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Gaussian Radial Basis Adaptive Backstepping Control for a Class of Nonlinear Systems

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Abstract: This study proposes a Gaussian Radial Basis Adaptive Backstepping Control (GRBABC) system for a class of n-order nonlinear systems. In the neural backstepping controller, a Gaussian radial basis function is utilized to on-line estimate of the system dynamic function. The adaptation laws of the control system are derived in the sense of Lyapunov function, thus the system can be guaranteed to be asymptotically stable. The proposed GRBABC is applied to two nonlinear chaotic systems which have the different order to illustrate its effectiveness. Simulation results verify that the proposed GRBABC can achieve favorable tracking performance by incorporating of $GRBF_{NN}$ identification, adaptive backstepping control techniques.

Key words: Adaptive control, backstepping control, chaotic system, Gaussian radial basis function neural network

INTRODUCTION

Recently, the adaptive neural control approach based on backstepping design has been developed for nonlinear uncertain systems without the requirement of matching conditions. In Kwan and Lewis (2000), Lewis *et al.* (2000) and Zhang *et al.* (2000a), stable neural controller design schemes were proposed for unknown nonlinear SISO systems via backstepping design technique. With the backstepping design technique, neural networks were mostly applied to approximate the unmatched and unknown nonlinearities and then to implement adaptive control methods using the conventional control technology.

The advantage of adaptive neural control based on backstepping methodology is that both the parameters and the nonlinear functions can be unknown and the uncertainties in systems need not satisfy the matching conditions (Chen *et al*., 2007).

Excellent contributions for backstepping control, using NNs, are presented by He and Jagannathan (2005), Jagannathan (1996, 2001), Jagannathan and Lewis, (1996a, b), Jagannathan *et al.* (1998), Jagannathan (2001), Lewis *et al.* (1998), Hsu *et al.* (2006), Alanis *et al.* (2007), Polycarpou (1996), Lin *et al.* (1998), Wang and Wang (1999) and Lin and Hs (2002, 2003). There, a multilayer NN controller is designed to deliver a desired tracking performance for the control of a class of partially unknown nonlinear system in discrete time; it includes a modified delta rule weight tuning.

In the past decade, backstepping design procedures have been intensively introduced by Choi and Farrell (2001), Kuljaca *et al.* (2003) and Lin and Hsu (2005a, b). The backstepping control is a systematic and recursive design methodology for nonlinear systems to offer a choice to accommodate the unmodeled nonlinear effects and the parameter uncertainties. The essence of backstepping design is to select recursively some appropriate functions of state variables as pseudocontrol inputs for lower dimension subsystems of the overall system. Each backstepping stage results in a new pseudocontrol design, expressed in terms of the pseudocontrol designs from preceding design stages. When the procedure is terminated, a feedback design for the true control input results, which achieves the original design objective by virtue of a final Lyapunov function, which is formed by summing up the Lyapunov functions associated with each individual design stage (Zhang *et al*., 2000b).

This study proposes a GRBABC system for a class of n-order nonlinear systems. This control system combines the Gaussian Radial Basis Function Neural Network ($GRBF_{NN}$) identification and adaptive backstepping control techniques.

The neural backstepping controller containing a $GRBF_{NN}$ identifier is designed in the sense of the backstepping control technique and the $GRBF_{NN}$ identifier is utilized to online estimate the system dynamic function. The adaptive laws of the GRBABC system are derived in the sense of Lyapunov function. Thus, the system can be guaranteed to be asymptotically stable.

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Finally, two chaotic systems (Duffing Oscillator system and lü system) are provided as the simulation examples to verify that the proposed GRBABC scheme can achieve favorable tracking performance with regard to unknown dynamic function.

MATERIALS AND METHODS

Design of ideal backstepping controller: Consider a class of n-order nonlinear systems:

$$x^{(n)} = f(x,\dot{x},\ddot{x},\ldots,x^{(n-1)}) + u \qquad (1)$$

where, x is the state trajectory of the system, which is assumed to be available for measurement, $f(x,\dot{x},\ddot{x},\ldots,x^{(n-1)})$ is an unknown real continuous function and u is the input of the system. The control objective is to find a control law so that the state trajectory x can track a trajectory command closely.

The Eq. 1 can be rewritten as the following state Eq.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_k &= x_{k+1} \\ &\vdots \\ \dot{x}_n &= f(x_1,x_2,x_3,\ldots,x_n) + u \end{aligned} \qquad (2)$$

Assuming that the parameters of the system Eq. 2 are known, the design of ideal backstepping controller is described step-by-step as follows.

