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Overall Processes Capability Index for Assembly Production Lines

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Abstract: This study develops a new method called Overall Processes Capability Index (OPCI) to measure the capability of assembly lines. After defining relation between percentage yield and process capability, the new definition of overall defective rate is developed for assembly lines in different cases (series and parallel processes). The interesting point is that a rejected item from one process can be assembled with one another rejected item of another process and creates one accepted assembled one. Then regarding to this point and with the aid of transforming new definition of overall defective rate to process yield, the overall processes capability of assembly production line is measured.

Key words: Process capability index, overall, defective rate, process yield, overall processes capability, assembly lines

INTRODUCTION

Nowadays manufacturers are so interested to understand the capability of their processes in order to improve them (Delaney and Phelan, 2009). Process capability can be defined as a measure of inherent variability in a process as compared to the specification requirements of the product (Montgomery, 2001; Kane, 1986; Bissell, 1990). Process Capability Indices (PCIs) such as C_a, C_p, C_{pk} and C_{pm} are quantitative measurements to evaluate the process capability (Montgomery, 2001; Kane, 1986).

A production line is a set of sequential operations established in a series, parallel or combination shape. In a production line, each process has its own process capability, that effects on the overall capability of the a production line. The most common process capability indices have been used in order to measure the capability of one process only, but manufacturers usually want to announce their overall processes capability, which must be based on their whole processes. Therefore, the aim of this study is to measure the overall processes capability of assembly production line. In the literature, there are few researches that have been focused directly to measure the capability of whole production line and this research aims to define a novel method to measure the overall process capability in assembly lines.

PROCESS CAPABILITY INDICES

Kane (1986) developed C_p and C_{pk} indices, which are commonly used in industry to evaluate single quality characteristic. These process capability indices assume that the quality characteristic is normally distributed and can be expressed as follows:

$$C_p = \frac{USL - LSL}{6\sigma} \quad (1)$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} \quad (2)$$

where, USL and LSL represent the upper and lower specification limits, respectively; μ represents the process mean and σ is the process standard deviation. In the case, that the customer provides a one-sided specification, indices C_{pu} and C_{pl} were developed. For processes with upper specification limit, C_{pu} and C_{pl} can be expressed as follows:

$$C_{pu} = \frac{USL - \mu}{3S} \quad (3)$$

$$C_{pl} = \frac{\mu - LSL}{3S} \quad (4)$$

The index C_{pu} compares the distance between the process mean and the upper specification limit with the

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upper half-width of the distribution. Similarly, C_{pl} compares the distance between the process mean and the lower specification limit with the lower half-width of the distribution. These indices show the relative size of the working margin, such as the closeness of the distribution to the specification limit.

Since, the indices C_p and C_{pk} do not take into account of the difference between the processes mean and its target value, Chan (1988) and Pearn (1992) considered this difference to develop indices C_{pm} and C_{pmk} as follows:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} \tag{5}$$

$$C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\} \tag{6}$$

where, T represents the target value of a quality characteristic. The process parameters μ and σ^2 are estimated from the sample mean and variance for $\bar{X}$ and S^2, when μ and σ^2 are unknown.

The above PCIs have been designed for single variable process, Chen (1994) and Boyles (1996) have explored several unilateral and bilateral specifications of process quality characteristics of multivariate process including. Because most of products especially assembled products, have several important quality characteristics, so each of characteristics must fall within specification limit in order to make a high quality product. Bothe (1992) studied about sequencing of multivariate process capability and indicated that the probability of each characteristic for a given product must first be determined to be within specification limits in a product capability study and then the combined probability that all characteristics are within specifications is calculated. Wu *et al.* (2004) developed a procedure to measure the process capability indices of a complete product with several characteristics. Teeravaraprug (2006a) used regression analysis in order to find the optimum process targets for multiple quality characteristics. Chen *et al.* (2006) proposed process capability analysis for a multi-process product. Moreover, Wang (2005) developed a procedure for constructing multi process capability indices for short-run production using the technique of principal component analysis. Moreover Teeravaraprug (2006b) proposed multi-product process mean with customer's loss consideration.

