# Journal of Applied Sciences

ISSN 1812-5654

science alert

ANSInet
an open access publisher
http://ansinet.com

Journal of Applied Sciences 9 (22): 4056-4060, 2009
ISSN 1812-5654


# Folding Model Analysis to Calculate Differential Cross Sections for the ($^{16}$O, $^{12}$C) System

A.I. Ass'ad
Department of Physics, Faculty of Applied Science, Al-Aqsa University,
Gaza Strip, Palestinian Authority, Palestine

**Abstract:** The ($^{16}$O, $^{12}$C) reaction has been studied on targets of $^{24}$Mg, $^{16}$O and $^{28}$Si and the differential cross sections of alpha transfer reactions have been evaluated in the framework of the Distorted Wave Born Approximation (DWBA) approach as a single-step process. The folding model potential is reproducing the large angle oscillatory structures of heavy-ion transfer reactions. This folding model can well reproduce the strong dependence of the total reaction cross section on the incident energy. The Gaussian potential is taken as the bound states of the transferred particle with the core nucleus forming the projectile or target and residual nuclei. The calculated angular distributions are found to be in a good agreement with the experimental data. The extracted spectroscopic factor is reasonable.

**Key words:** Folding model, differential cross section, alpha transfer reactions

## INTRODUCTION

The Distorted Wave Born Approximation (DWBA) calculations have been widely used in studying the direct nuclear reactions (Al-Farra, 2003; Ass'ad and Ashour, 2007). For most cases the DWBA is one step process (Mermaz *et al*., 1983). Therefore, several descriptions have been introduced to explain the general features of heavy-ion reactions (Bilwes *et al*., 1987; Farra and Ass'ad, 2004). The prominent gross structures of different transfer reactions have been analyzed in the whole angle region with qualitative agreement using both of surface transparent ion-ion potential and dynamic alpha particle transfer polarization potential (Filho *et al*., 1989). In some heavy-ion reactions, such as $^{24}$Mg($^{16}$O,$^{12}$C)$^{28}$Si oscillatory structure often appears in the whole angle region in the differential cross section (Linhua *et al*., 1985). This anomalous phenomenon, unexpected from ordinary DWBA theory, is explained by an alpha transfer process between two unidentical nuclear cores. The folding model can well reproduce the strong dependence of the total reaction cross section on the incident energy (Yang *et al*., 2002). Both zero-range and finite-range exchange parts of nucleon-nucleon NN interactions have been considered in the folding procedure (Zhang *et al*., 2009) to calculate the fusion cross section of $^{16}$O+$^{208}$Pb system. The differential cross sections of $^{16}$O-$^{16}$O and $^{12}$C-$^{12}$C system reactions (Jain and Shastry, 1979) have been calculated in terms of DWBA calculations, using double folding potential, where the nuclear part of the optical potential is quite sensitive to the shape of the density distribution. Elastic scattering of the two-neutron halo nucleus, $^{6}$He, on $^{12}$C target at 38.3 and 41.6 MeV/nucleon has been analyzed in the framework of the double-folding optical model (El-Azab *et al*., 2008). It has been shown that the obtained potential by folding a nucleon-nucleon interaction (Perez, 1973) into the ion densities gives a good description of the real part of the optical potential for $^{6}$Li-$^{6}$Li scattering between 9 and 16 MeV center of mass.

In the present study, the differential cross section of heavy-ion reactions with α-transfer reactions have been calculated in term of one step DWBA calculations using folding potential. The calculated differential cross section are fitted with the experimental data to extract spectroscopic factors.

**Nuclear optical potential [$V_{opt}(r)$]:** Here, the differential cross section for stripping reaction $^{16}$O-$^{12}$C system has been evaluated in the framework of one step DWBA calculations. The optical potential $V_{opt}(r)$ may thus be written as:

$$V_{opt}(r) = V_N(r) + V_C(r) + V_{S.O.}(r) \quad (1)$$

where, $V_N(r)$ is the complex nuclear part of the potential, $V_C(r)$ is the electrostatic potential (ESP) between the interacting bodies and $V_{S.O.}(r)$ is a spin-orbital interaction which is to be included where spin-orbital is important (Jain and Shastry, 1979).