Step 1: Define the tracking error:

$$e_1 = x_1 - x_d \qquad (3)$$

and the derivative of tracking error is defined as:

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_c = \alpha_1(x_1) - \dot{x}_d \qquad (4)$$

The α_1 can be viewed as a virtual control in the Eq. 4
Define a Lyapunov function as:

$$V_1 = \frac{1}{2}e_1^2 \qquad (5)$$

Differentiating Eq. 5 with respect to time and using Eq. 4, it is obtained that:

$$\dot{V}_1 = e_1.\dot{e}_1 = e_1(\alpha_1 - \dot{x}_d) \qquad (6)$$

Let:

$$\alpha_1(x_1) = -c_1e_1 + \dot{x}_d \qquad (7)$$

Then,

$$\dot{V}_1 = -c_1e_1^2 \qquad (8)$$

where, c_1 is a positive constant.

Step k: ($2 \le k \le n-1$)
Define

$$e_k = x_k - \alpha_{k-1} \qquad (9)$$

and the derivative of e_k is defined as:

$$\dot{e}_k = \dot{x}_k - \dot{\alpha}_{k-1} = \alpha_k - \dot{\alpha}_{k-1} \qquad (10)$$

where, The α_k can be viewed as a virtual control in the Eq. 10
Define a Lyapunov function as:

$$V_k = \sum_{i=2}^{k} V_{i-1} + \frac{1}{2}e_k^2 \qquad (11)$$

Differentiating Eq. 11 with respect to time and using Eq. 10, it is obtained that:

$$\dot{V}_k = \sum_{i=2}^{k} \dot{V}_{i-1} + e_k\dot{e}_k = \sum_{i=2}^{k} \dot{V}_{i-1} + e_k(\alpha_k - \dot{\alpha}_{k-1}) \qquad (12)$$

Let:

$$\alpha_k(x_1,x_2,\ldots,x_k) = -c_ke_k + \dot{\alpha}_{k-1} \qquad (13)$$

Then,

$$\dot{V}_k = -\sum_{i=2}^{k} c_ke_k^2 \qquad (14)$$

where, c_1, c_2,..., c_k are positive constant.

Step n:
Define

$$e_n = x_n - \alpha_{n-1} \qquad (15)$$

and the derivative of e_n is defined as:

$$\dot{e}_n = \dot{x}_n - \dot{\alpha}_{n-1} = f(x_1,x_2,x_3,\ldots,x_n) + u - \dot{\alpha}_{k-1} \qquad (16)$$

Define a Lyapunov function as:

$$V_n = \sum_{i=2}^{n} V_{i-1} + \frac{1}{2}e_n^2 \quad (17)$$

Differentiating Eq. 17 with respect to time and using Eq. 16, it is obtained that

$$\dot{V}_n = \sum_{i=2}^{k} \dot{V}_{i-1} + e_n\dot{e}_n = \sum_{i=2}^{n} \dot{V}_{i-1} + e_k(f + u - \dot{\alpha}_{n-1}) \quad (18)$$

Let:

$$u = -c_n e_n - f + \dot{\alpha}_{n-1} \quad (19)$$

Then,

$$\dot{V}_n = -\sum_{i=2}^{n} c_i e_i^2 \quad (20)$$

where, $c_1, c_2,..., c_n$ are positive constant.
Therefore, the ideal backstepping controller in Eq. 19 will asymptotically stabilize the system.

Design of Gaussian radial basis adaptive backstepping controller: Since the system dynamic function $f(x_1, x_2, x_3,..., x_n)$ may be unknown or perturbed in practical application, the ideal backstepping controller Eq. 19 cannot be precisely obtained. To solve this problem, a $GRBF_{NN}$ identifier is utilized to approximate the system dynamic function. The descriptions of the $GRBF_{NN}$ identifier and the design steps of the control system are described as follows.