From literature review, it is concluded that there are so many researches on multivariate process, multi-product process, multi characteristics process and multi-process product capability indices. Most of these studies invested on constructing a multivariate process capability in different cases such as short-run productions. On the other hand, some researches focused on measuring the process capability indices of a complete product that again assumed multi characteristics, however, there is no specific study on developing process capability indices, for whole processes as a production line. Therefore, in this study a procedure for measuring the processes capability indices for whole production line as a master process is developed.

MEASURING OVERALL PROCESSES CAPABILITY FOR ASSEMBLY LINES

In order to measure Overall Process Capability (OPC) in assembly lines, one innovative way is to calculate an overall defective rate for whole production line. Then this Overall Defective Rate (ODR) with the aid of process yield and its relation with PCIs can be used to measure the overall processes capability in assembly lines. If the process distribution X is normally distributed as $N(\mu, \sigma^2)$, then Wu *et al.* (2004) proved Eq. 7.

$$P_i = 2\Phi(3C_{pi}) - 1 \tag{7}$$

where, P_i is process yield (% yield) for ith process and C_{pi} is the capability index C_p of ith process.

Equation 7 can be employed for measuring the Overall Processes Capability (OPC), when the P_i represents the overall process yield and Overall Process Yield (OPY) can be obtained from overall defective rate as follows:

$$OPY = 1\text{-}ODR \tag{8}$$

$$OPY = 2\Phi(3 \times OPC) - 1 \tag{9}$$

In order to measure the overall processes capability in assembly lines, three cases were assumed:

- Overall processes capability in series assembly lines
- Overall processes capability in parallel assembly lines
- Overall processes capability in combined assembly lines (Numerical example)

Measuring overall processes capability in series assembly lines: Suppose a very simple assembly line with three series process such as Fig. 1.

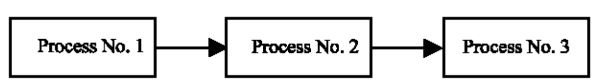

Fig. 1: Simple production line with three series processes

For measuring the process capability for series processes, we can use process yield simply. Because process yield represents the coverage of data within tolerance, so we can use Eq. 8 in order to measuring the whole process yield. Then with the aid of Eq. 9 overall processes capability in series assembly lines can be obtained as follows:

$$OP = \prod_{i=1}^{n} P_i \qquad (10)$$

where, OP is overall process, capability of assembly lines with n series processes.

The logical reason for Eq. 10 is that the output of first process, which is the input of second process, is % yield of first process. In other words, just accepted parts are allowed to be considered as the next process inputs. Therefore, the whole processes yield for a series assembly line is the multiplication of all processes yields.

Measuring overall processes capability in parallel assembly lines: Suppose an assembly line with 2 parallel processes and one process after them (Fig. 2).

In this case, Eq. 8 cannot be employed, because there is not a direct relation between process 1 and 2. In fact, the outputs of these processes together will affect on process 3. For example, assume that the statistical distributions of process 1 and 2 are normal and they have

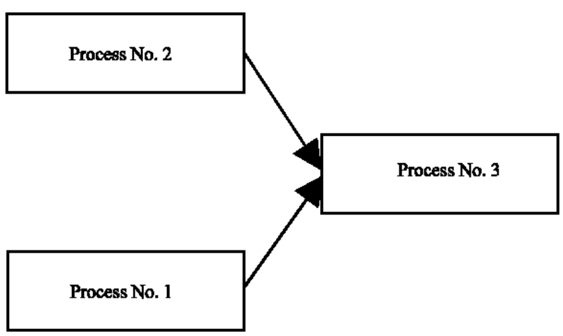

Fig. 2: A production line with two parallel processes and one process after them

similar standard deviation, defective rates and percentage yields, such as processes A and B in Fig. 3a and b. In assembly lines, one interesting point is that there is a different definition of rejects For example, a rejected item from process A can be assembled with one another rejected item of process B and creates one accepted assembled one in process C. Suppose process A produces screw and process B is producing nuts. Therefore, we have two outputs of processes A and B with the following specifications:

$$X_A = 22.75$$

$$X_B = 22.70$$

where, X_A and X_B are inner diameters of screw and nut.

