Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 9 (18): 3343-3349, 2009
ISSN 1812-5654

A New Method Based on Data Envelopment Analysis to Derive Weight Vector in the Group Analytic Hierarchy Process

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Abstract: In this study, we propose a new method based on Data Envelopment Analysis (DEA) for the weight vector derivation from the pair-wise comparison matrices in the group AHP that is called DEA-WDGD. In this method, we can use both interval importance weight and relative importance weight for each decision maker. In this method, solving only one Linear Programming (LP) model is enough for the local weights derivation of several pair-wise comparison matrices in the group decision making and there is no need to normalize the derived weight vector. Three numerical examples are examined. Also, the DEA-WDGD is compared with the DEA method which has been recently proposed for weights derivation in the group AHP. The results show that DEA-WDGD provides better weights. The Simple Additive Weighting (SAW) method is utilized to aggregate local weights without needing to normalize them.

Key words: Group decision making, linear programming, data envelopment analysis, analytic hierarchy process, weight vector

INTRODUCTION

Analytic Hierarchy Process (AHP), since invention has been a tool at the hands of decision makers and researchers; it is one of the most widely used Multiple Criteria Decision Making (MCDM) tools (Vaidya and Kumar, 2006). The AHP is used in almost all application related to MCDM in the last 20 years (Ho, 2008). How to weight vector derivation from pair-wise comparison matrices has being an important research topic in the AHP. Apart from Saaty's well-known Eigenvector Method (EM), quite a number of alternative approaches have been suggested such as the Weighted Least-Square Method (WLSM), the Logarithmic Least Squares Method (LLSM), the Geometric Least Squares Method (GLSM), the Fuzzy Programming Method (FPM), the gradient method (GEM) and so on (Wang and Chin, 2009). The utilization of integrated DEA-AHP approach is not new and there have been some utilizations of this approach. For example, Shang and Sueyoshi (1995) used it for the selection of flexible manufacturing system, Saen *et al*. (2005) applied this approach for efficiency evaluation of 18 Iranian research organizations, Azadeh *et al*. (2008) used combined DEA-AHP for railway system improvement and Jyoti Banwet and Deshmukh (2008) used the integrated DEA-AHP for the performance evaluation of Indian research and development organizations. Some studies have used this approach to solve the facility layout design problems (Yang and Kuo, 2003; Ertey *et al*., 2006). In these studies, AHP was often used for the evaluation of alternatives with respect to qualitative criteria and DEA for final ranking. In another kind of integrated DEA-AHP applications, AHP has been used for full ranking the DMUs used in DEA (Sinuany-Stern *et al*., 2000).

Most of the Multi Criteria Decision Making (MCDM) techniques require numerous parameters, which are difficult to be determined precisely requiring extensive sensitivity analysis. On the other hand, the main limitation in DEA is that standard formulation of DEA creates a separate linear program for each DMU. This will be computationally intensive when the number of DMUs is large (Raju and Kumar, 2006). Some studies have used DEA approach for weights derivation from pair-wise comparison matrices in AHP. Ramanathan (2006) has proposed DEAHP method. Wang *et al*. (2008a) has proposed a DEA model with Assurance Region (AR) for priority derivation in AHP. Recently, Wang and Chin (2009) have proposed two LP models for weight vector derivation in the AHP. Their models need to solve separate LP models for each criterion (alternative). Also, derived weights are not normalized and it only uses the relative importance weight for each decision maker.

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In this study, we propose a new method for the weight vector derivation of the pair-wise comparison matrices for alternatives or criteria in the group AHP situation, which uses the concept of variable weights of DEA. We called this method as the method based on the DEA for the weight vector derivation in the group decision making (DEA-WDGD). Using this method, solving only one LP model is enough for the local weights derivation of several pair-wise comparison matrices in the group decision making. Also, the obtained weights are normal and it is not necessary to be normalized again. We can use both the relative importance weight and the interval importance weight for identifying the possible weights for each decision maker.