$GRBF_{NN}$ Identifier: The network structure of the $GRBF_{NN}$ identifier is shown in Fig. 1, which can be considered as one layer feed forward neural network with nonlinear element. The $GRBF_{NN}$ output can perform the mapping according to:

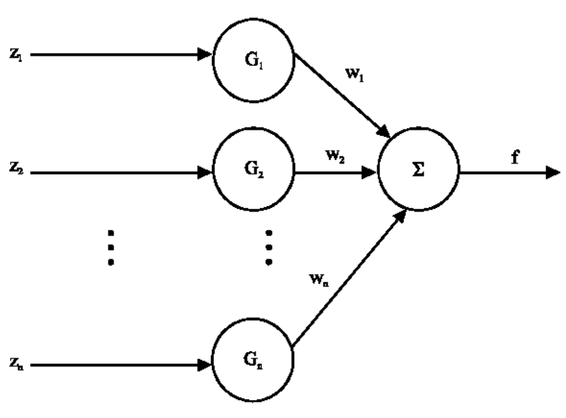

Fig. 1: Structure of GRBF neural network

$$f(z) = \sum_{j=1}^{n} w_j G_j(z_j, m_j, \sigma_j) \quad (21)$$

where, $z = [z_1, z_2,..., z_n]^T \in R^n$ is the input vector, $G_j(z_j, m_j, \sigma_j) \in R^n$, j = 12,..., n are the Gaussian radial basis function, $\alpha_j \in R$ is the spread of Gaussian function, n is the number of neurons. Each Gaussian radial basis function can be represented by:

$$G_j(z_j, m_j, \sigma_j) = \exp\left(\frac{z_j - m_j}{\sqrt{2}\sigma_j}\right)^2 \quad (22)$$

For ease notation, Eq. 21 can be expressed in compact vector forms as:

$$f(z, w, m, \sigma) = w^T G(z, m, \sigma) \quad (23)$$

where:
w = $[w_1, w_2,..., w_n]^T$
G = $[G_1, G_2,..., G_n]^T$
m = $[m_1, m_2,..., m_n]^T$
σ = $[\sigma_1, \sigma_2,..., \sigma_n]^T$

By the universal approximation theorem, there exists an ideal $GRBF_{NN}$ identifier f^* such that:

$$f = f^*(z) + \Delta = w^{*T} G(z, m^*, \sigma^*) \quad (24)$$

where, Δ denotes the approximation error and is assumed to be bounded. w^*, m^* and σ^* are the optimal parameter vectors of w, m and σ, respectively. In fact the optimal parameter vectors that are needed to best approximate a given nonlinear function are difficult to determine. Thus, an estimate function is defined as:

$$\hat{f}(z, \hat{w}, \hat{m}, \hat{\sigma}) = \hat{w}^T G(z, \hat{m}, \hat{\sigma}) \quad (25)$$

where, $\hat{w}$, $\hat{m}$ and $\hat{\sigma}$ are the estimated of w^*, m^* and σ^*, respectively. For notational convenience, denote $G^* = G(z, m^*, \sigma^*)$ and $\hat{G} = G(z, \hat{m}, \hat{\sigma})$.
Define

$$\tilde{w} = w^* - \hat{w} \quad (26)$$

Note that $\hat{m}$ and $\hat{\sigma}$ are assumed to be m^* and σ^*, respectively.

GRBABC System: The proposed GRBABC system is shown in Fig. 2.

The control law of the GRBABC is developed as follows:

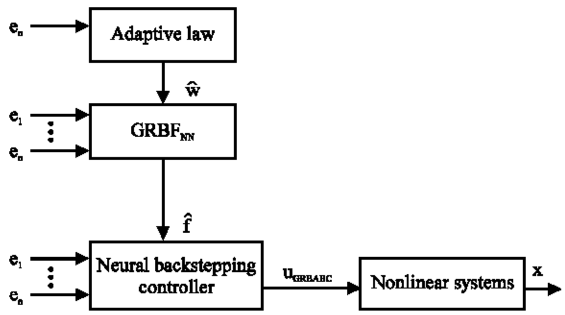

Fig. 2: GRBABC for nonlinear system

$$u_{GRBABC} = u_n \tag{27}$$

The neural backstepping controller is chosen as:

$$u_n = -\hat f + \dot\alpha_{n-1} - c_n e_n \tag{28}$$

where, the $GRBF_{NN}$ identifier $\hat f$ is designed to online estimate the system dynamic function f. Then, Theorems 1 and 2 show the properties of the proposed GRBABC system.