Both of these parts are rejected from the quality control department's point of view, but they can be assembled and make an accepted part in process 3. Because the maximum difference between them can be 0.6 mm (USL_B- USL_A = 23.4-22.8 = 0.6), so these two parts with 0.05 mm deference can be assembled. This example leads us to a new definition of defective rate in assembly lines. Note that these 2 parts sometimes are rejected, because $X_A < LSL_A$ and maybe it causes some problems such as loosing in jointing with other parts. So it is concluded that quality specifications out of specification limits in assembly processes, depend on their duty in the next processes can be rejected or accepted. Therefore, regarding to the duty of items in subsequent processes, another limit (L, U) is defined which is wider than specification limit (LSL, USL). Two simple rules are as follows:

- **Rule 1:** When specification of one part fall outside of the limit (L, U), so it is concluded that this part is certainly rejected
- **Rule 2:** When specification of one part is within (L, LSL) or (USL, U), so regarding the specifications of two parts and compare them with maximum distance, it can be concluded whether it is rejected or not

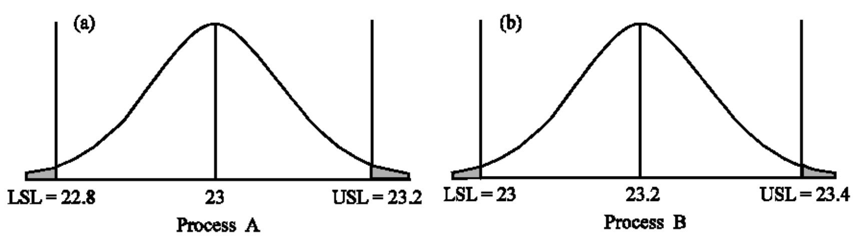

Fig. 3: Statistical distributions of process A and B

Since, in parallel assembly lines, the defective rate is different in concept, so calculating the overall defective rate is the first step of measuring the process capability. The overall defective rate for next process (assembling) will be occurred, when there are cases which two parts cannot be assembled. Some of them are $\{X_A>U_A\}$, $\{L_A<X_A,<LSL_A$ and $(|X_A-X_B|>$ Max $(|USL_A-LSL_B|$ and $|USL_B-LSL_A|)\}$ and so forth. So, calculating the overall defective rate with considering the all cases of occurrence of rejecting is complicated and considering the rate of accepted part is a better way. In this case with two parallel processes, rate of accepted parts is:

$$\text{Rate of accepted parts} = p\{L_A < X_A < U_A \ \text{ and } \ L_B < X_B < U_B \ \text{and} \ (|X_A - X_B| < \text{Max}(|USL_A - LSL_B| \ \text{ and } \ |USL_B - LSL_A|)\}$$

$$\text{Rate of accepted parts} = p\{L_A < X_A < U_A\} \times p\{L_B < X_B < U_B\} \times p\{(|X_A - X_B| < \text{Max}(|USL_A - LSL_B| \ \text{ and } \ |USL_B - LSL_A|)\}$$

$$= p\{(|X_A - X_B| < \text{Max}(|USL_A - LSL_B| \ \text{ and } \ |USL_B - LSL_A|)\} - p\{X_A > U_A\} - p\{X_A < L_A\} - p\{X_B > U_A\} - p\{X_B < L_B\}$$

$$= p\left\{\left(\frac{|X_A - X_B| - |(\mu_A - \mu_B)|}{\sqrt{\sigma_A^2 + \sigma_B^2}} < \frac{\text{Max}(|USL_A - LSL_B| \text{and} |USL_A - LSL_B|) - |(\mu_A - \mu_B)|}{\sqrt{\sigma_A^2 + \sigma_B^2}}\right)\right\}$$