Analytic hierarchy process: Analytic Hierarchy Process (AHP) was developed by Saaty (1980). The AHP is one the most popular MCDM tools for formulating and analyzing decisions (Ramanathan, 2006). The strength of the AHP lies in its ability to arrange complex multi-person and multi-attribute problems hierarchically and then to investigate each level of the hierarchy separately, combining the results as the analysis progresses (Franklin-Liu and Hai, 2005). The AHP was adopted in personal, social, manufacturing sector, political, engineering, education, industry, government, sports and management (Vadiya and Kumar, 2006).

Some key and basic steps involved in this methodology are as follow (Vadiya and Kumar, 2006):

- State the problem
- Broaden the objectives of the problem or consider all actors, objectives and its outcome
- Identify the criteria that influence the behavior
- Structure the problem in a hierarchy of different levels constituting goal, criteria, sub-criteria and alternatives
- Compare each element in the corresponding level and calibrate them in the numerical scale. This requires $n(n-1)/2$ comparisons, where n is the No. of elements with the considerations that diagonal elements are equal or 1 and the other elements will simply be the reciprocals of the earlier comparisons
- Perform calculations to find the maximum Eigen-value, Consistency Index (CI), Consistency Ratio (CR) and normalized values for each criterion/alternative
- If the maximum Eigen-value, CI and CR are satisfied then decision is taken based on the normalized values; else the procedure is repeated till these values lie in a desired range

Data envelopment analysis: Data Envelopment Analysis (DEA) is a nonparametric approach which is developed by Charnes *et al.* (1978) based on linear programming to evaluate relative efficiency of similar Decision Making Units (DMUs) and utilize multiple inputs to produce multiple outputs. The DEA application for assessing efficiency includes three stages (Golany and Roll, 1989): the first stage is to identify appropriate DMUs. Then, the inputs and outputs must be selected for the measurement of relative efficiency of DMUs. Finally, DEA model is applied to analyze the data.

Assume that there are N DMUs producing J outputs using I inputs. Let the mth DMU produce outputs y_{mj} using x_{mi} inputs. The resulting output-input structure of DMUs is shown in Table 1. The objective of the DEA models is to identify the DMU that produces the highest amounts of outputs by consuming lowest inputs. The efficiencies of other inefficient DMUs are obtained relative to the efficient DMUs and are assigned efficiencies score between zero and one. The efficiency scores are computed using mathematical programming (Ramanathan, 2006).

Weights derivation methods from pair-wise comparison matrices by using DEA approach: Here, we go over the papers that have used DEA for weights derivation from pair-wise comparison matrices.

Ramanathan (2006) used DEA for the local weights derivation from pair-wise comparison matrices for alternatives in the AHP. He proposed a new method and called it DEAHP. Let $A = (a_{ij})_{n\times n}$ be a pair-wise comparison matrix with $a_{ii} = 1$ and $a_{ij} = 1/a_{ij}$ and $W = (w_1,...,w_n)^T$ be its weight vector. The DEAHP has n outputs and one dummy constant input. Based on the input-oriented CCR model 1, the alternatives weights were calculated separately for each alternative using a separate linear programming model. This method was used for the aggregation of the local weights to get final weights. Using DEAHP for consistent matrices leads to estimating correct weights. Mehmet *et al.* (2007) used the DEAHP method in a supplier selection problem.

Table 1: DMUs, inputs and outputs for a DEA model

	Output 1	Output 2	...	Output J	Input 1	Input 2	...	Input I
DMU 1	y_{11}	y_{12}	...	y_{1J}	x_{11}	x_{12}	...	x_{1I}
DMU 2	y_{21}	y_{22}	...	y_{2J}	x_{21}	x_{22}	...	x_{2I}
...	...	...	...	...	...	...	...	...
DMU N	y_{N1}	y_{N2}	...	y_{NJ}	x_{N1}	x_{N2}	...	x_{NI}

