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Effective Parameters on Modulus of Subgrade Reaction in Clayey Soils

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Abstract: In Winkler model the subgrade soil is assumed to behave like infinite numbers of linear elastic springs, which the stiffness of the springs is named as the modulus of subgrade reaction. This modulus depends on some parameters like soil type, dimension, shape, embedment depth and type of foundation (Flexible or Rigid). The direct method to estimate the modulus of subgrade reaction is the plate load test. In this study the foundations on clayey soil are modeled with finite element software to investigate the validation of Terzaghi's formula and the effect of different parameters on subgrade reaction modulus. Due to obtained results, Terzaghi's formula does not consider the effect of soil consistency and the shape of foundation; therefore it results in uncertain values. As depth of embedment of foundation is increased, the modulus of subgrade reaction is increased. Flexural rigidity of foundation can improve the status of subgrade reaction modulus.

Key words: Winkler model, plate load test, modulus of subgrade reaction, mat foundation

INTRODUCTION

Soil has very complex mechanical behavior, because of its nonlinear, stress-dependant, anisotropic and heterogeneous nature. Hence, instead of modeling the subsoil in all its complexity, the subgrade is often replaced by a much simpler system called a subgrade reaction model. Winkler (1867) proposed a model that assumes the ratio between contact pressure, P, at any given point and the associated vertical settlement, y, is linear and given by the coefficient of subgrade reaction, K_s:

$$K_s = P/y \quad (1)$$

In this model, the subsoil is replaced by fictitious springs whose stiffness is equal to K_s.

Winkler (1867) assumed the soil medium as a system of identical but mutually independent, closely spaced, discrete and linearly elastic springs. But, the simplified assumptions which this approach is based on caused approximations.

One of the basic limitations is that, this model cannot transmit shear stresses which are derived from lack of spring coupling. Also, stress-strain behavior is assumed to be linear.

Many researches like Biot (1937), Terzaghi (1955), Vesic (1961), Horvath (1983a, b), Vallabhan and Daloglu (1999, 2000) and Vallabhan and Das (1989) have investigated the effective factors and determination approaches of K_s.

Geometry, dimensions of the foundation and soil layering are assigned to be the most important effective parameters on K_s. Generally, the value of subgrade modulus can be obtained in the following alternative approaches:

- Plate load test
- Consolidation test
- Triaxial test
- CBR test

Terzaghi (1955) made some recommendations about K_s for 1×1 ft rigid slab placed on a soil medium. However, the procedure to compute a value of K_s to use in a larger slab was not specified. Biot (1937) solved the problem for an infinite beam with a concentrated load resting on a 3D elastic soil continuum.

He found a correlation between continuum elastic theory and Winkler model, that the maximum moments in the beam are equated. Vesic (1961) matched the maximum displacement of the beam in both models and tried to develop a value for K_s with matching bending moments. He obtained the equation for K_s to use in the Winkler model.

Other works by Filonenko-Borodich (1940), Hetenyi (1946), Pasternak (1954) and Vlasov and Leont 'ev (1960)

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attempt to make the Winkler model more realistic by assuming some forms of interaction among the spring elements, that represent the soil continuum.

In this study, effective parameters on modulus of subgrade reaction in clayey soils are studied with use of numerical modeling. The studied parameters include:

- Size effect of foundation
- Effect of foundation's shape
- Effect of depth of foundation
- Effect of rigidity of foundation

Calibration of model: In this way for calibrating model, the studied case is based on results of 0.3 m plate load test on clayey soil by Consoli *et al*. (1998). Soil profile to 4 m deep shown in Fig. 1, in which cone measurements and the values of maximum shear modulus (G_0) obtained from cross-hole and down-hole tests are presented.

It is clear that both the CPT and shear modulus are fairly constant, which indicates that the homogeneity of the soil layer does not depend on strength. The water table is at about 4.0 m depth. The soil layer in finite element software is classified as low plasticity clay (CL) according to the unified soil classification system. Grain size data indicates 44% sand, 32% silt, 24% of clay.

The average bulk unit weight of soil ranged between 17.7 and 18.2 kN m^{-3}, the moisture content (ω) was

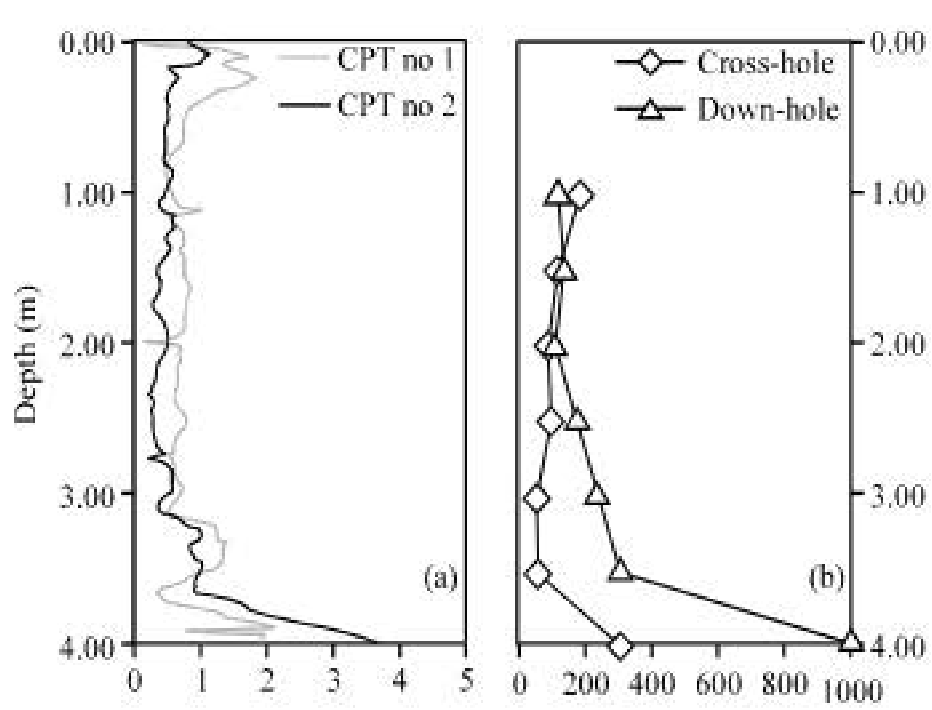

Fig. 1: Soil profile to 4 m deep, containing CPT tip resistance and shear moduli (G_0) values (Consoli *et al*., 1998). (a) q_c (Mpa) and (b) G_0 (MPa)

typically 24.5-26%, the degree of saturation (S) ranged around 78% and the void ratio (e) varied between 0.8-0.86.

