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New Process Capability Index using Taguchi Loss Functions

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Abstract: Classic process capability indices such as C_a, C_p and C_{pk} are well-known process capability indices, which are using widely. Since, process capability indices predict the capability of a process, they must have a significant relation with rate of rejects and losses. Studies showed that mostly process capability indices do not have a significant relation with rate of rejects or losses. Therefore, the loss-based indices are more appropriate and suitable indices to predict the capability of a process. In order to define a new loss-based process capability index, Taguchi loss functions were employed and this study proposed a novel process capability index called Taguchi-based Process Capability Index (TPCI). The methodology of this process capability index is based on standard rate of rejects for a capable process compared to other cases. Therefore, this study develops a new process capability index, which is Taguchi loss function-based and sensitive to losses. This new process capability index can provide a realistic and applicable metric to evaluate a process.

Key words: Process capability index, loss, loss-based process capability index, Taguchi loss functions, loss function, Taguchi-based process capability index

INTRODUCTION

Since, Process Capability Indices (PCIs) predict the process capability, it is expected that they have a significant relation with rate of rejects and losses. Studies showed that well-known process capability indices such as C_a, C_p and C_{pk} (Kane, 1986) do not have a significant relation with rate of rejects or losses. For example, Ramakrishnan *et al.* (2001) with an example showed that a higher C_p indicates a higher process quality, but a high quality process does not necessarily mean the fewer rate of rejects. Therefore, the process capability indices based on losses are more reliable and realistic than others. In order to define a new loss-based process capability index, loss functions such as Taguchi loss function can be employed. Nowadays loss functions not only have been used widely for predicting the losses but also for various purposes such as risks evaluation (Pan, 2007), decision-making (Kethley, 2008), quality engineering (Shu *et al.*, 2005), tolerances design (Naidu, 2008) and capability analysis (Hsieh and Tong, 2006). Leung and Spiring (2004) stated that with more error in production specifications, the loss functions decrease, thus regarding to close relation between loss functions and amount of losses and rate of rejects, these functions are more appropriate functions to design realistic loss function-based process capability indices.

There are some researches about new loss-based process capability indices. For instance, in a research work Hsieh and Tong (2006) constructed a measurable index incorporated the process capability indices philosophy and concept of quality loss function to analyze the process capability with the consideration of the qualitative response data. In their study, they invented new process capability index in order to measure the capability of a process. Moreover, Jeang and Chung (2008) proposed a process capability analysis based on minimum production cost and a quality loss. Scholars have used loss functions with the combination of a process capability or production cost to construct a new loss-based process capability index. In this study, we intend to construct an entirely loss-based process capability index to evaluate the process capability with the usage of Taguchi quality loss functions.

TAGUCHI LOSS FUNCTIONS

Taguchi (1986) developed his methods in loss functions for Japanese companies that were interested to improve their processes in order to implement total quality management. His method prepared a new approach to understanding and interpreting process information. Taguchi assumed the target as a base point and desired aim and he defined losses for all data departed from target. In other words, Taguchi losses can include even accepted products, which may cause customer dissatisfaction and loss of company reputation. Therefore, Taguchi loss functions detect the customer's desire to produce

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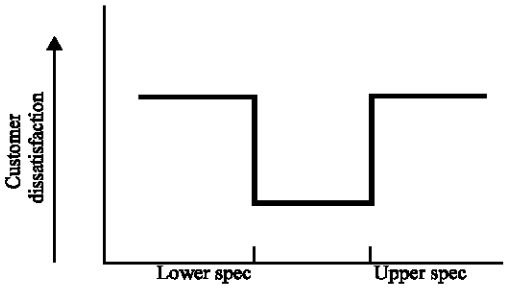