$$-p\left\{\frac{X_A-\mu_A}{\sigma_A} > \frac{U_A-\mu_A}{\sigma_A}\right\} - p\left\{\frac{X_A-\mu_A}{\sigma_A} < \frac{L_A-\mu_A}{\sigma_A}\right\} - p\left\{\frac{X_B-\mu_B}{\sigma_B} > \frac{U_B-\mu_B}{\sigma_B}\right\} - p\left\{\frac{X_B-\mu_B}{\sigma_B} < \frac{L_B-\mu_B}{\sigma_B}\right\}$$

$$\text{Rate of accepted parts} = \Phi\left(\frac{\text{Max}(|USL_A - LSL_B| \text{and} |USL_B - LSL_A|) - |(\mu_A - \mu_B)|}{\sqrt{\sigma_A^2 + \sigma_B^2}}\right)$$

$$-1 + \Phi\left(\frac{U_A - \mu_A}{\sigma_A}\right) - \Phi\left(\frac{L_A - \mu_A}{\sigma_A}\right) - 1 + \Phi\left(\frac{U_B - \mu_B}{\sigma_B}\right) - \Phi\left(\frac{L_B - \mu_B}{\sigma_B}\right) \quad (11)$$

Regarding to Eq. 9, the overall defective rate will be calculated as follows:

$$\text{ODR} = 1 - \text{Rate of accepted parts} \quad (12)$$

Then regarding to Eq. 8 and Eq. 9, Overall Processes Capability (OPC) will be calculated.

Measuring overall processes capability in combined assembly lines- a real example: XYZ Co. is a factory, which is producing plastic parts and helmet. One of its products is plastic container. This product consists of two main parts: body and cap. Each part is producing during an injection process, which has its specific defective rate. These two parts will be assembled in third process and will be packaged in process 4. Now company

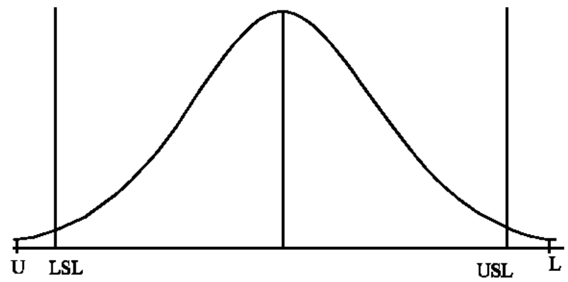

Fig. 4: New attitude to a typical process

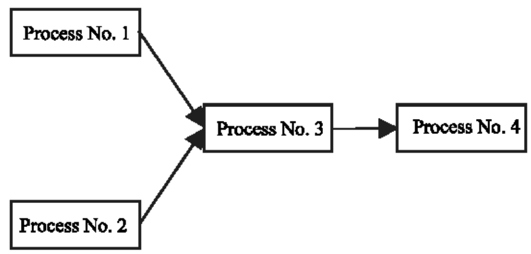

Fig. 5: Plastic containers of XYZ Co

Table 1: Results of data analysis for two injection processes

	Process No.	
Item (mm)	1	2
USL	23.200	23.400
Target	23.000	23.200
LSL	22.800	23.000
U	23.300	23.800
L	22.800	22.800
μ	23.045	23.185
σ	0.150	0.100

wants to know the overall processes capability for assembly line for this product. The production line is illustrated in Fig. 5. Injection 1 (process 1) is the process of producing body and injection 2 (process 2) is for cap. They will be assembled in the assembling process (process 3) and then packaged (process 4).

The main quality characteristic of these two parts is height of engagement. After gathering enough samples from height of engagement of these two injection processes, different statistical software can be used in order to analyze the data. The results have been shown in Table 1. In addition, the defective rate for processes 4 is 5%.

