Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 9 (19): 3587-3592, 2009
ISSN 1812-5654

Application of Homotopy Perturbation Method to Solve Combined Korteweg de Vries-Modified Korteweg de Vries Equation

[1]H. Mirgolbabaei and [2]D.D. Ganji
[1]School of Mechanic, Islamic Azad University, Jouybar Branch, Jouybar, Iran
[2]Department of Mechanical Engineering, Babol Nushirvani University of Technology, P.O. Box 484, Babol, Iran

Abstract: In this study, He's homotopy perturbation method is implemented for finding for solitary-wave solutions of the combined Korteweg de Vries-Modified Korteweg de Vries (KdV-MKdV) equation. Numerical solutions for the initial conditions has been obtained. The Homotopy Perturbation Method (HPM) deforms a difficult problem into a simple problem which can be easily solved. The results are compared with the exact solitary-wave solutions. The obtained solutions are compared with the Adomian's decomposition method. All the examples show that the results of the present method are in excellent agreement with those obtained by the Adomian's decomposition method.

Key words: The combined Korteweg, de Vries-Modified Korteweg, de Vries equation, homotopy perturbation method, Solitary-wave solution

INTRODUCTION

We consider solitary-wave solutions of the combined Korteweg de Vries-Modified Korteweg de Vries (KdV-MKdV) equation. In this study, we implemented the Homotopy Perturbation Method (Biazar, 2004; He, 2004) or finding the solutions of the combined KdV-MKdV equation. The numerical solution are compared with the ADM solution. It's remarkable accuracy is finally demonstrated for the combined KdV-MKdV equation.

The combined KdV-MKdV equation (Fan, 2003) will be handle more easily, quickly and elegantly by implementing the HPM rather than the traditional methods for the exact solutions as well as numerical solutions.

The KdV and MKdV equations are most popular soliton equations and have been extensively investigated. But the nonlinear terms of KdV and MKdV equations often simultaneously exist in practical problems such as fluid physics, physics and quantum field theory and consider the solution u(x, t) of the form the following so-called combined KdV-MKdV equation:

$$u_t + puu_x + qu^2u_x + L_x(u_x) = 0 \tag{1}$$

where, p and q are arbitrary constants and $L_x \equiv \partial^2/\partial x^2$.

This equation may represent the wave propagation of the bound particle, sound wave and thermal pulse (Mohamad, 1992). The explicit exact solutions of Eq. 1 have been found by Hirota bilinear method, inverse scattering and homogeneous balance method (Yu, 2000; Hong, 2000). Recently, a new method has proposed by Fan (2003). This proposed method will give a series of traveling wave solutions for Eq. 1 in a simple and unified way. Nonlinear phenomena play a crucial role in applied mathematics and physics.

This method established by He (1998, 2000a, b, 2005a, b, 2006a-c). The method has been used by many researchers (Ganji, 2006; Ganji and Sadighi, 2006; Ganji and Rajabi, 2006; Rafei *et al*., 2007; Hayat *et al*., 2004; Siddiqui *et al*., 2006; Abbasbandy, 2006a, b; Zhang and He, 2006; Koçak and Yıldırım, 2009; Berberler and Yıldırım, 2009) and the references therein to handle a wide variety of scientific and engineering applications: linear and nonlinear, homogeneous and inhomogeneous as well. It was shown by many authors that this method provides improvements over existing numerical techniques. With the rapid development of nonlinear science, many different methods were proposed to solve various Boundary-Value Problems (BVP) (Al-Hayani and Casasús, 2005), such as Homotopy perturbation method (HPM) and Variational Iteration Method (VIM) (He, 1999a; 2000a, b; Abdou and Soliman, 2005; Wazwaz, 2006; Sweilam and Khader, 2007; Ganji *et al*., 2008; Mirgolbabaei *et al*., 2009a, b; Omidvar *et al*., 2009). These methods give successive approximations of high accuracy of the solution. In this study, only a brief discussion of the Homotopy perturbation method will be emphasized, complete details of the method are found in many related works.