Theorem 1: Consider a nonlinear system represented by Eq. 1.The control system is designed as Eq. 27 where the neural backstepping controller is designed as Eq. 28, in which the adaptation law of the $GRBF_{NN}$ identifier is designed as:

$$\dot{\hat w} = -\dot{\tilde w} = e_n G \tag{29}$$

Proof 1:
Define a Lyapunov function as:

$$V_n = \sum_{i=1}^{n} \frac{1}{2} e_i^2 + \frac{1}{2}\tilde w^T \tilde w \tag{30}$$

Differentiating Eq. 30 with respect to time:

$$\dot V_n = \sum_{i=1}^{n} e_i \dot e_i + \tilde w^T \dot{\tilde w} \tag{31}$$

Let:

$$\sum_{i=1}^{n} e_i \dot e_i = \sum_{i=1}^{n-1} e_i \dot e_i + e_n \dot e_n \tag{32}$$

From Eq. 14 and 16, we have:

$$\sum_{i=1}^{n} e_i \dot e_i = -\sum_{i=1}^{n-1} c_i e_i^2 + e_n(f + u - \dot\alpha_{n-1}) \tag{33}$$

Substituting the Eq. 28 into Eq. 33,

$$\sum_{i=1}^{n} e_i \dot e_i = -\sum_{i=1}^{n-1} c_i e_i^2 + e_n(f - \hat f - c_n e_n) = -\sum_{i=1}^{n} c_i e_i^2 + e_n(f - \hat f) \tag{34}$$

Let $f = w^{*T}$ and $\hat f = \hat w^T G$

Then,

$$e_n(f - \hat f) = e_n(w^{*T}G - \hat w^T G) = e_n(w^{*T} - \hat w^T)G = e_n \tilde w^T G \tag{35}$$

Substituting the Eq. 35 into Eq. 34,

$$\sum_{i=1}^{n} e_i \dot e_i = -\sum_{i=1}^{n} c_i e_i^2 + e_n \tilde w^T G \tag{36}$$

Substituting the Eq. 36 into Eq. 31

$$\dot V_n = -\sum_{i=1}^{n} c_i e_i^2 + e_n \tilde w^T G + \tilde w^T \dot{\tilde w} = -\sum_{i=1}^{n} c_i e_i^2 + \tilde w^T (e_n G + \dot{\tilde w}) \tag{37}$$

IF the adaption law is obtained as follows:

$$\dot{\hat w} = -\dot{\tilde w} = e_n G \tag{38}$$

Then the differentiation of Lyapunov function will be negative.

$$\dot V_n = -\sum_{i=1}^{n} c_i e_i^2 \tag{39}$$

Therefore, the backstepping controller in Eq. 28 will asymptotically stabilize the system. Also the $GRBF_{NN}$ weights will converge to optimal values.

Theorem 2: Consider a nonlinear system represented by Eq. 1. The control system is designed as Eq. 27 where the neural backstepping controller is designed as Eq. 28, in which the adaptation law of the $GRBF_{NN}$ identifier is designed as:

$$\dot{\hat w} = e_n G + kw \tag{40}$$

where, $k \rangle\rangle 0$.

Proof 2:
Define a Lyapunov function as:

$$V_n = \sum_{i=1}^{n} \frac{1}{2} e_i^2 + \frac{1}{2}\hat w^T \hat w \tag{41}$$

Differentiating Eq. 41 with respect to time:

$$\dot V_n = \sum_{i=1}^{n} e_i \dot e_i + \hat w^T \dot{\hat w} \tag{42}$$

Substituting the Eq. 34 into Eq. 42,

$$\dot{V}_n = -\sum_{i=1}^{n} c_i e_i^2 + e_n(f - \hat{f}) + \hat{w}^T\dot{\hat{w}} \tag{43}$$

Let $\hat{f} = \hat{w}^T G$

Then,

$$\dot{V}_n = -\sum_{i=1}^{n} c_i e_i^2 + e_n f - e_n\hat{w}^T G + \hat{w}^T\dot{\hat{w}} = -\sum_{i=1}^{n} c_i e_i^2 + e_n f - \hat{w}^T(e_n G - \dot{\hat{w}}) \tag{44}$$

The adaption law is obtained as:

$$\dot{\hat{w}} = e_n G + kw \tag{45}$$

Then,

$$\dot{V}_n = -\sum_{i=1}^{n} c_i e_i^2 - k\|\hat{w}\|_2^2 + e_n f \tag{46}$$

Therefore, if $\sup\|e_n f\| \le k\|\hat{w}\|_2^2$, Then $\dot{V}_n \langle 0$.

The backstepping controller in Eq. 28 will asymptotically stabilize the system. Also the weights of $GRBF_{NN}$ would not diverge to infinity and we have a stable controller.

In general, the neural backstepping controller of Theorem 1 is same as Theorem 2, but the adaption law for $GRBF_{NN}$ weights training in Theorem 1 is different from Theorem 2.

