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Design of Variable Structure Synchronization Controller for Two Different Hyperchaotic Systems Containing Nonlinear Inputs

[1]Chao-Lin Kuo, [2]Cheng-Chi Wang and [3]Neng-Sheng Pai
[1]Department of Electrical Engineering,
[2]Department of Mechanical Engineering, Far-East University, No. 49,
Jung-Hwa Road, Hsin-Shih Town Tainan 744, Taiwan, Republic of China
[3]Department of Electrical Engineering, National Chin-Yi University of Technology,
Taichung 411, Taiwan, Republic of China

Abstract: This study introduces a variable structure technology for the synchronization of chaos between two different hyperchaotic systems with input nonlinearity. Based on Lyapunov stability theory, a sliding mode controller and some generic sufficient conditions for global asymptotic synchronization are designed such that the error dynamics of the hyperchaotic Rössler and hyperchaotic Chen systems satisfy stability in the Lyapunov sense in spit of the input nonlinearity. The numerical simulation results demonstrate the validity and feasibility of the proposed controller.

Key words: Hyperchaotic systems, Chaos synchronization, variable structure control, input nonlinearity

INTRODUCTION

Since, the ideal of synchronizing two identical chaotic systems from different initial conditions was first introduced by Carroll and Pecora, Chaos synchronization has gained a lot of attention among scientists from variety of research fields over the last few years (Carroll and Pecora, 1990, 1991; Chen and Dong, 1998). Chaos synchronization can be applied in the vast areas of physics and engineering science, especially in secure communication (Kocarev and Parlitz, 1995; Murali and Lakshmanan, 1998). In order to achieve the synchronization, a nonlinear controller that obtains signals from the master and slave systems and manipulates the slave system should be designed. Recently, many control methods have been developed to achieve Chaos synchronization between two identical chaotic systems with different initial conditions (Yassen, 2003; Liao, 1998; Fang *et al*., 1999; Yau *et al*., 2005, 2006; Yau, 2004). However, most of these methods are only applicable to the Chaos synchronization of two systems that are identical in every aspect and which contain only low dimensional attractors. This is in stark contrast to many real-word applications of the technology. In fact, in systems such as laser array, biological systems and cognitive processes, it is hardly the case that every component can be assumed to be identical. In the area of communications security for example, the adoption of higher dimensional chaotic systems as well as systems with more than one positive Lyapunov exponents has been proposed for use to generate more complex dynamics. Methods are therefore needed to synchronize chaotic systems that are both different and are of high dimensions. Moreover, when the controller is realized in practical physical systems, due to physical limitations of actuators, the nonlinearities in control input do exist. The presence of nonlinearities in control input may cause serious influence of system performance and decrease the system response. Besides, the nonlinearity in control input may cause the chaotic system perturbed to unpredictable results because the chaotic system is very sensitive to any system parameters. Therefore, its effect cannot be ignored in analysis of control design and realization for Chaos synchronization. Thus, the derivation of controller with input nonlinearity for Chaos synchronization is an important problem.

In the study, Ho and Hung (2002), Yassen (2005), Zhang *et al*. (2006) and Agiza and Yassen (2001) used active control techniques to synchronize two different chaotic systems are either only concerns some low dimension chaotic systems or the input nonlinearity is not discussed. In this case of input nonlinearity, the

Corresponding Author: Chao-Lin Kuo, Department of Electrical Engineering, Far-East University, No. 49, Jung-Hwa Road, Hsin-Shih Town Tainan 744, Taiwan, Republic of China Tel: 886-6-5979566/5400 Fax: 886-6-5977570

applications of above method are shown by Ho and Hung (2002), Yassen (2005), Zhang *et al.* (2006) and Agiza and Yassen (2001) are hard to achieve.

In this study, the goal is to force the two different hyperchaotic Rössler system and hyperchaotic Chen system to be synchronized even if they are subjected to input nonlinearity. The method of active sliding mode control law is applied to control the Chaos synchronization system. The technique requires two stages. The first stage is to select stable sliding surfaces for the desired dynamics and the second stage is to design a switching control law to achieve the stable sliding surfaces. Finally, numerical simulation is carried to confirm the validity of the proposed theoretical approach.