Corresponding Author: D. D. Ganji, Department of Mechanical Engineering, Babol Nushirvani University of Technology, P.O. Box 484, Babol, Iran

HOMOTOPY PERTURBATION METHOD

Basic idea of He's homotopy perturbation method: To show the basic ideas of this method, we consider the following nonlinear differential Equation:

$$A(u) - f(r) = 0, r \in \Omega \tag{2}$$

Considering the boundary conditions of:

$$B(u, \partial u/\partial n) = 0, \quad r \in \Gamma \tag{3}$$

where, A is a general differential operator, B a boundary operator, f(r) a known analytical function and Γ is the boundary of the domain Ω.

The operator A can be, generally divided into two parts of L and N, where L is the linear part, while N is the nonlinear one. Equation 2 can, therefore, be written as:

$$L(u)+N(u)-f(r) = 0 \tag{4}$$

By the homotopy technique, we construct a homotopy as $v(r, p):\Omega\times[0,1]\to R$ which satisfies:

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0,\ p \in [0,1],\ \ r \in \Omega \tag{5}$$

or

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \tag{6}$$

where, $p\in[0,1]$ is an embedding parameter and u_0 is an initial approximation of Eq. 2 which satisfy the boundary conditions. Obviously, considering Eq. 5 and 6, we will have:

$$H(v,0) = L(v)-L(u_0) = 0 \tag{7}$$

$$H(v,1) = A(v)-f(r) = 0 \tag{8}$$

The changing process of p from zero to unity is just that of v(r, p) from $u_0(r)$ to u(r). In topology, this is called deformation and $L(v)-L(u_0)$ and A(v)-f(r) are called homotopy.

According to HPM, we can first use the embedding parameter p as a small parameter and assume that the solution of Eq. 5 and 6 can be written as a power series in p:

$$v = v_0 + pv_1 + p^2v_2 + \ldots \tag{9}$$

Setting p = 1 results in the approximate solution of Eq. 2:

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \ldots \tag{10}$$

The combination of the perturbation method and the homotopy method is called the homotopy perturbation method, which lessens the limitations of the traditional perturbation methods. On the other hand, this technique can have full advantages of the traditional perturbation techniques.

The series 10 is convergent for most cases. However, the convergence rate depends on the nonlinear operator A(v).

The following opinions are suggested by He:

- The second derivative of N(v) with respect to v must be small because the parameter p may be relatively large, i.e., $p\to 1$
- The norm of $L^{-1}\,\partial N/\partial v$ must be smaller than one so that the series converges

Analysis of the method: For purposes of illustration of the HPM for solving the combined KdV-MKdV equation (Fan, 2003). Here, we will consider Eq. 1 for p, q = 1. We will show that how the HPM is computationally efficient.

We considered, the combined KdV-MKdV Eq. 1 with p, q = 1 has the solitary-wave solution of which is to be obtained. According to the HPM, we can construct a homotopy of Eq. 1 as follows:

$$(1-p)(v_t - v_{0,t}) + p(v_t + vv_x + v^2v_x + v_{xxx}) = 0 \tag{11}$$

and the initial approximations are as follows:

$$v_0\,(x, 0) = u(x, 0) \tag{12}$$

Substituting Eq. 9 and 10 into Eq. 8 and arranging the coefficients of p powers, we have:

$$\begin{aligned}&(v_{0,t}) + (v_0^2v_{0,x} + v_{1,t} + v_0v_{0,x} + v_{0,xxx})p + (v_1v_{0,x} + v_0v_{1,x}\\&+v_{2,t} + v_0^2v_{1,x} + 2v_0v_1v_{0,x} + v_{1,xxx})p^2 + \ldots = 0\end{aligned} \tag{13}$$

In order to obtain the unknowns v_i, I = 1, 2, 3,. . ., we must construct and solve the following system which includes three equations with three unknowns:

$$\begin{aligned}&v_{0,t} = 0\\&v_0^2v_{0,x} + v_{1,t} + v_0v_{0,x} + v_{0,xxx} = 0\\&v_1v_{0,x} + v_0v_{1,x} + v_{2,t} + v_0^2v_{1,x} + 2v_0v_1v_{0,x} + v_{1,xxx} = 0\end{aligned} \tag{14}$$