Ramanathan (2006)

$$\text{Max } w_0 = \sum_{j=1}^{n} a_{0j} v_j \quad \text{Subject to} \begin{cases} u_1 = 1, \\ \sum_{j=1}^{n} a_{ij} v_j - u_1 \le 0, & i = 1,\ldots,n, \\ u_1, v_j \ge 0, & j = 1,\ldots,n \end{cases} \tag{1}$$

Wang *et al*. (2008a) showed that the DEAHP method has some drawbacks and presented that the DEAHP may produce counterintuitive local weights for inconsistent pair-wise comparison matrices and the DEAHP is sometimes over insensitive to some comparisons in a pair-wise comparison matrix. To overcome these drawbacks of the DEAHP, They proposed a DEA method with Assurance Region (AR) for weights derivation in the AHP and finally SAW is used to get final weights.

Recently, Wang and Chin (2009) proposed a new DEA method for the weight vector derivation from a pair-wise comparison matrix, extended it to the group AHP situation and utilized the SAW to get final weights. They proposed model 2 for the weight vector derivation from a pair-wise comparison matrix and proposed model 3 in the group AHP situation. In these models, $A^k = (a_{ij}^k)_{n \times n}$ is a pair-wise comparison matrix provided by the k[th] Decision Makers (DM_k) (k = 1,...,m) and $h_k > 0$ is its relative importance weight. In model 2 and 3, the weights were calculated for each alternative or criterion by using a separate model.

$$\text{Max } w_0 = \sum_{j=1}^{n} a_{0j} z_j \quad \text{Subject to} \begin{cases} \sum_{j=1}^{n} (\sum_{i=1}^{n} a_{ij}) z_j = 1, \\ \sum_{j=1}^{n} a_{ij} z_j \ge n z_i, & i = 1,\ldots,n, \\ z_j \ge 0, & j = 1,\ldots,n \end{cases} \tag{2}$$

$$\text{Max } w_0 = \sum_{j=1}^{n} (\sum_{k=1}^{m} h_k a_{oj}^{(k)}) z_j \quad \text{Subject to} \begin{cases} \sum_{j=1}^{n} (\sum_{k=1}^{m} \sum_{i=1}^{n} h_k a_{ij}^{(k)}) z_j = 1, \\ \sum_{j=1}^{n} (\sum_{k=1}^{m} h_k a_{ij}^{(k)}) z_j \ge n z_i, & i = 1,\ldots,n, \\ z_j \ge 0, & j = 1,\ldots,n \end{cases} \tag{3}$$

In some studies, linguistic terms and ordinal numbers have used in the decision making to rank DMUs and this has not been based on the pair-wise comparison matrix. For example, Wang *et al*. (2007) proposed two LP models and a nonlinear programming (NLP) model to assess weights and utilized ordinal numbers to rank DMUs. Wang *et al*. (2008b) proposed an integrated DEA-AHP methodology to evaluate bridge risks of hundreds or thousands of bridge structures, based on the maintenance priorities. They utilized AHP only to determine the criteria weights. Linguistic terms such as high, medium, low and none were utilized to assess bridge risks under each criterion and DEA methodology to determine the value of the linguistic terms. Finally, they used the SAW method to get final weight for each bridge structure.

THE DEA-WDGD METHOD

Using the concept of DEA in DEA-WDGD method: In DEA models, DMUs lay in the rows and outputs and inputs indices lay in columns of datasheet table. The purpose is either to maximize outputs or to minimize inputs. Now, Let $A^k = (a_{ij}^k)_{n \times n}$ be a pair-wise comparison matrix which is provided by the DM_k with $a_{ii} = 1$, $a_{ij} = 1/a_{ij}$ for $j \ne i$ and (k = 1,...m). The concept of the new DEA method is shown in Table 2. In Table 2, ith row (i = 1,...,n) shows the ith criterion or alternative which is viewed as a DMU. Thus, the DEA-WDGD method will have n DMUs. Elements of A^k are the pair-wise comparisons of the DM_k and the sum of its ith row is shown in the kth column and ith row of Table 2 for (k = 1,..,m; i = 1,..,n). Each column of Table 2 is viewed as an output. Thus, the DEA-WDGD method will have m outputs.