Atterberg limits of the material were: liquid limit of 43% and plastic limit of 22% which yields a Plasticity Index (PI) of 21%.

The soil parameters used in Mohr-coulomb model in plaxis analysis are presented in Table 1.

The assumed dimensions of the model are shown in Fig. 2.

The comparison between obtained results from 3D analysis and the results by Consoli *et al*. (1998) is shown in Fig. 3.

As shown in Fig. 3a-c, there is little difference between numerical modeling results and mentioned case results and it shows appropriateness of numerical model.

Soil properties of tested models: In this study four kinds of clayey soils are considered, which their parameters are based on Bowles (1997) recommendations. The selected parameters for Mohr-coulomb soil behavior are shown in Table 2.

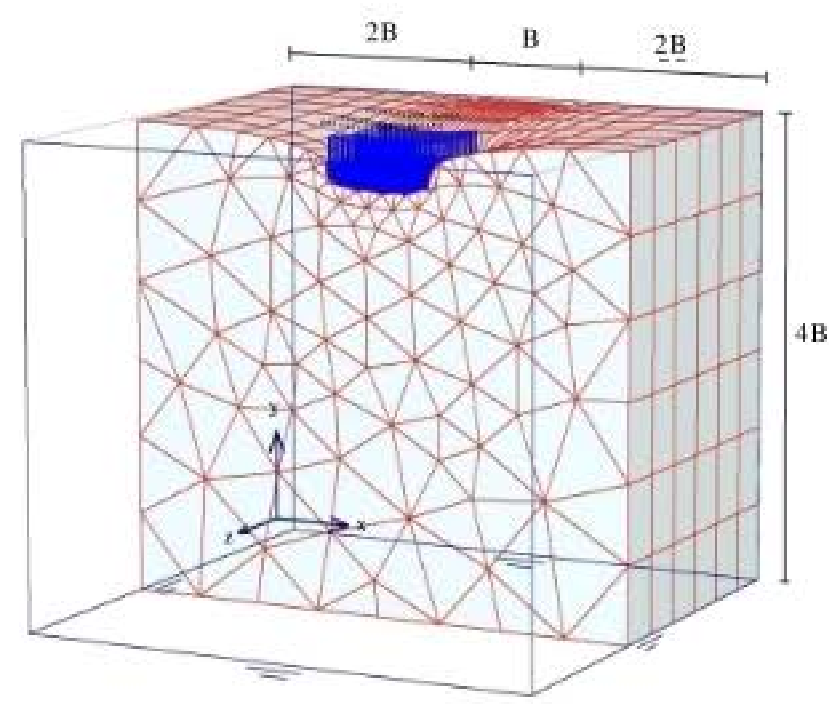

Fig. 2: Side dimensions of model

Table 1: Soil parameters (Consoli *et al*., 1998)

Parameters	Value
Modulus of elasticity (E)	10 (MPa)
Friction angle (Φ)	26°
Cohesion (C)	17 (KPa)
Poisson ratio	0.25 (assumed)
Soil unit weight	17.7 (kN m^{-3})

Table 2: Clayey soils properties (Bowles, 1997)

Parameters soil type	Allowable stress	Unit weight	E	Poisson ratio	Friction angle	Cohesion (kN m^{-2})
	(kN m^{-2})					
Medium clay	75	16	11250	0.30	18°	18.75
Stiff clay	150	18	22500	0.35	21°	37.50
Very stiff clay	300	19	45000	0.35	24°	75.00
Hard clay	500	20	60000	0.35	26°	100.00