SYSTEM DESCRIPTION AND PROBLEM FORMULATION

In this study, two different hyperchaotic systems included Rössler system and Chen system are described in the follows. In order to observe the synchronization behavior in these two systems, it is assumed that the hyperchaotic Rössler system drives the hyperchaotic Chen system. Therefore, the master and slave systems are shown in the follows:

$$\begin{cases} \dot{x}_1 = -x_2 - x_3 \\ \dot{x}_2 = x_1 + a_1 x_2 + x_4 \\ \dot{x}_3 = b_1 + x_1 x_3 \\ \dot{x}_4 = -c_1 x_3 + d_1 x_4 \end{cases} \quad (1)$$

Slave system:

$$\begin{cases} \dot{y}_1 = a_2(y_2 - y_1) + y_4 + \phi_1(u_1(t)) \\ \dot{y}_2 = d_2 y_1 - y_1 y_3 + c_2 y_2 + \phi_2(u_2(t)) \\ \dot{y}_3 = y_1 y_2 - b_2 y_3 + \phi_3(u_3(t)) \\ \dot{y}_4 = y_2 y_3 + \kappa y_4 + \phi_4(u_4(t)) \end{cases} \quad (2)$$

where, $\phi_1(u_1)$, $\phi_2(u_2)$, $\phi_3(u_3)$, $\phi_4(u_4)$ are the nonlinear control inputs attached in the slave system. Let the synchronization error vector state is:

$$e = [e_1 \quad e_2 \quad e_3 \quad e_4]^T = [x_1 - y_1 \quad x_2 - y_2 \quad x_3 - y_3 \quad x_4 - y_4]^T$$

Substitution Eq. 1 and 2 into the error state, the error dynamic equations can be obtained as follows:

$$\begin{cases} \dot{e}_1 = -x_2 - x_3 - a_2(y_2 - y_1) - y_4 - \phi_1(u_1) \\ \dot{e}_2 = x_1 + a_1 x_2 + x_4 - d_2 y_1 + y_1 y_3 - c_2 y_2 - \phi_2(u_2) \\ \dot{e}_3 = b_1 + x_1 x_3 - y_1 y_2 + b_2 y_3 - \phi_3(u_3) \\ \dot{e}_4 = -c_1 x_3 + d_1 x_4 - y_2 y_3 - \kappa y_4 - \phi_4(u_4) \end{cases} \quad (3)$$

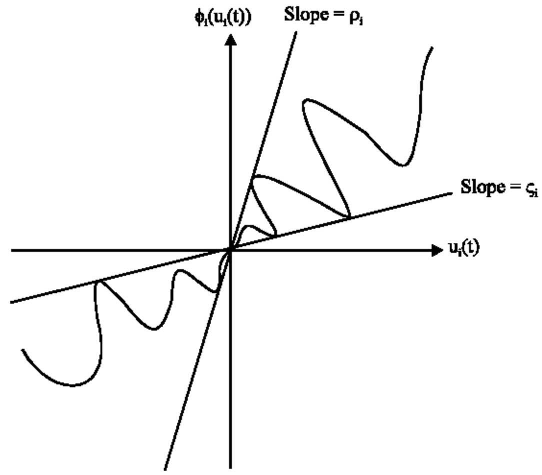

Fig. 1: A scalar nonlinear function $\phi_i(u_i(t))$ inside sector $[\varsigma_i, \rho_i]$, i = 1, 2, 3, 4

The $\phi_i(u_i(t)) \in C^1(R^n \to R)$ is a continues nonlinear function with $\phi_i(0) = 0$ and $u_i(t) \to \phi_i(u_i(t))$ is inside sector $[\varsigma_i, \rho_i]$ (i = 1, 2, 3, 4), i.e.,

$$\varsigma_i u_i^2 \le u_i \phi_i(u_i) \le \rho_i u_i^2 \quad (4)$$

where, ς_i and ρ_i are nonzero positive constants. A nonlinear function $\phi_i(u_i(t))$ is shown in Fig. 1.