We propose the following model for the local weights derivation from the pair-wise comparison matrices used in the group AHP:

$$\text{Max } \sum_{i=1}^{n} w_i \quad \text{Subject to} \begin{cases} \sum_{k=1}^{m} (\sum_{j=1}^{n} a_{ij}^k) v_k - w_i = 0, & i = 1,\ldots,n \\ \sum_{i=1}^{n} w_i \le 1, \\ v_1 - v_k = 0, & k = 2,\ldots,m, \\ w_i \ge 0;\ v_k \ge 0, & i = 1,\ldots,n;\ k = 1,\ldots m \end{cases} \tag{4}$$

where, w_i(i = 1,...,n) are the local weights of criteria or alternatives, v_k is the output weight of DM_k which is determined by model 4 for (k = 1,..,m) and a_{ij}^k (i, j = 1,...,n; k = 1,...,m) are elements of pair-wise comparison matrices. Since, the objective function maximizes the sum of weights, it reaches to its maximum value of the optimal

Table 2: The proposed DEA view of pair-wise comparison matrices in the group AHP

Criterion (Alternative)	Outputs with DEA view: DM DM_1	...	DM_m
Alternatives with DEA view			
1	$\sum_{j=1}^{n} a_{1j}^1$	...	$\sum_{j=1}^{n} a_{1j}^m$
⋮	⋮	...	⋮
i	$\sum_{j=1}^{n} a_{ij}^1$	...	$\sum_{j=1}^{n} a_{ij}^m$
⋮	⋮	...	⋮
n	$\sum_{j=1}^{n} a_{nj}^1$	...	$\sum_{j=1}^{n} a_{nj}^m$

solution. So, the resulted weights will be normalized. In this LP model, Z is maximized under the condition that the same weights are used in evaluating all DMUs, this principle is in accordance of DEA. One of the features of This LP model is that it will be used only once for the weight vector derivation from pair-wise comparison matrices in the group AHP.

The DEA-WDGD method with considering the importance of decision makers' opinions

Relative importance weights for Dms: Let $A^k = (a_{ij}^k)_{n\times n}$ be a pair-wise comparison matrix provided by the DM_k (k = 1...,m) and h_k be its relative importance weight that satisfying:

$$\sum_{k=1}^{m} h_k = 1$$

Thus:

$$v_1 = h_1,\ v_2 = h_2, \ldots,\ v_k = h_k, \ldots,\ v_{m-1} = h_{m-1},\ v_m = h_m \quad (5)$$

We can not use the Eq. 5 in model 4 because v_k is variable in model 4. So, Eq. 5 are expressed equivalently as:

$$\begin{cases} h_1 v_2 - h_2 v_1 = 0, \\ h_2 v_3 - h_3 v_2 = 0, \\ \vdots \\ h_{m-1} v_m - h_m v_{m-1} = 0 \end{cases} \quad (6)$$

Now, we can replace the linear constraints (6) instead of constraints ($v_1 - v_k = 0$, k = 2,...,m) of model 4.

Interval importance weight for Dms: Let $A^k = (a_{ij}^k)_{n\times n}$ be a pair-wise comparison matrix provided by the DM_k (k = i,...,m) and h_k be its interval importance weight that $\alpha_k \le h_k \le \beta_k$ with α_k, $\beta_k \in [0, 1]$. With considering the first and second constraints of the model 4,

$$\sum\nolimits_{k=1}^{m} v_k \ne 1$$

and will be less than 1. Thus, h_k can be expressed equivalently as:

$$(v_k / \sum\nolimits_{k=1}^{m} v_k)$$

in model 4 which is normalized weight importance. Therefore:

Let $$h_k = \frac{v_k}{\sum_{k=1}^{m} v_k}$$

If $\alpha_k \le h_k \le \beta_k$, then it can be expressed equivalently as:

$$\begin{cases} \sum_{k=1}^{m} \beta_k v_k - v_k \ge 0, \\ \sum_{k=1}^{m} \alpha_k v_k - v_k \le 0 \end{cases} \quad (7)$$

Now, we should replace the linear constraints (7) instead of constraints ($v_1 - v_k = 0$, k = 2,...,m) of model 4.