Regarding to Eq. 11, rate of accepted parts is calculated as follows:

$$= \Phi\left(\frac{0.2 - 0.14}{0.064}\right) - 1 + \Phi\left(\frac{23.3 - 23.045}{0.15}\right) - \Phi\left(\frac{22.8 - 23.045}{0.15}\right)$$

$$-1 + \Phi\left(\frac{23.8 - 23.185}{0.1}\right) - \Phi\left(\frac{22.8 - 23.185}{0.1}\right) = 0.898$$

Table 2: Results of affection of increasing C_{pA} and C_{pB} on OPC

ID	σ_1	σ_2	OPC	C_{pA}	C_{pB}
1	0.25	0.20	0.579	0.267	0.333
2	0.24	0.19	0.614	0.278	0.351
3	0.23	0.18	0.649	0.290	0.370
4	0.22	0.17	0.683	0.303	0.392
5	0.21	0.16	0.717	0.317	0.417
6	0.20	0.15	0.751	0.333	0.444
7	0.19	0.14	0.783	0.351	0.476
8	0.18	0.13	0.814	0.370	0.513
9	0.17	0.12	0.844	0.392	0.556
10	0.16	0.11	0.873	0.417	0.606
11	0.15	0.10	0.899	0.444	0.667
12	0.14	0.09	0.923	0.476	0.741
13	0.13	0.08	0.944	0.513	0.833
14	0.12	0.07	0.962	0.556	0.952
15	0.11	0.06	0.977	0.606	1.111
16	0.10	0.05	0.987	0.667	1.333
17	0.09	0.04	0.994	0.741	1.667
18	0.08	0.03	0.998	0.833	2.222

Then overall defective rate and %Yield are:

$$ODR = 1\text{-}0.898 = 0.102$$

$$\%Yield = 1\text{-}ODR = \text{Rate of accepted parts} = 0.898$$

Now defective rates for processes 1, 2 and 3 have been calculated and regarding to 5% defective rate of process 4 and Eq. 10 overall % yield is:

$$OP = \prod_{i=1}^{n} P_i = 0.898 \times 0.95 = 0.853$$

$$\%Yield = 2\Phi(3C_p) - 1$$

$$0.853 = 2\Phi(3C_p) - 1$$

$$\Phi(3C_p) = \frac{1 + 0.853}{2} = 0.926$$

$$3C_p = 1.47$$

$$\text{Overall Process Capability (OPC)} = C_p = 0.49$$

The overall process control for this assembly production line is 0.49, where C_p for processes A and B are 0.44 and 0.66. In order to clarify the usage of overall processes capability index, with fixing the $\bar{X}_1$ and $\bar{X}_2$ and changing their standard deviation OPC has been compared with C_{pA} and C_{pB} and results have been shown in Table 2.

CONCLUSION AND FUTURE RESEARCH

Nowadays manufacturers and suppliers want to point out the capability of their whole assembly production line, because it is so useful for them to present their potential. Therefore, this study develops a new method in order to measure the process capability of a production line in case of assembly lines. This process capability is defined for whole assembly line and called overall processes capability index. Regarding to relation between percentage yield and process capability, the Overall Defective Rate (ODR) developed for assembly lines in different cases (series and parallel processes). Then with a novel attitude to defective rate in assembly lines, a method for measuring overall processes capability index was developed. In a real example presented, it was obvious that OPC has a moderate and reasonable behavior among C_p of other processes. In the other hand, because OPC is rejected-based, so it is more realistic factor to measure the capability of assembly lines. With this method, manufacturers and suppliers will be able to present their capability of their assembly lines. The proposed index was defined based on C_p, yet there are other PCIs, which can be employed in future studies. Wu ***et al.*** (2009) have a yield comparison between PCIs, which can be employed in order to construct other overall, process capability indices. Moreover, in this study we assumed simple assembly line which each process has one-quality characteristic, while in future research multi lines with multivariate processes or multi-product processes can be considered.

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