Now, the sliding surfaces suitable for the application can be defined as:

$$S_i = e_i + \int_0^t \lambda_i e_i(\tau) d\tau, \; i = 1, 2, 3, 4 \quad (5)$$

where, $S_i(t) \in R$ and λ_i is the design parameters which can be determined later. For the existence of the sliding mode (Slotine and Li, 1991), it is necessary and sufficient that:

$$S_i = e_i + \int_0^t \lambda_i e_i(\tau) d\tau = 0, \; i = 1, 2, 3, 4 \quad (6)$$

and

$$\dot{S}_i = \dot{e}_i + \lambda_i e_i = 0, \; i = 1, 2, 3, 4 \quad (7)$$

Therefore, the following sliding mode dynamics can be obtained as:

$$\dot{e}_i = -\lambda_i e_i, \; i = 1, 2, 3, 4 \quad (8)$$

Obviously, if the design parameters $\lambda_i > 0$, i = 1, 2, 3, 4, the stability of Eq. 6 are surely guaranteed, that is $\lim_{t \to \infty} e_i(t) \to 0$. Thus, the slave system will be derived to master system by designing the appropriate signal control inputs $u_i(t)$, i = 1, 2, 3, 4. Meanwhile, it is worthy of that the values of parameters $\lambda_i>0$, i = 1, 2, 3, 4, are also relative to the speed of system response.

SLIDING MODE CONTROL LAW WITH INPUT NONLINEARITY

We choose a control law of the form:

$$u_i = \gamma_i \eta_i \operatorname{sign}(S_i), \; \gamma_i > \frac{1}{\varsigma_i}, \; i = 1,2,3,4 \qquad (9)$$

Where:

$$\begin{cases} \eta_1 = |-x_2 - x_3 - a_2(y_2 - y_1) - y_4 + \lambda_1 e_1| \\ \eta_2 = |x_1 + a_1 x_2 + x_4 - d_2 y_1 + y_1 y_3 - c_2 y_2 + \lambda_2 e_2| \\ \eta_3 = |b_1 + x_1 x_3 - y_1 y_2 + b_2 y_3 + \lambda_3 e_3| \\ \eta_4 = |-c_1 x_3 + d_1 x_4 - y_2 y_3 - \kappa y_4 + \lambda_4 e_4| \end{cases}$$

Based on the control law (Eq. 9), the reaching condition $s(t)\dot{s}(t) < 0$ is guaranteed in the following theorem, that is, the proposed Eq. 9 will derive the Eq. 3 with nonlinear inputs onto the sliding mode s(t) = 0.

Theorem 1: Consider the error dynamics Eq. 3 with input nonlinearities. The hitting condition of the sliding mode is satisfied, if the control $u_i(t)$ is given by Eq. 9 for i = 1, 2, 3, 4.

Proof: Letting the Lyapunov function of the system be:

$$V = \frac{1}{2}\left(S_1^2 + S_2^2 + S_3^2 + S_4^2\right)$$

Then its derivative with respect to time is:

$$\begin{aligned} \dot{V} &= S_1\dot{S}_1 + S_2\dot{S}_2 + S_3\dot{S}_3 + S_4\dot{S}_4 \\ &= S_1(\dot{e}_1 + c_1 e_1) + S_2(\dot{e}_2 + c_2 e_2) + S_3(\dot{e}_3 + c_3 e_3) + S_4(\dot{e}_4 + c_4 e_4) \\ &= S_1(-x_2 - x_3 - a_2(y_2 - y_1)y_4 + \lambda_1 e_1 - \phi_1(u_1)) \\ &\quad + S_2(x_1 + a_1 x_2 + x_4 - d_2 y_1 + y_1 y_3 - c_2 y_2 + \lambda_2 e_2 - \phi_2(u_2)) \\ &\quad + S_3(b_1 + x_1 x_3 - y_1 y_2 + b_2 y_3 + \lambda_3 e_3 - \phi_3(u_3)) \\ &\quad + S_4(-c_1 x_3 + d_1 x_4 - y_2 y_3 - \kappa y_4 + \lambda_4 e_4 - \phi_4(u_4)) \\ &\le |S_1||-x_2 - x_3 - a_2(y_2 - y_1)y_4 + \lambda_1 e_1| - S_1\phi_1(u_1) \\ &\quad + |S_2||x_1 + a_1 x_2 + x_4 - d_2 y_1 + y_1 y_3 - c_2 y_2 + \lambda_2 e_2| - S_2\phi_2(u_2) \\ &\quad + |S_3||b_1 + x_1 x_3 - y_1 y_2 + b_2 y_3 + \lambda_3 e_3| - S_3\phi_3(u_3) \\ &\quad + |S_4||-c_1 x_3 + d_1 x_4 - y_2 y_3 - \kappa y_4 + \lambda_4 e_4| - S_4\phi_4(u_4) \\ &\le \eta_1|S_1| - \varsigma_1\gamma_1\eta_1|S_1| + \eta_2|S_2| - \varsigma_2\gamma_2\eta_2|S_2| \, \eta_2|S_2| - \varsigma_3\gamma_3\eta_3|S_3| \\ &\quad + \eta_4|S_4| - \varsigma_4\gamma_4\eta_4|S_4| \\ &\le (1 - \gamma_1\varsigma_1)\eta_1|S_1| + (1 - \gamma_2\varsigma_2)\eta_2|S_2| \\ &\quad + (1 - \gamma_3\varsigma_3)\eta_3|S_3| + (1 - \gamma_4\varsigma_4)\eta_4|S_4| \end{aligned} \qquad (10)$$