Fig. 1: Traditional issue to customer dissatisfaction

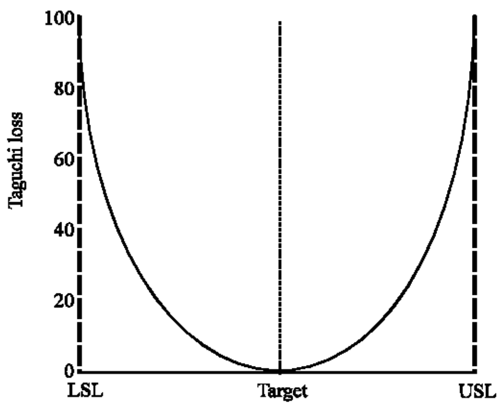

Fig. 2: Two-sided equal-specifications Taguchi loss function

products that are more homogeneous. Regarding to this approach, in addition to traditional costs of re-work, other losses such as scraps, warranty and services costs and cost of inhomogeneous are assumed. Before Taguchi's definition, traditional quality as shown in Fig. 1 was defined by good or bad. If the specification was within specification limits, the product was good; otherwise, it was marked as a reject.

This view assumed that a product is either good or bad and is uniformly good between the upper and lower specifications, however there is no sharp cut-off in the real world. Taguchi's curve is centered on the target value, which provides the best performance in the customer's eyes. Identifying this best value is not necessarily a simple task and is often the designer's best guess (Lofthouse, 1999). This loss function is given by Eq. 1 as follows:

$$L(y) = k(y-m)^2 \tag{1}$$

where, L(y) is the loss associated with a particular value of quality character y, m is the target value of the specification; k is the loss coefficient, whose value is constant depending on the cost at the specification limits and the width of the specification.

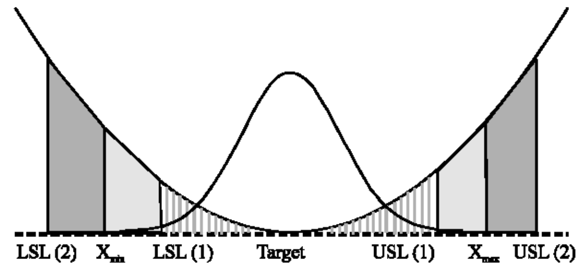

Fig. 3: Narrow and wide tolerances for a typical process

Since, PCIs measure the capability of a process with the comparison of inherent variability and capability of a process with the specification requirements of the product, the place of USL and LSL has a significant relationship with all PCIs. A typical process can be a super capable process when the tolerance is so narrow, while this process can be a very poor process when the tolerance is wide. For example as it is shown in Fig. 3, if the tolerance is LSL(1) to USL(1), then this typical process is not capable, but if the tolerance is LSL(2) to USL(2) the process will be capable.

Through analyzing Fig. 1 with Taguchi loss function, a standard rate of rejects or losses can be predicted for a capable process. On evaluating the process in Fig. 3, if LSL and USL adjust on X_{min} and X_{max}, the C_p of this process is 1 and the process is capable. For this process, we expect losses, which are located in the area under Taguchi loss function, but when the tolerance is narrower or wider than the expected Taguchi losses, it would be changed. With this point of view, in order to create a new Taguchi-based process capability index (TPCI), we divided the area under Taguchi loss function within the specification limits to the area under Taguchi loss function within X_{min} and X_{max} as follows:

$$TPCI = \frac{\text{Area of behind of Taguchi loss function within the specifications limits}}{\text{Area of behind of Taguchi loss function between } X_{min} \text{ and } X_{max}}$$

$$TPCI = \frac{\int_{LSL}^{USL} (x-T)^2 dx}{\int_{X_{min}}^{X_{max}} (x-T)^2 dx} = \frac{\sigma^2 (\int_{LSL}^{USL} \frac{(x-\mu+\mu-T)^2}{\sigma^2}) dx}{\sigma^2 (\int_{X_{min}}^{X_{max}} \frac{(x-\mu+\mu-T)^2}{\sigma^2}) dx}$$

$$TPCI = \frac{\sigma^2 (\int_{LSL}^{USL} \frac{(x-\mu)^2}{\sigma^2} dx + 2\sigma^2 \int_{LSL}^{USL} \frac{(x-\mu)(\mu-T)}{\sigma^2} dx + \sigma^2 \int_{LSL}^{USL} \frac{(\mu-T)^2}{\sigma^2} dx}{\sigma^2 (\int_{X_{min}}^{X_{max}} \frac{(x-\mu)^2}{\sigma^2} dx + 2\sigma^2 \int_{X_{min}}^{X_{max}} \frac{(x-\mu)(\mu-T)}{\sigma^2} dx + \sigma^2 \int_{X_{min}}^{X_{max}} \frac{(\mu-T)^2}{\sigma^2} dx}$$