In Theorem 1, the $GRBF_{NN}$ weights will converge to optimal values. Although, in Theorem 2, the weights of $GRBF_{NN}$ would not diverge to infinity and we have a stable controller.

Note that Theorem 2 is more generalized than Theorem 1 and provided both theorems properties.

RESULTS AND DISCUSSION

Description of chaotic systems: Dynamic chaos is a very interesting nonlinear effect which has been intensively studied during the last three decades.

Chaos control can be mainly divided into two categories (Chen and Dong, 1998; Feng *et al*., 2007; Zhang *et al*., 2004; Xiau and Jun 2003): one is the suppression of the chaotic dynamical behavior and the other is to generate or enhance chaos in nonlinear system. Nowadays, different techniques and methods have been proposed to achieve chaos control. For instance, entrainment and migration control, optimal control method, stochastic control method, robust control method, adaptive control method, variable structure method, neural network control method and so on (Ueta and Yet, 1999; Chen and Lu, 2002; Wang and Ge, 2002; Lu and Zhang, 2001; Chen *et al*., 2002; Liu *et al*., 2003; Feng *et al*., 2007; Harb *et al*., 2007).

Chaotic phenomena can be found in many scientific and engineering fields such as biological systems, electronic circuits, power converters, chemical systems and so on (Chen, 1999). Since the pioneering study of Ott *et al.* (1990), Park (2006) and Yongguang and Suochun (2003) proposed the well-known OGY control method, the control of chaotic systems has been widely studied. Recently, numerous backstepping control design procedures have been proposed to achieve chaotic control (Hsu *et al*., 2006; Guan and Chen, 2003; Yassen, 2006). The key idea of backstepping design is to select recursively some appropriate functions of state variables as virtual control inputs for lower dimension subsystems of the overall system (Krstic *et al*., 1995; Wai *et al*., 2002; Lin *et al*., 2005).

Chaotic systems have been known to exhibit complex dynamical behavior. The interest in chaotic systems lies mostly upon their complex, unpredictable behavior and extreme sensitivity to initial conditions as well as parameter variations.

For some chaotic systems, since the dynamic characteristics of the control system are nonlinear and the precise models are difficult to obtain, the model-based control approaches are difficult to be implemented (Peng *et al*., 2007).

Simulation results: In this section, the proposed GRBABC technique is applied to control two nonlinear chaotic systems: a Duffing Oscillator system (Example 1) and a Lü system (Example 2). It should be emphasized that development of the GRBABC does not require the knowledge of the system dynamic function.

Chaotic systems have been known to exhibit complex dynamical behavior. Several control techniques have been proposed for the chaotic systems (Lian *et al*., 2002). However, some of them cannot achieve favorable control performance and some of them require system dynamic function.

Duffing oscillator system: Consider a second-order chaotic system such as well known Duffing's equation describing a special nonlinear circuit or a pendulum moving in a viscous medium under control (Lian *et al*., 2002).

$$\ddot{x} = -p\dot{x} - p_1 x - p_2 x^3 + q\cos(\omega t) + u = f(x,\dot{x}) + u \tag{47}$$

where, p, p_1, p_2 and q are real constants. t is the time variable and ω is the frequency. $f(x,\dot{x})$ is the system