From Eq. 10, if the first two approximations are considered, we will obtain:

$$u(x,t) = \lim_{p\to 1} v\ (x,t) = \sum_{k=0}^{k=2} v_k(x,t) \qquad (15)$$

Application: Firstly, we consider the solutions of Eq. 1 with the initial condition (Fan, 2003):

$$u(x,0) = \alpha + \gamma \operatorname{sech}(kx) \qquad (16)$$

where p, q are any real number,

$$k = \sqrt{\beta}, \alpha = -\frac{p}{2q}$$

and

$$\gamma = \sqrt{\frac{6\beta}{q}}$$

To calculate the terms of the homotopy series Eq. 15 for $u(x, t)$ we substitute the initial condition Eq. 16 and 9 into the system Eq. 14 and finally using Maple, the solutions of the equation can be obtained as follows:

$$v_0(x,t) = \alpha + \gamma \operatorname{sech}(kx) \qquad (17)$$

$$\begin{aligned} v_1(x,t) = {} & \frac{2\gamma kt}{\cosh(4kx) + 4\cosh(2kx) + 3}(\alpha^2 \sinh(3kx) + \sinh(kx)\alpha^2 \\ & + 4\gamma\alpha \sinh(2kx) + \alpha \sinh(3kx) + \alpha \sinh(kx) + 2\gamma \sinh(2kx) \\ & + 4\gamma^2 \sinh(kx) + k^2 \sinh(3kx) - 23k^2 \sinh(kx)) \end{aligned}$$

In this manner the other components can be easily obtained.

With initial conditions Eq. 16, the solitary wave solution of Eq. 1 is in full agreement with the ones constructed by Dogan Kaya (He, 1999a, b). To examine the accuracy and reliability of the HPM for the combined KdV-MKdV equation, we can also consider the different initial value (Fan, 2003):

$$u\ (x,0) = \alpha + \gamma \tanh(kx) \qquad (18)$$

where p, q are any real number,

$$k = \sqrt{\beta}, \alpha = -\frac{p}{2q}$$

and

$$\gamma = \sqrt{\frac{6\beta}{q}}$$

To calculate the terms of the homotopy serie (Eq. 15), we substitute the initial condition Eq. 18 and 9 into system Eq. 14 and finally using Maple, the solutions of equation can be obtained. Following this procedure as in the first example, we obtain the solutions:

$$v_0(x,t) = \alpha + \gamma \tanh(kx)$$

$$\begin{aligned} v_1(x,t) = {} & -\gamma kt\alpha + \gamma kt\alpha \tanh^2(kx) - \gamma^2 kt \tanh(kx) + \gamma^2 kt \tanh^3(kx) \\ & + 2\gamma k^3 t - 8\gamma k^3 t \tanh^2(kx) + 6\gamma k^3 t \tanh^4(kx) - \gamma kt\alpha^2 \\ & + \gamma kt\alpha^2 \tanh^2(kx) - 2\gamma^2 kt\alpha \tanh(kx) + 2\gamma^2 kt\alpha \tanh^3(kx) \\ & - \gamma^3 kt \tanh^2(kx) + \gamma^3 kt \tanh^4(kx) \end{aligned}$$

In this manner the other components can be easily obtained. In this case, the solitary wave solution of Eq. 1 are in full agreement with the ones constructed by Dogan Kaya (He, 1999a, b).