$$= \frac{\sigma^2 (\int_{LSL}^{USL} \frac{(x-\mu)^2}{\sigma^2} dx + 2\sigma(\mu-T) \int_{LSL}^{USL} \frac{(x-\mu)}{\sigma} dx + \sigma^2 \int_{LSL}^{USL} \frac{(\mu-T)^2}{\sigma^2} dx}{\sigma^2 (\int_{X_{min}}^{X_{max}} \frac{(x-\mu)^2}{\sigma^2} dx + 2\sigma(\mu-T) \int_{X_{min}}^{X_{max}} \frac{(x-\mu)}{\sigma} dx + \sigma^2 \int_{X_{min}}^{X_{max}} \frac{(\mu-T)^2}{\sigma^2} dx}$$

where, USL and LSL represent the upper and lower specification limits, respectively; μ represents the process mean, T is target and σ is the process standard deviation.

In order to solve this integral, since $x \sim N(\mu, \sigma^2)$, it is concluded that

$$\frac{(x-\mu)^2}{\sigma^2} \approx \chi_1^2$$

where, χ_1^2 is Chi-squared distribution with one degree of freedom. The probability density function of χ_1^2 is:

$$f(x,1) = \begin{cases} \dfrac{1}{2^{1/2}\Gamma(1/2)} x^{-1/2} e^{-x/2} & \text{for } x > 0, \\ 0 & \text{for } x \le 0, \end{cases}$$

where, Γ denotes the Gamma function. In addition, we know:

$$\frac{(x-\mu)}{\sigma} \approx N(0,1)$$

where, N denotes normal distribution. Therefore, the TPCI is given as:

$$TPCI = \frac{\sigma^2 \left(\int_{LSL}^{USL} \frac{1}{2^{1/2}\Gamma(1/2)} x^{-1/2} e^{-x/2} dx + 2\sigma(\mu - T)(\phi(\frac{USL-\mu}{\sigma}) - \phi(\frac{LSL-\mu}{\sigma})) + (\mu - T)^2 (USL - LSL)\right.}{\sigma^2 \left(\int_{X_{min}}^{X_{max}} \frac{1}{2^{1/2}\Gamma(1/2)} x^{-1/2} e^{-x/2} dx + 2\sigma(\mu - T)(\phi(\frac{X_{max}-\mu}{\sigma}) - \phi(\frac{X_{min}-\mu}{\sigma})) + (\mu - T)^2 (X_{max} - X_{min})\right.}$$

In this study we assume that sample size is at least 30, but if the sample size is small

$$\frac{(x-\mu)}{S} \approx t$$

must be replaced by

$$\frac{(x-\mu)}{S} \approx N(0,1)$$

In order to calculate above integral we used MATLAB software. Since this integral is so complicated, MATLAB software suggested an approximate method to calculate. This method uses erf function (standard normal cumulative distribution function), which can calculate the integral approximately as follows:

$$TPCI = \frac{\sigma^2 \frac{1}{2^{1/2}\pi}\left(-2x^{1/2} e^{-\frac{1}{2}x} + \pi^{1/2} 2^{1/2} \operatorname{erf}(\frac{1}{2} 2^{1/2} x^{1/2}) \right]_{LSL}^{USL} + 2\sigma(\mu - T)(\phi(\frac{USL-\mu}{\sigma}) - \phi(\frac{LSL-\mu}{\sigma})) + (\mu - T)^2 (USL - LSL)}{\sigma^2 \frac{1}{2^{1/2}\pi}\left(-2x^{1/2} e^{-\frac{1}{2}x} + \pi^{1/2} 2^{1/2} \operatorname{erf}(\frac{1}{2} 2^{1/2} x^{1/2}) \right]_{X_{min}}^{X_{max}} + 2\sigma(\mu - T)(\phi(\frac{X_{max}-\mu}{\sigma}) - \phi(\frac{X_{min}-\mu}{\sigma})) + (\mu - T)^2 (X_{max} - X_{min})}$$

NUMERICAL EXAMPLE

XYZ Co. is a factory producing plastic parts and helmets. One of its products is plastic container. This product has 2 main parts. First part is body and the second part is cap. Each part is being produced by an injection process and admittedly in each process there is a specific rate of rejects. Finally, in last process these two parts will be assembled. Now the company wants to estimate process capability of its injection process. Thickness is the key quality characteristic of this process and the specifications of data are shown in Table 1.