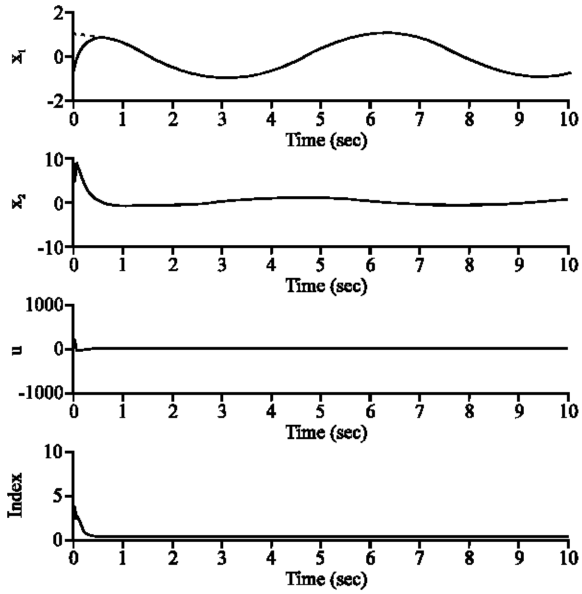

Fig. 3: Simulation results of Duffing Oscillator system, Theorem 1, q = 0.62

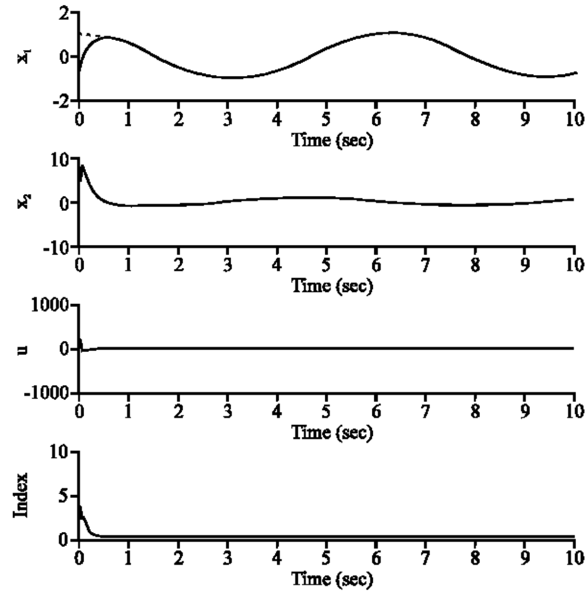

Fig. 4: Simulation results of Duffing Oscillator system, Theorem 1, q = 1.95

dynamic function where $p = 0.4$, $p_1 = -1.1$, $p_2 = 1.0$, $\omega = 1.8$ and $q = 0.62$, $q = 1.95$ and $q = 7$. u is the control effort.

The system dynamic function would be online estimated by the $GRBF_{NN}$ identifier. The structure of $GRBF_{NN}$ is shown in Fig. 1. A $GRBF_{NN}$ identifier with five hidden nodes is utilized to approach the system dynamic function of the chaotic system.

In addition, the control parameters are selected as $c_1 = 5$ and $c_2 = 60$. The trajectory command is set as $x_d = \cos(t)$.

The simulation results of the GRBABC with consider Eq. 29 for q = 0.62, q = 1.95 and q = 7 are shown in Fig. 3-5.

The simulation results of the GRBABC with consider Eq. 40 for q = 0.62, q = 1.95 and q = 7 are shown in Fig. 6-8, respectively.

The performance index I is defined as $I = \sqrt{e_1^2 + e_2^2}$. The performance index I is shown that the proposed GRBABC can achieve favorable tracking performance.

Figure 5-8 are shown that the results have good performance compare to other papers like (Lian *et al.*, 2002). These results are converged to desirable trajectory command in 1 sec; however, the results of other papers are converged to that desirable trajectory command in 4 sec. Consider that the control effort is limited.

These results are shown that the better tracking performance can be achieved by using Theorem 1 compare to Theorem 2.

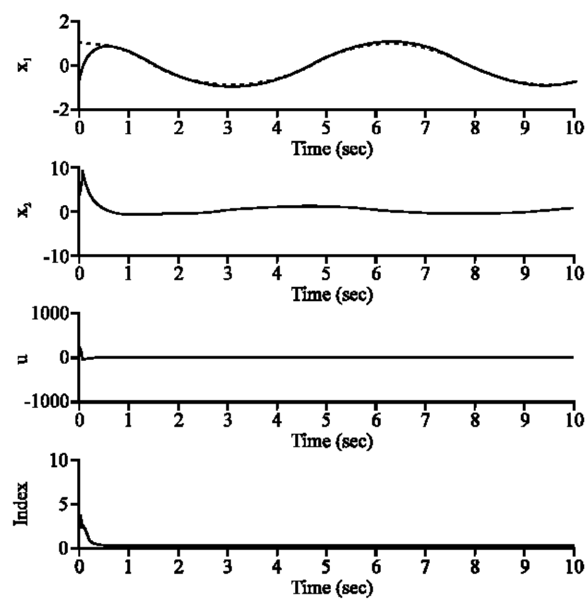

Fig. 5: Simulation results of Duffing Oscillator system, Theorem 1, q = 7

Lü system: Consider a third -order chaotic system such as well known Lü equation describing (Tan *et al.*, 2003).

$$\begin{aligned} \dot{x}_1 &= a\ (x_2 - x_1) \\ \dot{x}_2 &= -x_1 x_3 + c x_2 \\ \dot{x}_3 &= x_1 x_2 - b x_3 + u \end{aligned} \tag{48}$$

where, a = 36, b = 3, c = 20 and u is the control effort. The system Eq. 48 can be rewritten as the following:

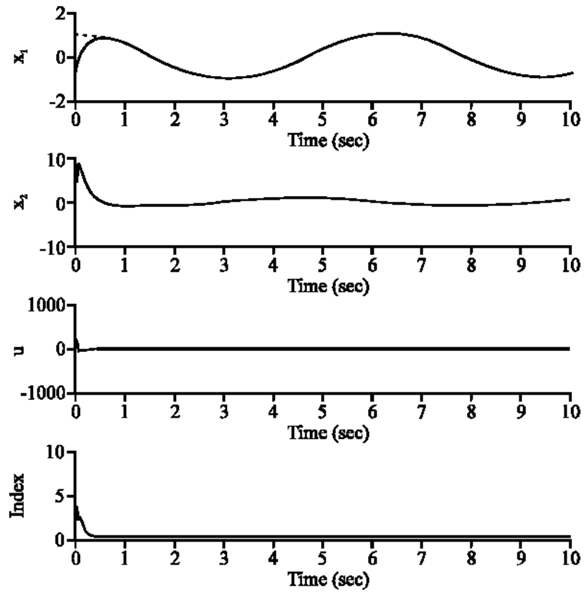

Fig. 6: Simulation results of Duffing oscillator system, Theorem 1, $q = 0.62$

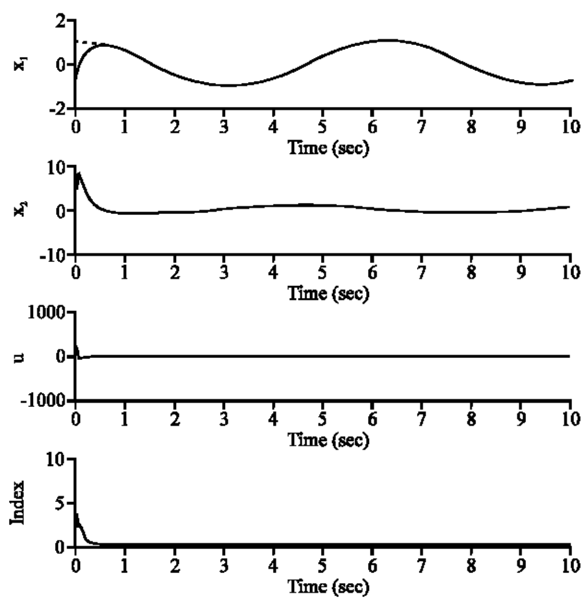

Fig.7: Simulation results of Duffing oscillator system, Theorem 2, $q = 1.95$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= f(x_1, x_2, x_3) + U \end{aligned} \tag{49}$$

And

$$f(x_1, x_2, x_3) = a_1x_1 + a_2x_2 + a_3x_1x_3 + a_4x_2x_3 + a_5x_1^2x_2 \tag{50}$$

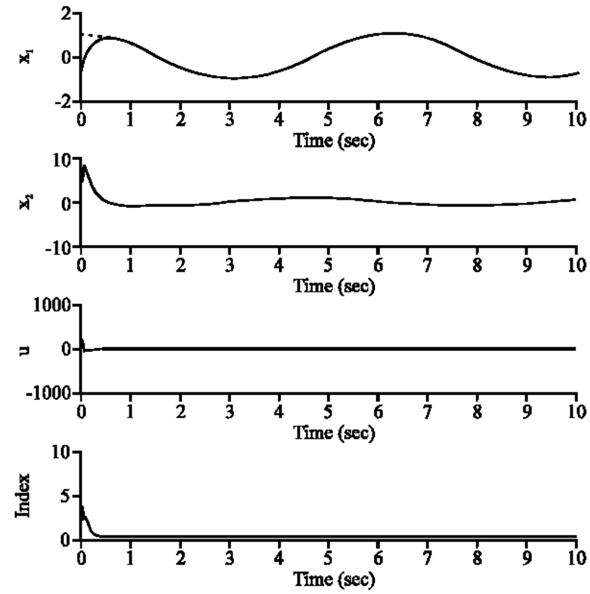

Fig. 8: Simulation results of Duffing oscillator system, Theorem 2, $q = 7$

where, $a_1 = -46656$, $a_2 = 35136$, $a_3 = 1980$, $a_4 = -1296$ and $a_5 = -36$. U is as the following:

$$U = a_5x_1u \tag{51}$$

The system dynamic function would be online estimated by the $GRBF_{NN}$ identifier. The structure of $GRBF_{NN}$ is shown in Fig. 1. A $GRBF_{NN}$ identifier with five hidden nodes is utilized to approach the system dynamic function of the chaotic system.

