178 | 0.629 | 0.0068 |
| OLD | 0.0586 | 0.0181 | 0.0317 | 0.0133 | 0.662 | -0.0160 |
| Clinch | 0.0211 | 0.0256 | 0.0208 | 0.0243 | 0.626 | -0.0014 |

Table 3: Comparison of the measured and estimated dispersion coefficients K ($m^2$ $sec^{-1}$)

| | Seo and Baek's Eq. | | Maghrebi's model | | Bogle's Eq. | | Measured |
|---|---|---|---|---|---|---|---|
| Case | $K_{Eq.1}$ | $K_{Eq.2}$ | $K_{Eq.1}$ | $K_{Eq.2}$ | $K_{Eq.1}$ | $K_{Eq.2}$ | K |
| WGA41 | 57.60 | 28.20 | 36.00 | 17.64 | -1131.9 | -554.6 | 15.70 |
| WGA60 | 17.90 | 8.80 | 4.30 | 2.10 | -65.9 | -32.3 | 5.40 |
| WGB38 | 69.00 | 33.81 | 62.98 | 30.86 | 299.1 | 146.6 | 33.90 |
| WGB57 | 0.72 | 0.35 | 1.40 | 0.69 | 13.8 | 6.8 | 0.15 |
| OLD | 51.90 | 25.40 | 20.63 | 10.11 | -516.2 | -252.9 | 12.20 |
| Clinch | 8.50 | 4.20 | 8.00 | 3.90 | -13.8 | -6.8 | 8.55 |

In Table 3, the measured and estimated dispersion coefficient at six river cross-sections are shown. The calculated dispersion coefficients are given based on three different depth-average velocities. Equation 2, with the assumption of $W_1$ = 0.7 W which is proposed by Fischer (1975), is usually considered as the most common equation for calculation of K. The calculated value of K based on this equation is compared with the measured data (Table 3). From Table 3, it is seen that the calculated K based on Eq. 2 is about half of the value obtained by Eq. 1. In all cases, the dispersion coefficients predicted by Seo and Baek's equation and Maghrebi's model are reasonable and comparable with field data. However, Bogle's equation predicts negative velocities in the vicinity of the banks, the dispersion coefficient predicted by this equation are unrealistic and much larger than the measured ones. Among all sections, the calculated K at two of them is significantly different from the measured ones (WGB57 and Clinch). It seems that the reverse flow at WGB57 is responsible for that. Also, at Clinch Station due to the shallow river section, the measured K is about twice larger than the calculated ones. This feature of flow is explained in detail by the storage transient mechanism and solute transport in the previously performed work (Bencala, 1983; Bencala and Walters, 1983).

## CONCLUSIONS

Estimation of dispersion coefficient in natural stream has many applications in river engineering. The shape of the stream cross-sections influences the dispersion coefficient. Most of the rivers have a large ratio of width-to-depth, say more than 10. This makes the evaluation of the dispersion coefficient very simple, as it can be expected to find that the transverse velocity profile is more than one hundred times as important in producing longitudinal dispersion as the vertical profile. A quantitative estimate of the dispersion coefficient in a real stream can be obtained by neglecting the vertical profile entirely. This leads to vertically uniform concentrations. Thus, a depth-averaged approach would be adequate. For estimation of the dispersion coefficient in natural stream having large width-to-depth ratios the integral formula proposed by Fischer (Eq. 1) should be evaluated. However, due to the lack of the detailed information on the velocity profile and cross-sectional geometry, a number of empirical formulas have been proposed to evaluate the longitudinal dispersion coefficient. Equation 2 is an empirical equation, which is proposed on the similar assumption as Eq. 1. The results of this study show that the magnitude of the longitudinal dispersion coefficient obtained from Eq. 1 is about twice larger than the one evaluated by using Eq. 2. Although, a comparison of the results extracted from Eq. 1 and 2 has not appeared in previous works, it is concluded that Eq. 2 is able to produce more accurate results.

To deal with Eq. 2, the depth-averaged transverse velocity profile is required. Some of these equations, such as the one proposed by Bogle (1997) (Eq. 6), are only applicable to symmetrical cross-sections. However, due to any kind of irregularity in the cross-section, the depth-averaged velocity profile will not remain symmetric any more. Some equations have a good flexibility to fit the unsymmetrical measured data obtained from unsymmetrical cross-sections. Equation 7, introduced by Seo and Baek (2004) has considerable flexibility in producing symmetrical and asymmetrical velocity distributions to fit the measured velocity data.

The results of this study show that the proposed model for the production of isovel contours by Maghrebi (2003) can be considered as an appropriate, fast and easy model to predict transverse depth-average velocity profiles. The longitudinal dispersion coefficient that results from the isovel contours is close to the measured longitudinal dispersion coefficient. The coefficient obtained by the use of Eq. 6 for irregular cases such as asymmetrical cross-sections are unrealistic.