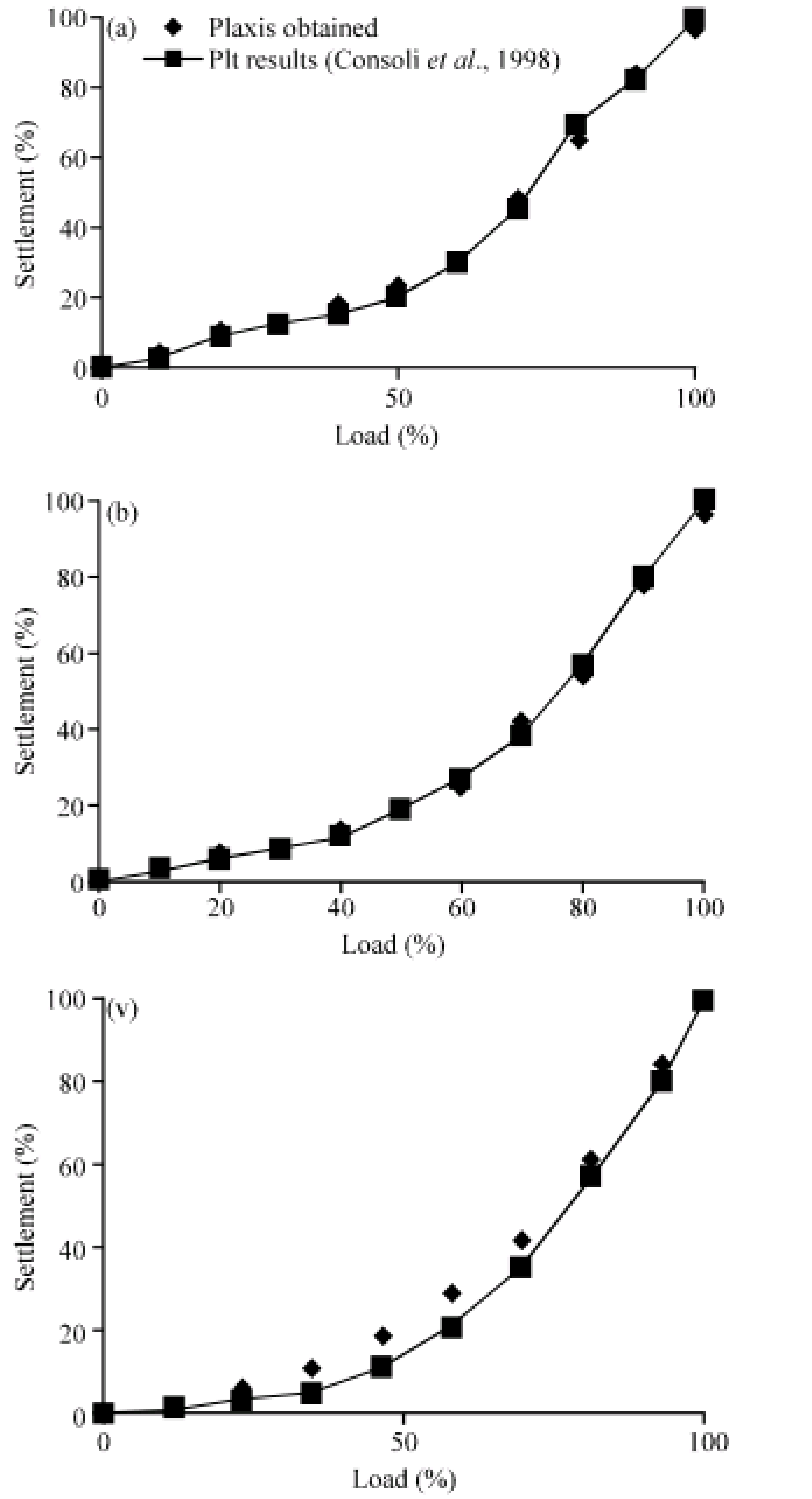

Fig. 3: Comparison of calibration results. (a) 0.3 m plate; (b) 0.45 m plate and (c) 0.7 m concrete footing

RESULTS

Size effect of foundation on K_s: Vertical settlement analysis of foundations with different widths (0 to 18 m) was performed with plaxis 3D software. The vertical settlement (y) for each analysis was obtained and the load-settlement graphs were plotted, then the secant modulus of each graph (K_s) was determined.

The Terzaghi's equation for estimating the modulus of subgrade reaction with plate load test results is:

$$K_s = K_{s1} \times \frac{0.3}{B} \tag{2}$$

In Eq. 2, K_s is subgrade reaction modulus for prototype foundation, K_{s1} is subgrade reaction modulus

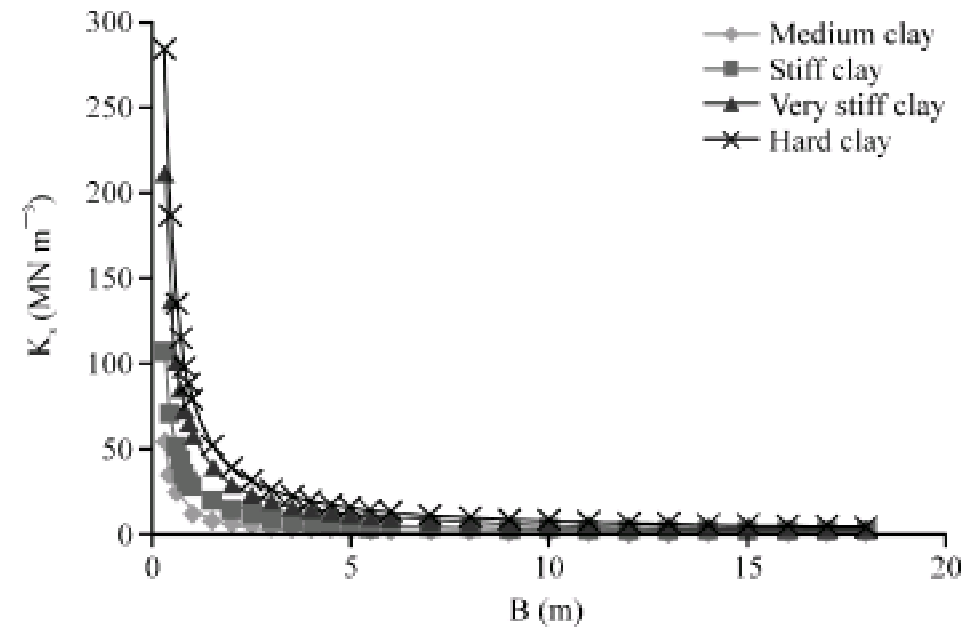

Fig. 4: Decreasing of K_s with side dimension of foundation

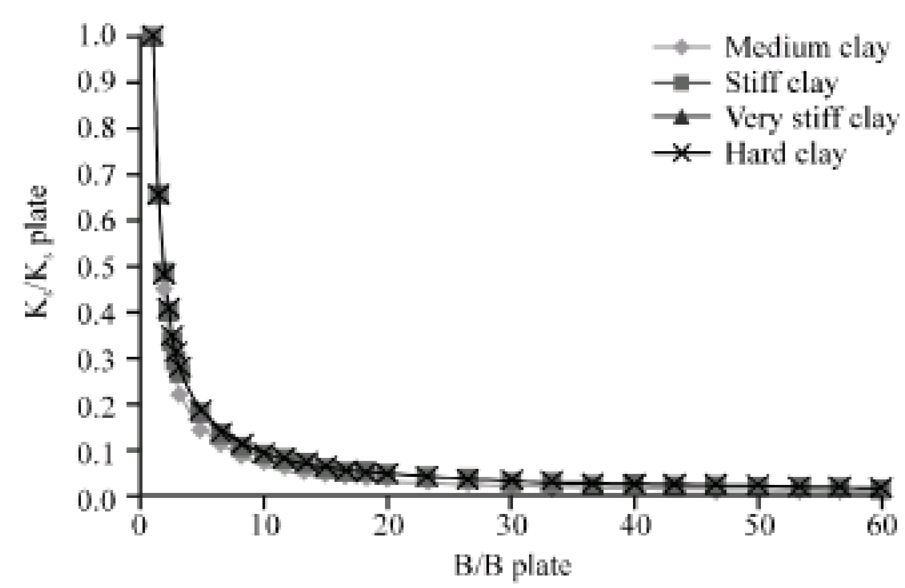

Fig. 5: Normalized K_s with normalized width

based on plate load test results and B is the side dimension of foundation.