In addition, the control parameters are selected as $c_1 = 5$, $c_2 = 5$ and $c_3 = 5$. The trajectory command is set as: $x_d = \cos(t)$.

The simulation results of the GRBABC with consider Eq. 29 are shown in Fig. 9.

The simulation results of the GRBABC with consider Eq. 40 and k = 5 are shown in Fig.10.

The performance index I is defined as $I = \sqrt{e_1^2 + e_2^2 + e_3^2}$. The performance index I is shown that the proposed GRBABC can achieve favorable tracking performance.

Figure 9 and 10 are shown that the results have good performance compare to other papers like that (Tan *et al*., 2003). Consider that the control effort is limited.

These results are shown that the better tracking performance can be achieved by using Theorem 1 compare to Theorem 2.

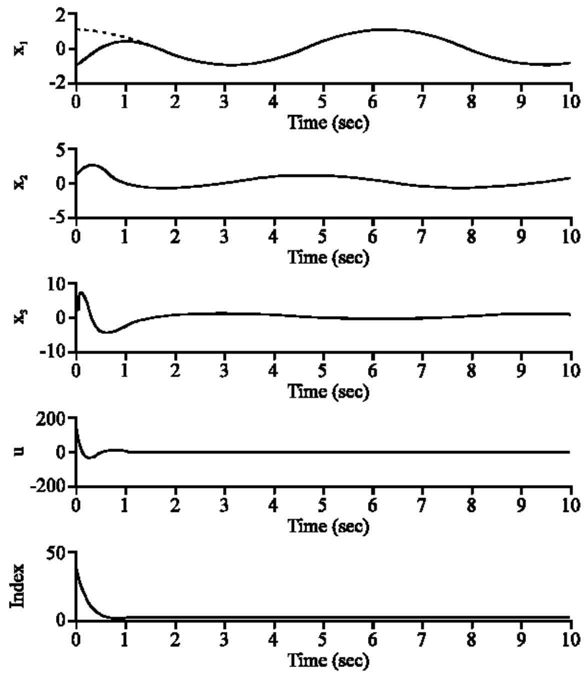

Fig. 9: Simulation results of Lü system, Theorem 1

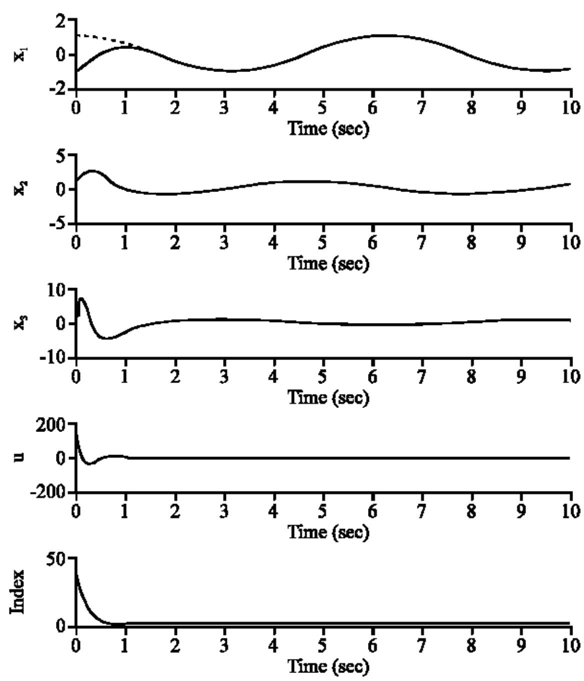

Fig. 10: Simulation results of Lü system, Theorem 2

CONCLUSIONS

For some systems, since the dynamic characteristics of the control system are nonlinear and the precise models are difficult to obtain, the model-based control approaches are difficult to be implemented. To overcome this drawback, a novel GRBABC system has been proposed.

In the neural backstepping controller, a $GRBF_{NN}$ identifier is utilized to online estimate the system dynamic function. The two adaptive laws of the GRBABC system are synthesized using the two type Lyapunov functions so that the asymptotic stability of the control system can be guaranteed.

Finally, two chaotic systems (Duffing Oscillator and Lü systems) are simulated to illustrate the effectiveness of the proposed design method. Simulation results verified that the proposed GRBABC system with adaption law Eq. 29 can achieve favorable tracking performance of these nonlinear systems.

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