NUMERICAL EXAMPLES

We provide three numerical examples to illustrate the potential applications of the DEA-WDGD in the group AHP situation. The first example compares the DEA-WDGD with Wang and Chin (2009) method and shows that derived weights of the DEA-WDGD are better than it. The second one compares derived weights from DEA-WDGD with Saaty's eigenvector method. The third example utilizes the interval importance weights for DMs.

Example 1: Consider four pair-wise comparison matrices about the relative importance of five decision criteria in the group AHP, which are borrowed from Wang and Chin (2009). Four DMs are provided by four DMs which are shown as follow. Each DM has relative importance weight (h_k).

$$A^{(1)} = \begin{bmatrix} 1 & 1 & 3 & 4 & 1 \\ 1 & 1 & 1 & 1/2 & 1/3 \\ 1/3 & 1 & 1 & 1/2 & 1/2 \\ 1/4 & 2 & 2 & 1 & 1/2 \\ 1 & 3 & 2 & 2 & 1 \end{bmatrix}, \quad A^{(2)} = \begin{bmatrix} 1 & 8 & 1 & 2 & 2 \\ 1/8 & 1 & 1/8 & 1/3 & 1/5 \\ 1 & 8 & 1 & 2 & 2 \\ 1/2 & 3 & 1/2 & 1 & 1 \\ 1/2 & 5 & 1/2 & 1 & 1 \end{bmatrix},$$

$$A^{(3)} = \begin{bmatrix} 1 & 8 & 1 & 1 & 1 \\ 1/8 & 1 & 1/8 & 1/5 & 1/8 \\ 1 & 8 & 1 & 2 & 1 \\ 1 & 5 & 1/2 & 1 & 1 \\ 1 & 8 & 1 & 1 & 1 \end{bmatrix}, \quad A^{(4)} = \begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 1/2 & 1 & 1/2 & 1 & 1 \\ 1 & 2 & 1 & 1/2 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

This example is solved by DEA-WDGD method and Wang and Chin method (model 3 in this study). The derived weights by these methods are shown in Table 3. In order to compare the quality of local weights derived by different methods, we use Fitting Performance (FP) index, which is measured by the following Euclidean distance (Wang *et al*., 2008a):

$$FP = \sqrt{\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}(a_{ij} - w_i / w_j)^2}, \qquad a_{ij} \in E \quad (8)$$

The FP value of a consistent comparison matrix is zero because its elements can be written as ratios of local

Table 3: The local weights produced by different methods for example 1

Sample	Relative importance weights of four DMs (%)	Method	Local weights w_1	w_2	w_3	w_4	w_5	FP
1	50, 30, 15, 5	Wang and Chin	0.3091	0.0906	0.2054	0.1662	0.2287	0.378
		DEA-WDGD	0.3015	0.0768	0.2154	0.1674	0.2388	0.354
2	50, 16.7, 16.7, 16.7	Wang and Chin	0.2967	0.1006	0.1912	0.1762	0.2353	0.300
		DEA-WDGD	0.2909	0.0882	0.1991	0.1778	0.2440	0.290
3	25, 25, 25, 25	Wang and Chin	0.2803	0.0863	0.2291	0.1845	0.2198	0.229
		DEA-WDGD	0.2813	0.0750	0.2398	0.1762	0.2277	0.144

Table 4: The derived local weights and the ranks for alternatives by different methods

Alternative	Eigenvector method	Rank	DEA-WDGD method	Rank
A_1	0.3185	1	0.2799	1
A_2	0.1669	4	0.1651	4
A_3	0.2349	2	0.2570	2
A_4	0.1938	3	0.2221	3
A_5	0.0859	5	0.0759	5
FP	0.3250		0.3500	

Table 5: The local weights and output weights for example 3

Local weights w_1	w_2	w_3	w_4	w_5	Output weights v_1	v_2	v_3	v_4
0.2663	0.0740	0.2373	0.1853	0.2370	0.0053	0.0027	0.0106	0.0080

weights. For inconsistent matrices, the FP value will not be zero but the smaller value of FP shows the higher quality of the derived weights. In Eq. 8, w_i and w_j are the derived weights of each method and E is the geometric mean of pair-wise comparison matrices.