Based on obtained results, the modulus of subgrade reaction (K_s) decreased as the side dimension of plate increased (Fig. 4). With normalizing the obtained K_s with plate load tests' K_s in its own soil type; all graphs in Fig. 5 are almost the same.

The decreasing manner of K_s with increasing foundations' width is the same between all kinds of clayey soils according to their consistencies, but as it can be seen in Fig. 6, as the consistency of clayey soil decreases, the difference between Terzaghi's equation and obtained K_s is increased.

It can be concluded from this figure that the Terzaghi's equation is not suitable for low consistent clayey soils. The first drop of K_s in this figure shows that the Terzaghi's equation, as Bowles mentions, deteriorates when foundation dimension is 3 times of plate dimension. As shown in Fig. 4, for a constant side dimension, the modulus of subgrade reaction increases as soil consistency does. According to Fig. 6, for a dimension higher than 3 times of plate dimension, the modified equation could be used:

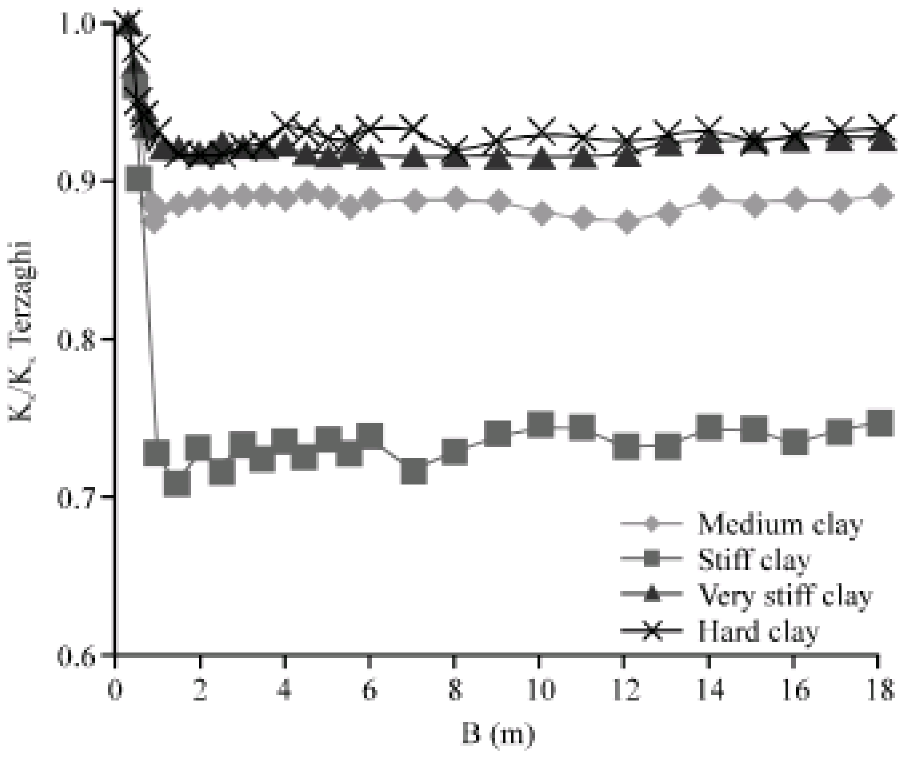

Fig. 6: Consistency effect of clayey soil on K_s values

Table 3: Modification coefficient

Soil type	n
Medium clay	0.75
Stiff clay	0.85
Very stiff clay	0.90
Hard clay	0.95

$$K_s = K_{s1} \times n \times \frac{0.3}{B} \quad (3)$$

In this equation, n is the modification coefficient and the other parameters were previously discussed. Different amounts of n are presented in Table 3.

Effect of foundations' shape on K_s: As it is shown in previous section, it seems that Terzaghi's equation is suitable for stiff clay; therefore this type of soil is selected in foregoing analysis.

Terzaghi's equation, for estimating the modulus of subgrade reaction of rectangular foundation based on plate load test results, is:

$$K_s = K_{s1} \times \left(\frac{0.5+m}{1.5m}\right) \quad (4)$$

In Eq. 4, K_s is the modulus of subgrade reaction for prototype foundation, K_{s1} is modulus of subgrade reaction based on plate load test results and m is the side dimensions ratio of rectangle foundation (B/L).

For investigating the shape effect of foundation on modulus of subgrade reaction, the selected value of m is 1 for square foundation and 1.5 to 5 for rectangle and 10 for strip foundation with same side dimension (B) in all kinds of shape. The selected dimensions are presented in Table 4.

The normalized results of numerical analysis with plate's data are presented in Fig. 7. It is shown that the modulus of subgrade reaction for square shape is the highest value, because with increasing the dimensions of foundation, the value of settlement with constant load intensity in square foundation becomes the lowest. Therefore, the modulus of subgrade reaction for strip foundation is the lowest and for square foundation is the highest value.