The distribution of data and related Taguchi loss function are showed in Fig. 4. Since, LSL and USL are located between X_{min} and X_{max}, we expect so many rejects and high loss. Therefore, for this example TPCI must be less than 1.

Table 1: Specification of injection data (example 1)

Item (cm)	Injection
USL	12.4000
Target	12.3500
LSL	12.3000
$\bar{X}$	12.3000
SD	0.0982
X_{max}	12.5750
X_{min}	12.0250

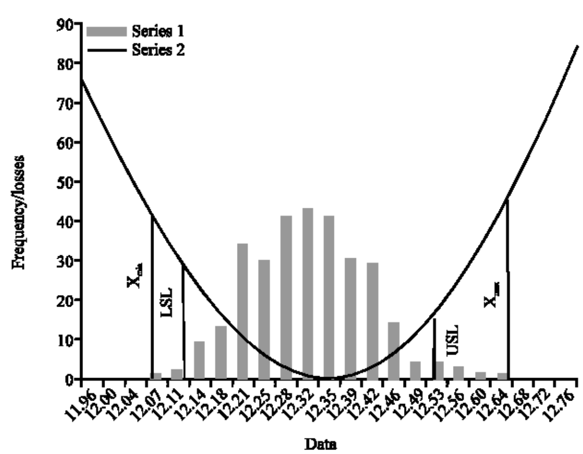

Fig. 4: Distribution of data and related Taguchi loss function for example 1

Table 2: Specification of injection data (example 2)

Item (cm)	Injection
USL	12.7600
Target	12.3500
LSL	12.0000
$\bar{X}$	12.3000
SD	0.0982
X_{max}	12.5750
X_{min}	12.0250

$$TPCI = \frac{\sigma^2 \frac{1}{2^{1/2}\pi}\left(-2x^{1/2}e^{-\frac{1}{2}x} + \pi^{1/2}2^{1/2}\mathrm{erf}(\frac{1}{2}2^{1/2}x^{1/2})\right]_{LSL}^{USL} + 2\sigma(\mu-T)(\phi(\frac{USL-\mu}{\sigma}) - \phi(\frac{LSL-\mu}{\sigma})) + (\mu-T)^2(USL-LSL)}{\sigma^2 \frac{1}{2^{1/2}\pi}\left(-2x^{1/2}e^{-\frac{1}{2}x} + \pi^{1/2}2^{1/2}\mathrm{erf}(\frac{1}{2}2^{1/2}x^{1/2})\right]_{X_{min}}^{X_{max}} + 2\sigma(\mu-T)(\phi(\frac{X_{max}-\mu}{\sigma}) - \phi(\frac{X_{min}-\mu}{\sigma})) + (\mu-T)^2(X_{max}-X_{min})}$$

$$= \frac{0.098^2 \frac{1}{2^{1/2}\pi}\left(-2x^{1/2}e^{-\frac{1}{2}x} + \pi^{1/2}2^{1/2}\mathrm{erf}(\frac{1}{2}2^{1/2}x^{1/2})\right]_{12.3}^{12.4} + 2*0.098(12.3-12.35)(\phi(\frac{12.4-\mu}{0.098}) - \phi(\frac{12.3-\mu}{0.098})) + (12.3-12.35)^2(12.4-12.3)}{0.098^2 \frac{1}{2^{1/2}\pi}\left(-2x^{1/2}e^{-\frac{1}{2}x} + \pi^{1/2}2^{1/2}\mathrm{erf}(\frac{1}{2}2^{1/2}x^{1/2})\right]_{12.025}^{12.575} + 2*0.098(12.3-12.35)(\phi(\frac{12.575-12.3}{0.098}) - \phi(\frac{12.025-12.3}{0.098})) + (12.3-12.35)^2(12.575-12.025)}$$

TCPI=0.916

In second example, we assume the same process while XYZ Co. wants to work with another customer with narrower tolerance. The specifications of data are shown in Table 2.