As for model 4 and Eq. 6 the LP model for the sample 1 of Table 3 can be written as follow:

$$\text{Max } w_1 + w_2 + w_3 + w_4 + w_5,$$

$$\text{Subject to} \begin{cases} 10v_1 + 14v_2 + 12v_3 + 6v_4 - w_1 = 0, \\ 3.8v_1 + 1.8v_2 + 1.6v_3 + 4v_4 - w_2 = 0, \\ 3.3v_1 + 14v_2 + 13v_3 + 5.5v_4 - w_3 = 0, \\ 5.8v_1 + 6v_2 + 8.5v_3 + 6v_4 - w_4 = 0, \\ 9v_1 + 8v_2 + 12v_3 + 5v_4 - w_5 = 0, \\ w_1 + w_2 + w_3 + w_4 + w_5 \le 1 \\ 0.3v_1 - 0.5v_2 = 0, \\ 0.15v_2 - 0.3v_3 = 0, \\ 0.05v_3 - 0.15v_4 = 0, \\ w_i \ge 0;\ v_k \ge 0, \qquad i = 1,...,5;\ k = 1,...,4 \end{cases}$$

As can be shown From Table 3, the weights derived from DEA-WDGD have better FP values comparing with Wang and chin's model.

Example 2: Consider four pair-wise comparison matrices $A^{(1)}$, $A^{(2)}$, $A^{(3)}$, $A^{(4)}$ that are comparisons about the relative importance of five alternatives with respect to a criterion. The DEA-WDGD and the EM are used to derive weights and a comparison between the results is provided. In the EM, the geometric mean is used to aggregate four matrices.

$$A^{(1)} = \begin{bmatrix} 1 & 1 & 3 & 4 & 2 \\ 1 & 1 & 1/2 & 2 & 5 \\ 1/3 & 2 & 1 & 3 & 6 \\ 1/4 & 1/2 & 1/3 & 1 & 1 \\ 1/2 & 1/5 & 1/6 & 1 & 1 \end{bmatrix}, \quad A^{(2)} = \begin{bmatrix} 1 & 2 & 1/3 & 1/2 & 2 \\ 1/2 & 1 & 1/6 & 1/3 & 1/2 \\ 3 & 6 & 1 & 1/2 & 4 \\ 2 & 3 & 2 & 1 & 5 \\ 1/2 & 2 & 1/4 & 1/5 & 1 \end{bmatrix},$$

$$A^{(3)} = \begin{bmatrix} 1 & 3 & 5 & 7 & 5 \\ 1/3 & 1 & 1 & 3 & 2 \\ 1/5 & 1 & 1 & 5 & 5 \\ 1/7 & 1/3 & 1/5 & 1 & 1 \\ 1/5 & 1/2 & 1/5 & 1 & 1 \end{bmatrix}, \quad A^{(4)} = \begin{bmatrix} 1 & 2 & 3 & 1/2 & 2 \\ 1/2 & 1 & 1 & 1/2 & 5 \\ 1/3 & 1 & 1 & 1/6 & 1 \\ 2 & 2 & 6 & 1 & 7 \\ 1/2 & 1/5 & 1 & 1/7 & 1 \end{bmatrix}$$

As can be shown from Table 4, two methods have equal ranks and the derived weights are not too far from each other. Eigenvector method is a nonlinear method but the DEA-WDGD method is formulated as an LP model that is much easier to derive weights than EM.