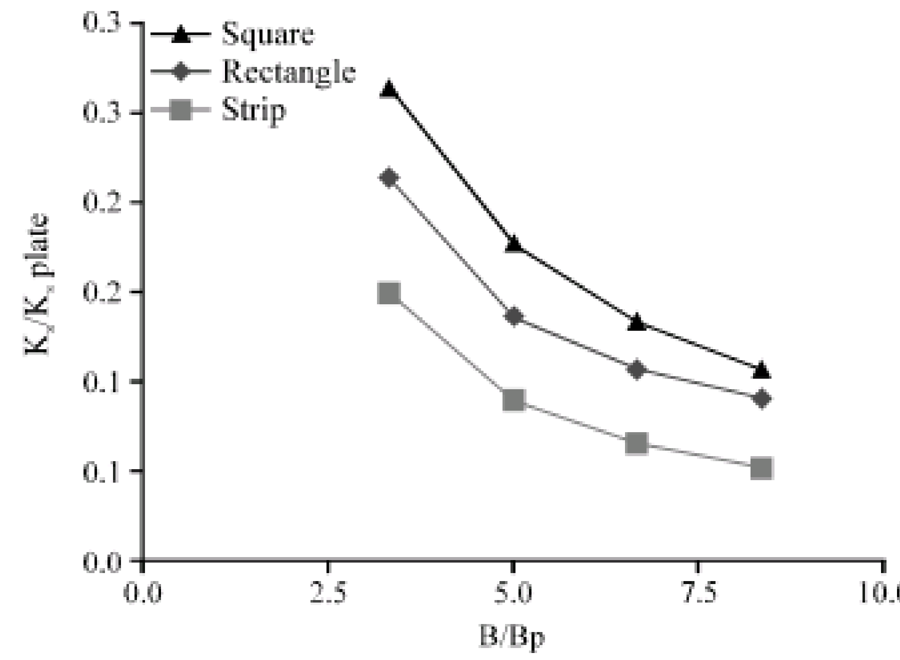

Fig. 7: Variation of modulus of subgrade reaction with shape

Table 4: Selected dimensions for comparison between shapes of foundations

Foundation type	Square	Rectangle	Strip
Dimensions (m)	1×1	1×3	1
	1.5×1.5	1.5×4.5	1.5
	2×2	2×6	2
	2.5×2.5	2.5×7.5	2.5

As shown in Fig. 7, as the width of each foundation shape is increased, the modulus of subgrade reaction is decreased.

Due to obtained results, it can be concluded that there is uncertainty about the Terzaghi's equation, because this equation ignores the effect of foundation's loading area and is based only on dimensions ratio (m).

As shown in Fig. 8, this uncertainty is correct and with increasing dimensions in constant m, the modulus of subgrade reaction has a very high drop down.

Maybe Terzaghi's equation is suitable for the rectangle foundation with 30 cm side dimension; because his equation result is near to 30 cm plate's result. This difference between rectangle foundations with constant m and constant load intensity, which is because of settlement increase with increasing the dimensions of foundation, is ignored in Terzaghi's equation.

As a result, the modified equation for rectangular and stripe foundations are presented:

$$\text{Rectangular:} \quad K_s = K_{s1} \times \left(\frac{5\times m}{4+m}\right) \times \left(\frac{Bp}{Lrf}\right)^{1.1} \quad (5)$$

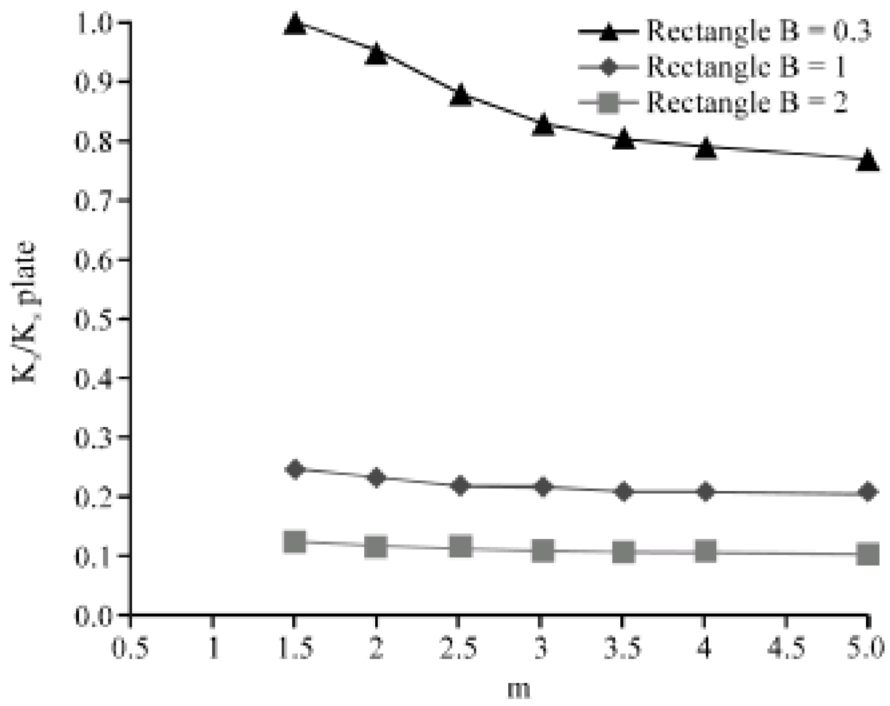

Fig. 8: Variation of K_s with different side dimension ratio (m)

$$\text{Stripe:} \quad K_s = K_{s1}\times\left(\frac{m+0.5}{1.5m}\right)\times\left(\frac{Bp}{Bsf}\right)^{1.3} \tag{6}$$

In these equations B_p is plate's width, L_{rf} is the length of rectangular foundation, B_{sf} is width of stripe foundation and other parameters were previously described.

In Fig. 9a-c, the appropriateness of suggested equations in stripe and rectangular foundations is presented. Figure 9a presents the accuracy of suggested equation against Terzaghi's equation in stripe foundation. In Fig. 9b and c weakness of Terzaghi's formula is presented against analysis results and suggested equation. As mentioned, Terzaghi's equation is not suitable for using in stripe and rectangular foundations of high width.

Embedment depth effect of foundation on K_s: As shown in Fig. 10, with increasing the depth of embedment, the experienced stress by soil is increased and the magnitude of foundation settlement with constant load intensity is decreased, therefore the value of subgrade modulus is increased.