Where:

$$\begin{aligned} & u_i\phi_i(u_i) \ge \varsigma_i u_i^2 \\ & \Rightarrow \gamma\eta_i \operatorname{sigu}(S_i)\phi(u_i) \ge \varsigma_i \gamma^2 \eta_i^2 \operatorname{sign}^2(S_i) \\ & \Rightarrow \gamma\eta_i |S_i| S_i \phi(u_i) \ge \varsigma_i \gamma\eta_i |S_i||S_i|, \qquad \text{For } i = 1,2,3,4 \\ & \Rightarrow S_i\phi(u_i) \ge \varsigma_i \gamma_i \eta_i |S_i| \\ & \Rightarrow -S_i\phi(u_i) \le -\varsigma_i \gamma_i \eta_i |S_i| \end{aligned}$$

Therefore, if:

$$\gamma_i > \frac{1}{\varsigma_i}, \quad \text{For } i = 1,2,3,4 \qquad (11)$$

then $\dot{V} < 0$, confirming the presence of reaching condition. Thus the proof is achieved completely.

Remarks: The controller designed in this study is robust. Therefore, we can increase the value of η_i to overcome the effect of disturbances which are bounded. The performance of proposed algorithm is still kept under the disturbance.

NUMERICAL SIMULATIONS

In this simulation, the 4th order Runge-Kutta algorithm was used to solve the sets of differential equations related to the master and slave systems with a time grid of 0.0001. We selected the parameters of the hyperchaotic Rössler system as a_1 = 0.25, b_1 = 3, c_1 = 0.5, d_1 = 0.05 and the parameters of the hyperchaotic Chen systems as a_2 = 35, b_2 = 3, c_2 = 12, d_2 = 7, κ = 0.5. The initial values of hyperchaotic Rössler and Chen systems are $x(0) = [x_1(0) \;\; x_2(0) \;\; x_3(0) \;\; x_4(0)] = [-15 \;\; -10 \;\; 20 \;\; 15]$, $y(0) = [y_1(0) \;\; y_2(0) \;\; y_3(0) \;\; y_4(0)] = [10 \;\; 15 \;\; 10 \;\; 5]$. In the synchronization example, we selected $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 2$ to result in stable sliding modes and the nonlinear inputs are defined as:

$$\phi_i(u_i(t)) = [0.6 + 0.3 \cdot \cos(u_i(t))]u_i(t), \; i = 1,2,3,4 \qquad (12)$$

Furthermore, it is assumed that the slope of nonlinear sectors in these three synchronization examples are $\varsigma_1 = \varsigma_2 = \varsigma_3 = \varsigma_4$ and $\rho_1 = \rho_2 = \rho_3 = \rho_4 = 0.9$ and the parameters $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 5$ are selected to satisfy the Eq. 11. The time responses of the hyperchaotic Chen system controlled by the hyperchaotic Rössler system is shown in Fig. 2a-d. It can be see that the slave system synchronizes with the master system in spite of input nonlinearity. Obviously, the synchronization errors converge asymptotically to zero after the control is active at time t = 10 sec in Fig. 3.