The distribution of data and related Taguchi loss function are shown in Fig. 5. Since, LSL and USL is located between Xmin and Xmax, we expect few rejects and low loss. Therefore, in this example new PCI is more than 1.

$$TPCI = \frac{\sigma^2 \frac{1}{2^{1/2}\pi}\left(-2x^{1/2}e^{-\frac{1}{2}x} + \pi^{1/2}2^{1/2}\mathrm{erf}(\frac{1}{2}2^{1/2}x^{1/2})\right]_{LSL}^{USL} + 2\sigma(\mu-T)(\phi(\frac{USL-\mu}{\sigma}) - \phi(\frac{LSL-\mu}{\sigma})) + (\mu-T)^2(USL-LSL)}{\sigma^2 \frac{1}{2^{1/2}\pi}\left(-2x^{1/2}e^{-\frac{1}{2}x} + \pi^{1/2}2^{1/2}\mathrm{erf}(\frac{1}{2}2^{1/2}x^{1/2})\right]_{X min}^{X max} + 2\sigma(\mu-T)(\phi(\frac{Xmax-\mu}{\sigma}) - \phi(\frac{Xmin-\mu}{\sigma})) + (\mu-T)^2(Xmax-Xmin)}$$

$$= \frac{0.098^2 \frac{1}{2^{1/2}\pi}\left(-2x^{1/2}e^{-\frac{1}{2}x} + \pi^{1/2}2^{1/2}\mathrm{erf}(\frac{1}{2}2^{1/2}x^{1/2})\right]_{12}^{12.76} + 2*0.098(12.3-12.35)(\phi(\frac{12.76-\mu}{0.098}) - \phi(\frac{12-\mu}{0.098})) + (12-12.35)^2(12.76-12)}{0.098^2 \frac{1}{2^{1/2}\pi}\left(-2x^{1/2}e^{-\frac{1}{2}x} + \pi^{1/2}2^{1/2}\mathrm{erf}(\frac{1}{2}2^{1/2}x^{1/2})\right]_{12.025}^{12.575} + 2*0.098(12.3-12.35)(\phi(\frac{12.575-12.3}{0.098}) - \phi(\frac{12.025-12.3}{0.098})) + (12.3-12.35)^2(12.575-12.025)}$$

TCPI=1.126

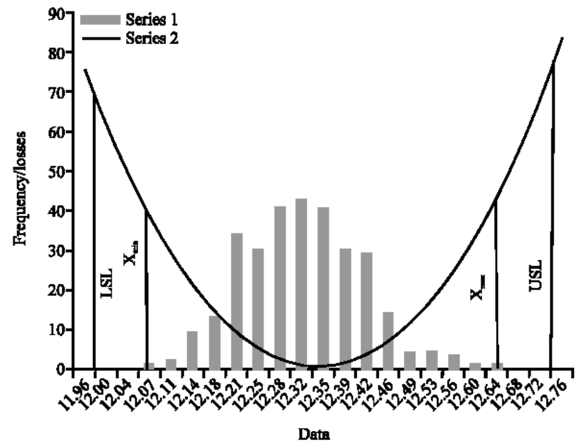

Fig. 5: Distribution of data and related Taguchi loss function for example 2

CONCLUSION

In this study, a novel PCI (TPCI) was designed based on Taguchi loss functions. The logical idea was to compare standard loss for a capable process with other cases. When LSL and USL adjusted on X_{min} and X_{max}, the C_p of the process will be 1 and this process will be capable. In this case we expect some losses, which is located the area under Taguchi loss function (standard losses), but when the tolerance was narrower or wider than the expected Taguchi losses, it would be changed. With this point of view, we divided the area under Taguchi loss function between tolerance to the area under Taguchi loss function within X_{min} and X_{max}. While new loss-based PCIs usually constructed with the combination of a process capability or production cost to construct a new loss-based process capability index, this new loss-based PCI was entirely loss-based PCI constructed based on usage of Taguchi quality loss functions. This new PCI as showed with an example, is more realistic and sensitive to the losses compared with other PCIs.

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