But there is another aim of investigating this effect which is estimating the modulus of subgrade reaction in depth, with results of plate load test in surface. For this reason the numerical analysis is performed on stiff clay and the selected embedment depths are 0 to 5 m.

Equation 7 provides the best fit for correlation between modulus of subgrade reaction in surface (K_{s0}) and in depth (K_{sd}), as shown in Fig. 11.

$$\frac{Ksd}{Kso} = \frac{qo}{qo-\lambda D} \tag{7}$$

where, q_0 is load intensity in surface (kN m^{-2}), D is depth in meter and γ is the unit weight of the soil in (kN m^{-3}).

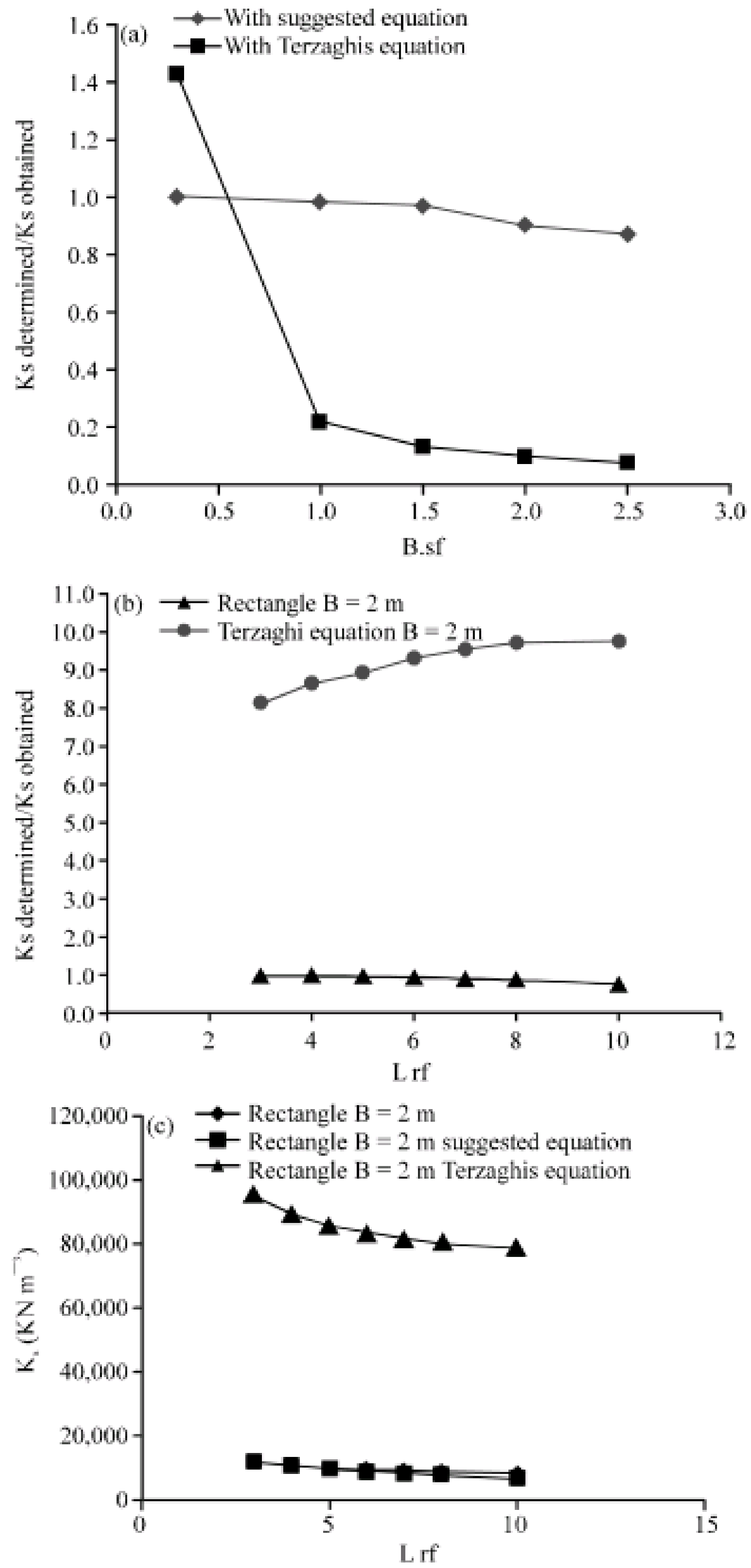

Fig. 9: Comparison between obtained results (a) suggested and Terzaghi's equation; (b) Stripe foundation and (c) 2 m width Rectangular foundation

Rigidity effect of foundation on K_s determination: When the flexural rigidity of the footing is taken into account, a solution is based on some forms of a beam on an elastic foundation; this may be the classical Winkler solution in which the foundation is considered as a bed of springs.

As shown in literature (Bowles, 1997) there is a parameter that determines if a foundation should be analyzed on the basis of the conventional rigid procedure or as a beam on elastic foundation:

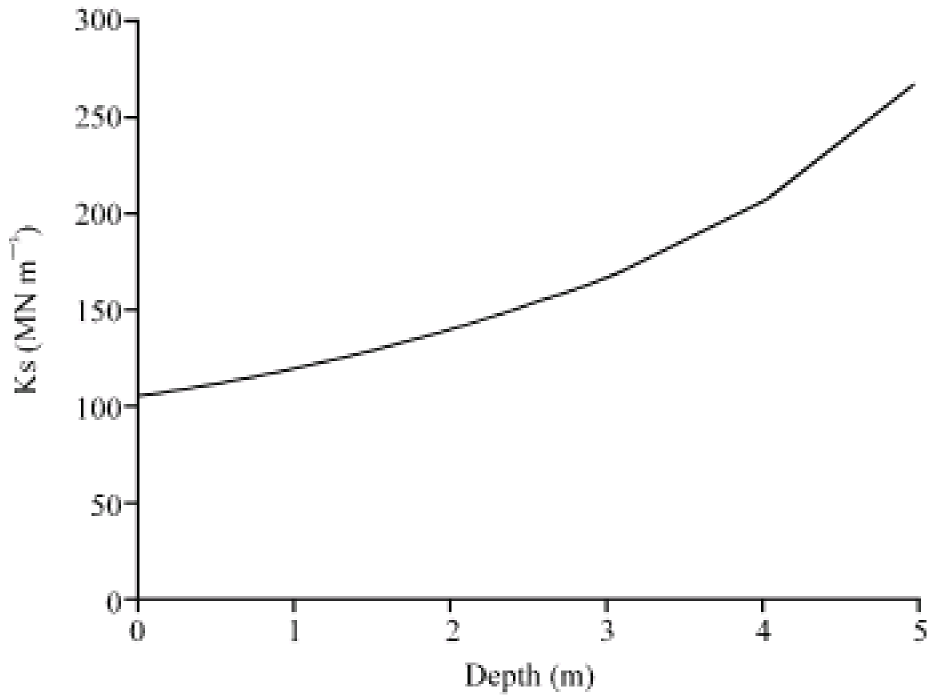

Fig. 10: Effect of embedment depth of foundation on K_s

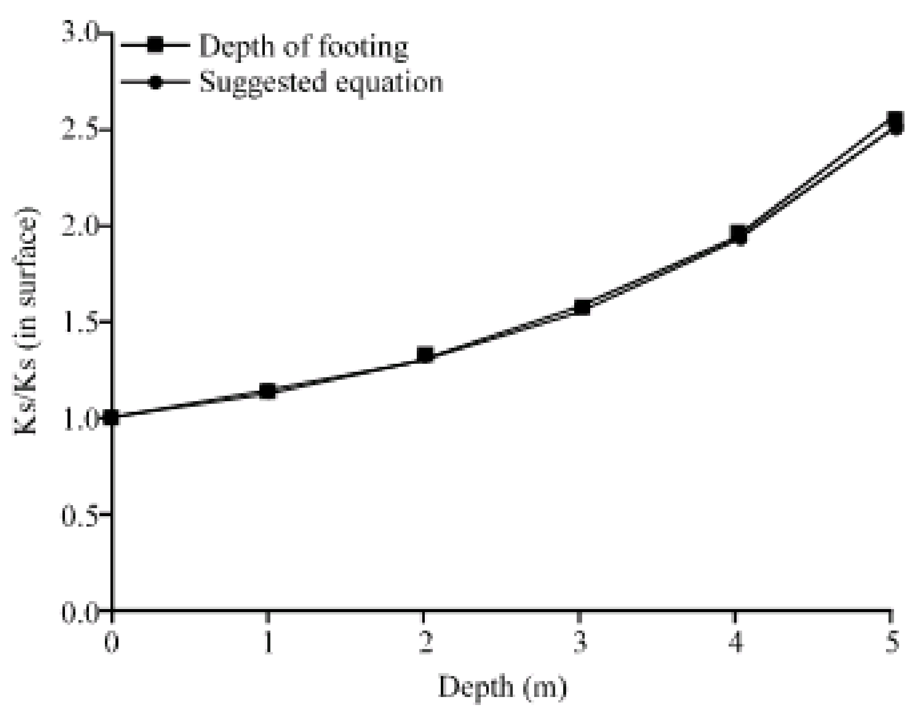

Fig. 11: Comparison of suggested equation and numerical analysis results

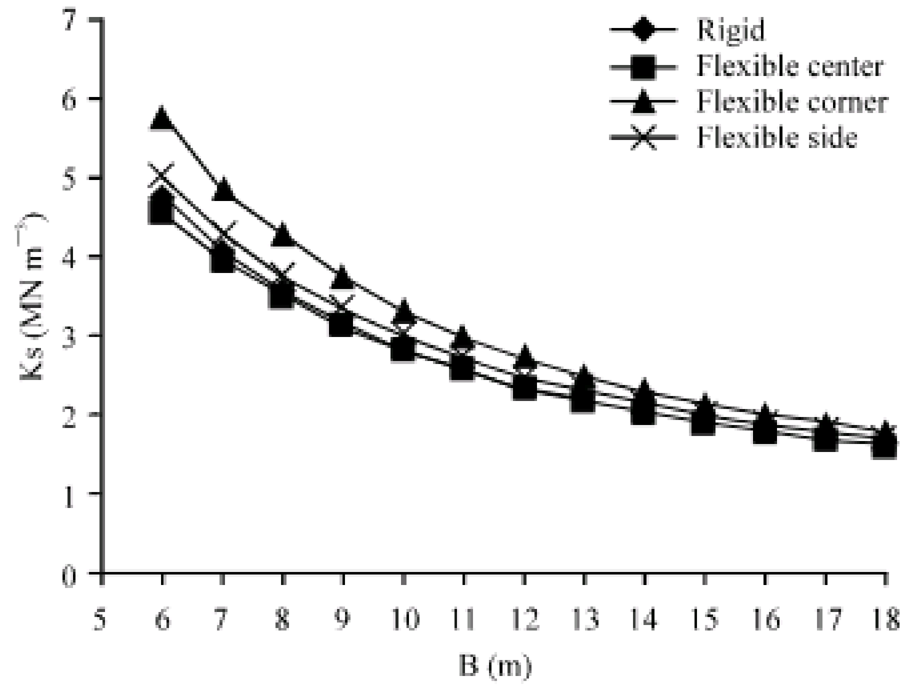

Fig. 12: Flexural effect of foundation on K_s

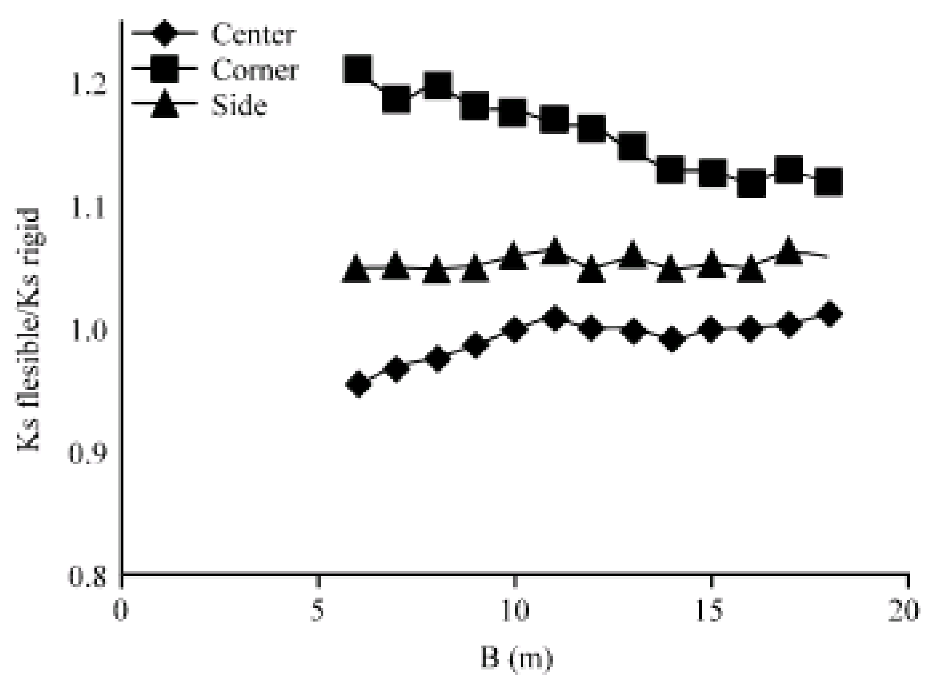

Fig. 13: Comparison between Flexible K_s and Rigid K_s

$$\lambda = \sqrt{\frac{K_s \times B}{4EI}} \tag{8}$$

Where:

B: Width of foundation (m)
E: Elasticity modulus of foundations material (KN m^{-2})
K_s: Modulus of subgrade reaction obtained from 0.3 m plate bearing test (kN m^{-3})

The criterion of selecting the foundation as rigid or flexible is:

- Rigid members: $\lambda L<(\pi\backslash 4)$
- Flexible members: $L\gamma>\pi$

L: Length of foundation (m)

According to this criterion, the maximum height of foundation for being flexible was produced and the numerical analysis was done on stiff clay soil with different foundation's width.

As shown in Fig. 12, as the width of flexible foundation is increased, like the rigid foundation, the modulus of subgrade reaction is decreased.

The ratio between flexible K_s and rigid K_s is presented in Fig. 13 for different points of foundation. As shown, the modulus of subgrade reaction in corner has the highest value and the side modulus of subgrade reaction is the middle value while the center value is the lowest one. This difference is known as dishing effect of flexible foundation. Figure 14 presents a dishing shape of numerical model after analysis.

As shown the corners settlement of flexible foundation has minimum value and the center point has a maximum value. As can be concluded from Fig. 11 there is little difference between modulus of subgrade reaction in rigid foundation and in center of flexible one.

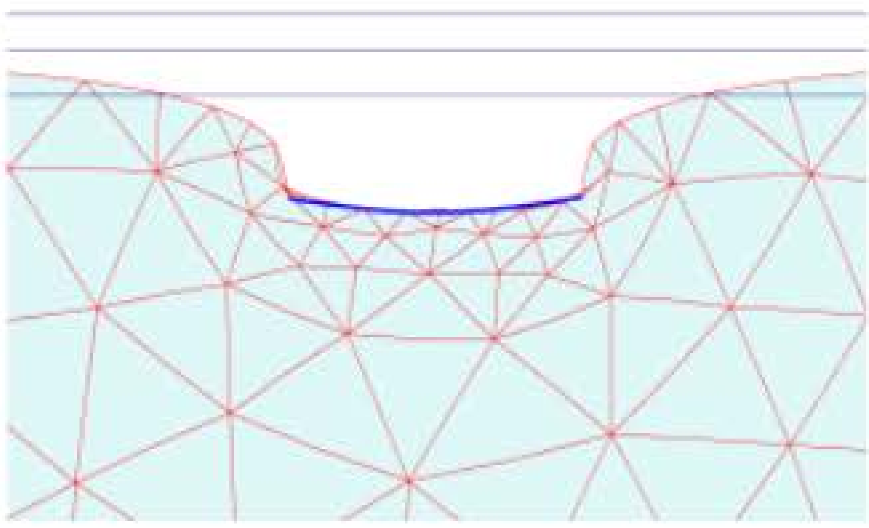

Fig. 14: Dishing shape of footing

CONCLUSIONS