Example 3: Consider four pair-wise comparison matrices $A^{(1)}$, $A^{(2)}$, $A^{(3)}$, $A^{(4)}$ in example 1 and suppose Interval importance weights for DMs as follow:

$$0.2 \le h_1 \le 0.3,$$
$$0.1 \le h_2 \le 0.3,$$
$$0.3 \le h_3 \le 0.4,$$
$$0.2 \le h_4 \le 0.4$$

Before solving model 4 to derive weights we should replace the following constraints instead of constraints ($v_1 - v_k = 0$, k = 2,...,m) of model 4. The results are shown in Table 5.

$$\begin{cases} \sum_{k=1}^{m} 0.2v_k - v_1 \le 0, & \sum_{k=1}^{m} 0.3v_k - v_1 \ge 0, \\ \sum_{k=1}^{m} 0.1v_k - v_2 \le 0, & \sum_{k=1}^{m} 0.3v_k - v_2 \ge 0, \\ \sum_{k=1}^{m} 0.3v_k - v_3 \le 0, & \sum_{k=1}^{m} 0.4v_k - v_3 \ge 0, \\ \sum_{k=1}^{m} 0.2v_k - v_4 \le 0, & \sum_{k=1}^{m} 0.4v_k - v_4 \ge 0 \end{cases}$$

AGGREGATION OF LOCAL WEIGHTS TO GET FINAL WEIGHTS

Here, we propose the aggregation of local weights to get final weights by DEA-WDGD. In the Wang and Chin (2009) model the sum of resulted weights may be more than one because they are not normalized. However, it is necessary to normalize them before aggregation to have the AHP final weights but when we use DEA-WDGD to derive local weights, there is no need to normalize local weights before aggregation and this is one of the superiority of present model. A hierarchical structure in AHP is shown in Fig. 1 that has m criteria and n alternatives. Let $(w_1,...,w_m)$ be the local weights of m criteria that all have been derived by DEA-WDGD and $w_{1j},...,w_{nj}$ be the local weights for m alternatives with respect to the jth criterion (j = 1,...,m). The final weights are shown in the last column of Table 6.

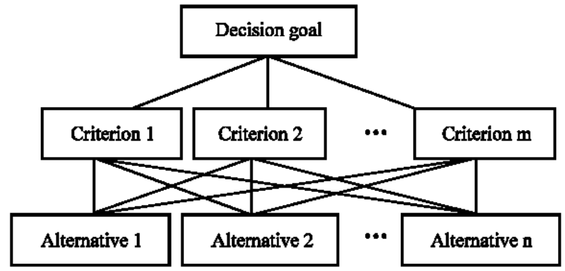

Fig. 1: A hierarchical structure by analytic hierarchy process

Table 6: Aggregation of local weights to get final weights

Alternative	Criteria w_1	w_2	...	w_m	Final weights
A_1	w_{11}	w_{12}	...	w_{1m}	$\sum_{j=1}^{m} w_{1j}w_j$
A_2	w_{21}	w_{22}	...	w_{2m}	$\sum_{j=1}^{m} w_{2j}w_j$
⋮	⋮	⋮	⋮	⋮	⋮
A_n	w_{n1}	w_{n2}	...	w_{nm}	$\sum_{j=1}^{m} w_{nj}w_j$

CONCLUSIONS

In this study, we proposed the DEA-WDGD method to derive weight vector in the group AHP situation by utilizing DEA approach. Some numerical examples were provided and we showed that the derived weights of DEA-WDGD are better than Wang and Chin (2009) model. After comparing the derived weights of DEA-WDGD with eigenvector method it was shown that the derived weights of DEA-WDGD are acceptable. The new DEA-WDGD method has the following good characteristics:

- The DEA-WDGD with utilizing the DEA approach is formulated as an LP model and is much easier than eigenvector method to derive weights in the group AHP
- In the DEA-WDGD, solving one LP model is sufficient for the local weights derivation of several pair-wise comparison matrices in the group AHP situation
- The DEA-WDGD method Utilize the simple additive weighting method for aggregation of local weights without the need to normalization
- The DEA-WDGD can use both the interval importance weight and the relative importance weight for each decision maker

Further researches can extend the DEA-WDGD to handle fuzzy AHP and enable it to derive weights from only one pair-wise comparison matrix.