DISCUSSION

Comparing the results with the ADM solution: To demonstrate the convergence of the HPM, the results of the numerical example are presented and only few terms are required to obtain accurate solutions. The accuracy of the HPM for the combined Korteweg de Vries-Modified Korteweg de Vries (KdV-MKdV) equation is controllable and absolute errors are very small with the present choice of t and x. These results are listed in Table 1 and 2; it is seen that the implemented method achieves a minimum accuracy of five and maximum accuracy of nine significant

Table 1: The HPM results for u(x, t) for the first three approximations in comparison with the analytical solutions when for the solitary wave solutions with the initial conditions Eq. 16 of Eq. 1, respectively

u_{exact}-$u_{homotopy}$	(x, t)
7.5E-09	(0.1,0.1)
1.91E-08	(0.1,0.2)
3.47E-08	(0.1,0.3)
1.3E-08	(0.2,0.1)
3.02E-08	(0.2,0.2)
5.13E-08	(0.2,0.3)
1.87E-08	(0.3,0.1)
4.13E-08	(0.3,0.2)
6.79E-08	(0.3,0.3)

Table 2: The HPM results for u(x, t) for the first three approximations in comparison with the analytical solutions when for the solitary wave solutions with the initial conditions Eq. 18 of Eq. 1, respectively

(x, t)	u_{exact}-$u_{homotopy}$
(0.1,0.1)	6.1228E-06
(0.1,0.2)	1.2245E-05
(0.1,0.3)	1.83666E-05
(0.2,0.1)	6.1222E-06
(0.2,0.2)	1.22438E-05
(0.2,0.3)	1.83647E-05
(0.3,0.1)	6.1216E-06
(0.3,0.2)	1.22425E-05
(0.3,0.3)	1.83628E-05

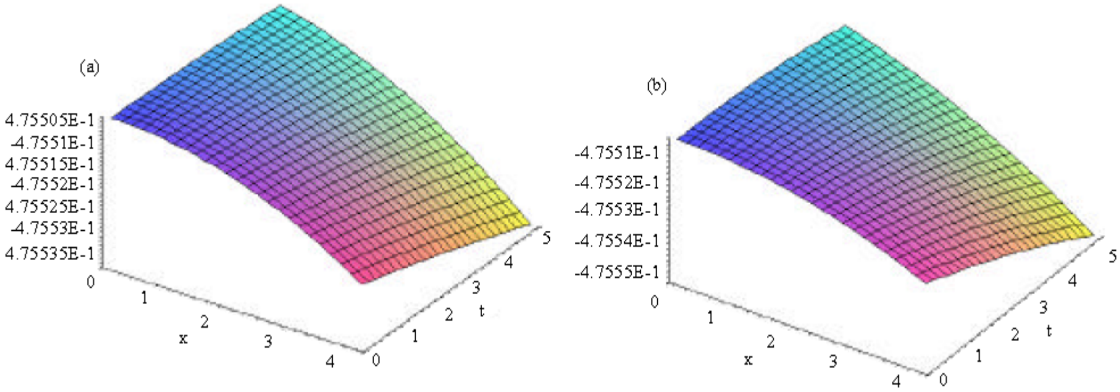

Fig. 1: The HPM results for u(x, t), shown in(a), in comparison with the ADM result, shown in (b), when k = 0.01, p, q = 1, for the solitary wave solution with the initial conditions Eq. 18 of Eq. 1