## NOTATIONS

The following symbols are used in this study:

| | | |
|---|---|---|
| A | : | Cross-sectional area of the stream |
| $A_q$ | : | Regression coefficient |
| $c_1$, $c_2$ | : | Constants related to the boundary roughness and flow regime, respectively |
| ds | : | Vector notation along the wetted perimeter |
| ds | : | Element notation along the wetted perimeter |
| $E_t$ | : | Cross-sectional mean value of the transverse mixing coefficient |
| $f(r_M)$ | : | Velocity function |
| H | : | Mean depth of flow |
| h | : | Local depth of flow |
| I | : | Dimensionless integral |
| K | : | Longitudinal dispersion coefficient |
| m | : | Constant |
| R | : | Correlation coefficient |
| r | : | Velocity deviation intensity ratio |
| $r_M$ | : | Distance from a point in the river section to the boundary element |
| $S_r$ | : | Sum of the squares of the residuals between the data points and the mean |
| $S_t$ | : | Sum of squares of the residuals between the data points and the predicted data by model |
| U | : | Mean velocity |
| u | : | Streamwise velocity vector |
| u | : | Streamwise velocity at a point in the channel section |
| $u_d$ | : | Depth-averaged velocity |
| $u'$ | : | Deviation of the depth-averaged velocity from the cross-sectional mean velocity $u_d$-U |
| $u_*$ | : | Shear velocity |
| $\sqrt{u'^2}$ | : | Intensity of velocity deviation in the streamwise direction |
| U(z, y) | : | Normalized point velocity = u(z, y)/U |
| W | : | Stream width |
| $W_1$ | : | Characteristic length associated with shear due to the transverse velocity distribution |
| z | : | Lateral coordinate measured from the left bank of the stream |
| z' | : | Normalized lateral coordinate measured from the left bank of the stream |
| $\alpha$, $\beta$ | : | Constant parameters |
| $\Gamma$ | : | Gamma function |
| $\varepsilon_t$ | : | Transverse mixing coefficient |
| $\theta$ | : | Angle between the positional vector and the boundary elemental vector |

**REFERENCES**


Bencala, K.E., 1983. Simulation of solute transport in mountain pool-and-riffle stream with a kinetic mass transfer model for sorption. J. Water Resour. Res., 19: 732-738.

Bencala, K.E. and R.A. Walters, 1983. Simulation of solute transport in mountain pool-and-riffle stream: A transient storage model. J . Water Resour. Res., 19: 718-724.

Bogle, G.V., 1997. Stream velocity profiles and longitudinal dispersion. J. Hydraulic Eng., 123: 816-820.

Chen, C.L., 1991. Unified theory on power laws for flow resistance. J. Hydraulic Eng., 117: 371-389.

Deng, Z.Q., V.P. Singh, L. Bengtsson and D.D. Adrian, 2002. Longitudinal dispersion coefficient in single-channel streams. J. Hydraulic Eng., 128: 901-916.

Elder, J.W., 1959. The dispersion of a marked fluid in turbulent shear flow. J. Fluid Mechanics, 5: 544-560.

Fischer, H.B., 1975. Discussion of 'Simple method for predicting dispersion in stream' by R.S. McQuivey and T.N. Keefe. J. Environ. Eng. Div. Am. Soc. Civil Eng., 101: 453-455.

Fischer, H.B., E.J. List, R.C.Y. Koh, J. Imberger and N.H. Brooks, 1979. Mixing in Inland and Coastal Waters. Academic Press, New York, ISBN: 9780122581502.

Maghrebi, M.F., 2003. Discharge estimation in flumes using a new technique for the production of isovel contours. Proceedings of International Conference on Civil and Environment Engineering ICCEE, Oct. 23-24, Hiroshima, Japan, pp: 147-156.

Maghrebi, M.F., 2006. Application of the single point measurement in discharge estimation. J. Adv. Water Resour., 29: 1504-1514.

Maghrebi, M.F. and J.E. Ball, 2006. New method for estimation of discharge. J. Hydraulic Eng., 132: 1044-1051.

Seo, I.W. and T.S. Cheong, 1998. Predicting longitudinal dispersion coefficient in natural streams. J. Hydraulic Eng., 124: 25-32.

Seo, I.W. and K.O. Baek, 2004. Estimation of the longitudinal dispersion coefficient using the velocity profile in natural streams. J. Hydraulic Eng., 130: 227-236.

Sooky, A.A., 1969. Longitudinal dispersion in open channels. J. Hydraulic Div. Am. Soc. Civil Eng., 95: 1327-1346.

Tayfur, G. and V.P. Singh, 2005. Predicting longitudinal dispersion coefficient in natural streams by artificial neural network. J. Hydraulic Eng., 131: 991-1000.

Taylor, G.I., 1954. Dispersion of matter in turbulent flow through a pipe. Proc. Royal. Soc. London, Ser. A, Math. Physical Sci., 223: 446-468.

Yen, B.C., 2002. Open channel flow resistance. J. Hydraulic Eng., 128: 20-39.