REFERENCES

Azadeh, A., S.F. Ghaderi and H. Izadbakhsh, 2008. Integration of DEA and AHP with computer simulation for railway system improvement and optimization. Applied Math. Comput., 195: 775-785.

Charnes, A., W.W. Cooper and E. Rhodes, 1978. Measuring the efficiency of decision making units. Eur. J. Operat. Res., 2: 429-444.

Ertey, T., D. Ruan and U.R. Tuzkaya, 2006. Integrating data envelopment analysis and analytic hierarchy for the facility layout design in manufacturing systems. Inform. Sci., 176: 237-262.

Franklin-Liu, F.H. and L.H. Hai, 2005. The voting analytic hierarchy process method for selecting supplier. Int. J. Prod. Econ., 97: 308-317.

Golany, B. and Y. Roll, 1989. An application procedure for DEA. Omega, 17: 237-250.

Ho, W., 2008. Integrated analytic hierarchy process and its applications. A literature review. Eur. J. Operat. Res., 186: 211-228.

Jyoti Banwet, D.K. and S.G. Deshmukh, 2008. Evaluating performance of national R&D organizations using integrated DEA-AHP technique. Int. J. Prod. Perform. Manage., 57: 370-388.

Mehmet, S., S.C.L. Koh, S. Zaim, M. Demirbag and E. Tatoglu, 2007. An application of data envelopment analytic hierarchy process for supplier selection: A case study of BEKO in Turkey. Int. J. Prod. Res., 45: 1973-2003.

Raju, K.S. and D.N. Kumar, 2006. Ranking irrigation planning alternatives using data envelopment analysis. Water Resour. Manage., 20: 553-566.

Ramanathan, R., 2006. Data envelopment analysis for weight derivation and aggregation in the analytic hierarchy process, Comput. Opert. Res., 33: 1289-1307.

Saaty, T.L., 1980. The Analytic Hierarchy Process. McGraw Hill, New York, ISBN: 0070543712.

Saen, F.R., A. Memariani and F.H. Lotfi, 2005. Determining relative efficiency of slighty non-homogeneous decision making units by data envelopment analysis: A case study in IROST. Applied Math. Comput., 165: 313-328.

Shang, J. and T. Sueyoshi, 1995. A unified framework for the selection of a flexible manufacturing system. Eur. J. Oper. Res., 85: 297-315.

Sinuany-Stern, Z., M. Abraham and H. Yossi, 2000. An AHP/DEA methodology for ranking decision making units. Int. Trans. Opert. Res., 7: 109-124.

Vaidya, O.S. and S. Kumar, 2006. Analytic hierarchy process: An overview of applications. Eur. J. Operat. Res., 169: 1-29.

Wang, Y.M., K.S. Chin and J.B. Yang, 2007. Three new models for preference voting and aggregation. J. Opert. Res. Soc., 58: 1389-1393.

Wang, Y.M., K.S. Chin and G.K.K. Poon, 2008a. A data envelopment analysis method with assurance region for weight generation in the analytic hierarchy process. Decis. Support Syst., 45: 913-921.

Wang, Y.M., J. Liu and T.M.S. Elhag, 2008b. An integrated AHP-DEA methodology for bridge risk assessment. Comput. Ind. Eng., 54: 513-525.

Wang, Y.M. and K.S. Chin, 2009. A new data envelopment analysis method for priority determination and group decision making in the analytic hierarchy process. Eur. J. Opert. Res., 195: 239-250.

Yang, T. and C. Kuo, 2003. A hierarchical DEA-AHP methodology for the facilities layout design problem. Eur. J. Opert. Res., 147: 128-136.