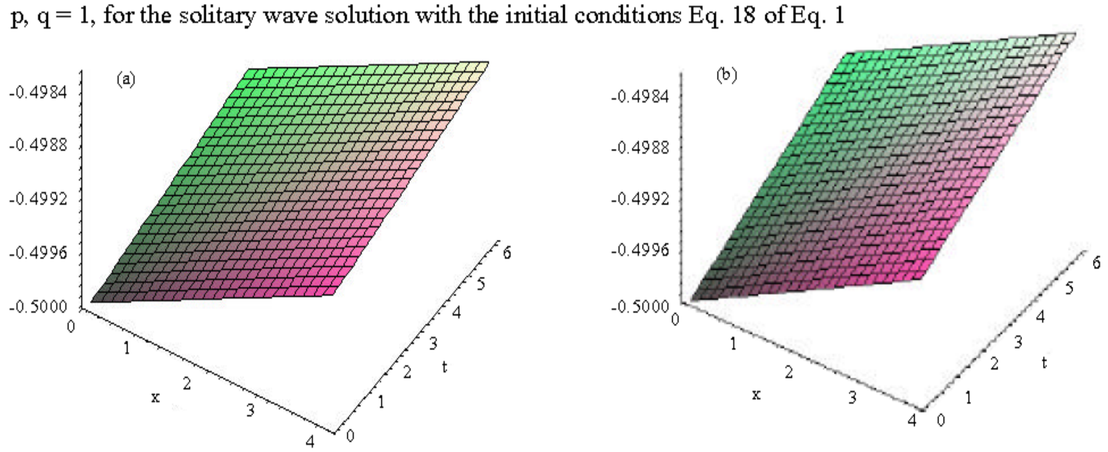

Fig. 2: The HPM results for u(x, t), shown in (a), in comparison with the ADM result, shown in (b), when k = 0.01, p, q = 1, for the solitary wave solution with the initial conditions Eq. 18 of Eq. 1

values for Eq. 1, for the first three approximations. Both the exact results and the approximate solutions obtained for the first three approximations are plotted in Fig. 1a, b and 2a, b. There are no visible differences in the two solutions of each pair of diagrams.

CONCLUSIONS

The homotopy perturbation method (HPM) was used for finding soliton solutions of a combined Korteweg de Vries-Modified Korteweg de Vries (KdV-MKdV) equation with initial conditions. It can be concluded that the HPM is very powerful and efficient technique in finding exact solutions for wide classes of problems.

It is worth pointing out that the HPM presents a rapid convergence for the solutions. The obtained solutions are compared with the Adomian's decomposition method. All the examples show that the results of the present method are in excellent agreement with those obtained by the Adomian's decomposition method. The HPM has got many merits and much more advantages than the Adomian's decomposition method. This method is to overcome the difficulties arising in calculation of Adomian polynomials. Also the HPM does not require small parameters in the equation, so that the limitations of the traditional perturbation methods can be eliminated and also the calculations in the HPM are simple and straightforward. The reliability of the method and the reduction in the size of computational domain give this method a wider applicability. The results show that the HPM is a powerful mathematical tool for solving systems of nonlinear partial differential equations having wide applications in engineering.

REFERENCES

Abbasbandy, S., 2006a. Homotopy perturbation method for quadratic Riccati differential equation and comparison with Adomian's decomposition method. Applied Math. Comput., 172: 485-490.

Abbasbandy, S., 2006b. Iterated He's homotopy perturbation method for quadratic Riccati differential equation. Appl. Math. Comput., 175: 581-589.

Abdou, M.A. and A.A. Soliman, 2005. Variational iteration method for solving Burger's and coupled Burger's equation. J. Comput. Applied Math., 181: 245-251.

Al-Hayani, W. and L. Casasús, 2005. Approximate analytical solution of fourth order boundary value problems. Numer. Algorithms, 40: 67-78.

Berberler, M.E. and A. Yıldırım, 2009. Hes homotopy perturbation method for solving the shock wave equation. Appl. Anal., (In Press).

Biazar, J., E. Babolian and R. Islam, 2004. Solution of the system of ordinary differential equations by adomian decomposition method. Applied Math. Comput., 147: 713-719.

Fan, E., 2003. Uniformly constructing a series of explicit exact solutions to nonlinear equations in mathematical physics. Chaos Solitons Fractals, 16: 819-839.

Ganji, D.D., 2006. The application of Hes homotopy perturbation method to nonlinear equations arising in heat transfer. Phys. Lett. A, 355: 337-341.

Ganji, D.D. and A. Rajabi, 2006. Assessment of homotopy-perturbation and perturbation methods in heat radiation equations. Int. Commun. Heat Mass Transfer, 33: 391-400.

Ganji, D.D. and A. Sadighi, 2006. Application of he's homotopy-prturbation method to nonlinear coupled systems of reaction-diffusion equations. Int. J. Nonl. Sci. Num. Simu., 7: 411-418.

Ganji, D.D., H. Mirgolbabaei, Me. Miansari and Mo. Miansari, 2008. Application of homotopy perturbation method to solve linear and non-linear systems of ordinary differential equations and differential equation of order three. J. Applied Sci., 8: 1256-1261.