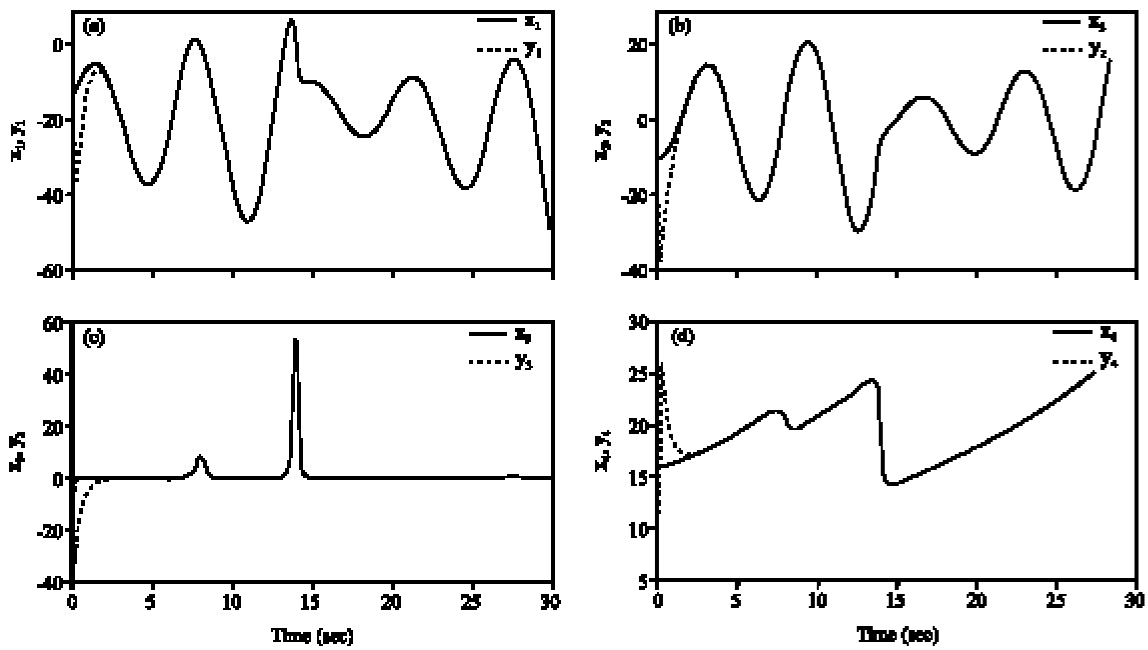

Fig. 2: The time history of controlled hyperchaotic Rössler (x_1, x_2, x_3, x_4) and Chen (y_1, y_2, y_3, y_4) chaotic systems: (a) x_1, y_1 versus time t, (b) x_2, y_2 versus time t, (c) x_3, y_3 versus time t and (d) x_4, y_4 versus time t

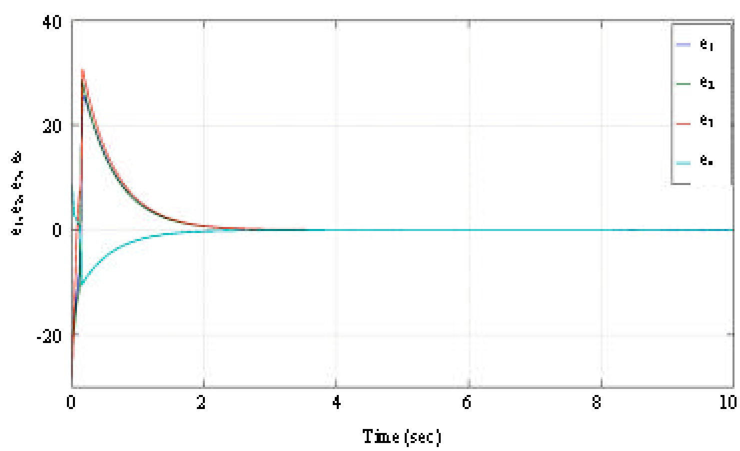

Fig. 3: The synchronization time response of error dynamics of controlled hyperchaotic Rössler and Chen systems

CONCLUSION

In this study, we introduced a sliding mode control technique to synchronize the hyperchaotic Rössler system and hyperchaotic Chen system. Based on Lyapunov stability theorem, an effective control method for synchronizing different chaotic systems has been proposed using variable structure design. The proposed sliding mode control enables stabilization of synchronization error dynamics to zeros asymptotically in spit of input nonlinearity. Numerical simulation results are presented to verify the effectiveness of the proposed synchronization technique. The main feature of this approach is that it gives the flexibility to construct a control law so that the control strategy can be easily extended to any dimensional chaotic systems.

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