At present study, shallow foundations on clayey soils are modeled with use of finite element software to investigate the validation of Terzaghi's formula. The effects of different parameters on subgrade reaction modulus are studied and the following results are obtained: Modulus of subgrade reaction is decreased as the side dimension of foundation is increased. It seems that the Terzaghi's equation as Bowles shows deteriorates when foundation dimension is 3 times of plate dimension. As the consistency of clayey soil is decreased the difference between obtained modulus of subgrade reaction and Terzaghi's one is increased. The modulus of subgrade reaction has a direct relation with clayey soils consistencies. It means that as the consistency of soil decreases the modulus of subgrade reaction decreases. With constant width and with constant load intensity, the modulus of subgrade reaction in strip foundation has the lowest value while the square foundation has the highest value. Due to obtained results, Terzaghi's equation for estimating the modulus of subgrade reaction for rectangular foundation is not recommended for use in both rectangular and strip foundation. As depth of embedment of foundation is increased, the modulus of subgrade reaction is increased and that's because of decreasing the foundations settlement value.

The following equation is proposed for estimation the modulus of subgrade reaction in specific depth of embedment from plate load test results in surface:

$$\frac{K_{sd}}{K_{so}} = \frac{q_o}{q_o - \lambda D}$$

Flexural effect of foundation is evaluated. The modulus of subgrade reaction in center of flexible foundation has the same value with same width rigid foundation.

The highest value of modulus of subgrade reaction is in the corner of flexible foundation and the side points have middle value of modulus of subgrade reaction in compare with center point and corner points.

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