Hayat, T., M. Khan and S. Asghar, 2004. Homotopy analysis of MHD flows of an Oldroyd 8-constant fluid. Acta Mech., 168: 213-32.

He, J.H., 1998. Approximate analytical solution for seepage flow with fractional derivatives in porous media. J. Comput. Math. Applied Mech. Eng., 167: 57-68.

He, J.H., 1999a. Homotopy perturbation technique. J. Comput. Math. Appl. Mech. Eng., 17: 257-262.

He, J.H., 1999b. Variational iteration method-a kind of nonlinear analytical technique: Some examples. Int. J. Nonlinear Mech., 34: 699-708.

He, J.H., 2000a. A coupling method of homotopy technique and perturbation technique for nonlinear problems. Int. J. Non-Linear Mech., 351: 37-43.

He, J.H., 2000b. Variational iteration method for autonomous ordinary differential systems. Applied Math. Comput., 114: 115-123.

He, J.H., 2004. The homotopy perturbation method for nonlinear oscillators with discontinuities. Applied Math. Comput., 151: 287-292.

He, J.H., 2005a. Application of homotopy perturbation method to nonlinear wave equations. Chaos Solitons Fractals, 26: 695-700.

He, J.H., 2005b. Periodic solutions and bifurcations of delay-differential equations. Phys. Lett. A., 347: 228-230.

He, J.H., 2006a. Homotopy perturbation method for solving boundary value problems. Phys. Lett. A, 350: 87-88.

He, J.H., 2006b. New Interpretation of homotopy perturbation method. Int. J. Modern Phys. B, 20: 2561-2568.

He, Ji-H., 2006c. Some asymptotic methods for strongly nonlinear equations. Int. J. Modern Phys. B., 20: 1141-1199.

Hong, W.P., 2000. New types of solitary-wave solutions from the combined KdV-MKdV equation. NUOVO CIMENTO B, 115: 117-118.

Koçak, H. and A. Yıldırım, 2009. Numerical solution of 3D Greens function for the dynamic system of anisotropic elasticity. Phys. Lett. A, 373: 3145-3150.

Mirgolbabaei, H., D.D. Ganji and H. Taherian, 2009a. Soliton solution of the Kadomtse-Petviashvili equation by homotopy perturbation method. World J. Modell. Simulat., 5: 38-44.

Mirgolbabaei, H., D.D. Ganji, M.M. Etghani and A. Sobati, 2009b. Adapted variational iteration method and axisymmetric flow over a stretching sheet. World J. Modell. Simulat., (In Press).

Mohamad, M.N.B., 1992. Exact solutions to the combined KdV and MKdV equation. Math. Meth. Applied Sci., 15: 73-78.

Omidvar, M., A. Barari, H. Mirgolbabaei and D.D. Ganji, 2009. Solution of diffusion equations using homotopy perturbation and variational iteration methods. MOEJ Int. J. Sci. Technol., (In Press).

Rafei, M., D.D. Ganji, H. Danialia and H. Pashaei, 2007. The variational iteration method for nonlinear oscillators with discontinuities. J. Sound Vibration, 305: 614-620.

Siddiqui, A.M., R. Mahmood and Q.K. Ghori, 2006. Thin film flow of a third grade fluid on a moving belt by Hes homotopy perturbation method. Int. J. Nonlin. Sci. Numer. Simulat., 7: 7-14.

Sweilam, N.H. and M.M. Khader, 2007. Variational iteration method for one dimensional nonlinear thermoelasticity. Chaos Solitons Fract., 32: 145-149.

Wazwaz, A.M., 2006. The variational iteration method for rational solutions for KdV, K(2, 2), Burgers and cubic Boussinesq equations. J. Comput. Applied Math., 207: 18-23.

Yu, J., 2000. Exact solitary wave solutions to a combined KdV and MKdV equation. Math. Meth. Applied Sci., 23: 1667-1670.

Zhang, L.N. and J.H. He, 2006. Homotopy perturbation method for the solution of the electrostatic potential differential equation. Math. Problems Eng., 2006: 1-6.