Table 1: Optical potential parameters used in the DWBA calculations

| Reaction | $V_0$ (Me) | $R_v$ (fm) | $a_v$ (fm) | $W_0$ (Me) | $R_w$ (fm) | $a_w$ (fm) | $r_c$ (fm) | References |
|---|---|---|---|---|---|---|---|---|
| $^{16}O+^{24}Mg$ | 13.1 | 1.39 | 0.40 | 62.0 | 0.93 | 0.43 | 1.20 | Chengqun *et al*. (1988) |
| $^{16}O+^{16}O$ | 17.0 | 1.35 | 0.49 | 4.0 | 1.35 | 0.49 | 1.25 | Guozhu *et al*. (1984) |
| $^{16}O+^{28}Si$ | 10.0 | 1.35 | 0.61 | 23.4 | 1.23 | 0.55 | 1.25 | Linhua *et al*. (1985) |

In this study, we shall consider the problem of evaluating the potentials $V_N(r)$ and $V_C(r)$ for nucleus-nucleus system and Coulomb potential, respectively. The total optical potential is thus (Jain and Shastry, 1979).

$$V_{opt}(r) = (V + iW)\, V_N(r) + V_C(r) \qquad (2)$$

where, V and W are strength parameters of the real and imaginary parts, respectively (Yosio and Taro, 1984), given by:

$$V = \frac{-V_o}{1+\exp\{(r-R_v)/a_v\}}$$
$$W = \frac{-W_o}{1+\exp\{(r-R_w)/a_w\}} \qquad (3)$$

where, the parameters $V_o$, $R_v$ and $a_v$ are the strength, radius and diffuseness of the real potential, while the parameters $W_o$, $R_w$ and $a_w$ describe the imaginary part which are determined by fitting scattering reaction of the corresponding interaction of two heavy-ions.

The necessary parameters of the optical potential (Linhua *et al*., 1985; Chengqun *et al*., 1988; Guozhu *et al*., 1984) are shown in Table 1. The Coulomb potential due to a uniform charge sphere of radius $R_c$ is given by:

$$V_C(r) = \begin{cases} Z_i Z_j e^2 \left(3 - r^2/R_C^2\right)/2R_C ; & r \le R_C \\ Z_i Z_j e^2/r \quad ; & r \ge R_C \end{cases} \qquad (4)$$

The interaction radii have the form:

$$R_x = r_x \left(A_i^{1/3} + A_j^{1/3}\right) \qquad \text{for } x = v, w, c \qquad (5)$$

where, $r_v = r_w = 1.18$ fm and $r_c$ is shown in Table 1.
where, $V_N$ (r) is the nucleus-nucleus potential reaction (Jain and Shastry, 1979) and given by:

$$V_N(r) = 6315\frac{e^{-4r}}{4r} - 1961\frac{e^{-2.5r}}{2.5r} \qquad (6)$$

## NUMERICAL CALCULATIONS AND RESULTS AND DISCUSSION

To show, how sensitivity of the folding optical potential effects the differential cross section, we studied the effect of the folding optical potential as follows: The differential cross section has been numerically carried out for $^{24}Mg(^{16}O, ^{12}C)^{28}Si$ reaction at 27.8, 36.2 MeV, $^{16}O(^{16}O, ^{12}C)^{20}Ne$ reaction at 22.75 Mev and $^{28}Si(^{16}O, ^{12}C)^{32}S$ reaction at 26.23 Mev. The nuclear interactions describing the particle-nucleus bound states are represented by double Gaussian potentials (Chengqun *et al*., 1988).

Table 2: Parameters of Gaussian potential

| Reaction | $V_R$ (MeV) | $a_R$ (fm) | $V_A$ (MeV) | $a_A$ (fm) | $a_{HO}$ (fm) | References |
|---|---|---|---|---|---|---|
| $^{16}O+^{24}Mg$ | 310 | 0.93 | -285 | 2.50 | 1.55 | Chengqun *et al*. (1988) |
| $^{16}O+^{16}O$ | 209 | 1.35 | -210 | 1.80 | 1.50 | Guozhu *et al*. (1984) |
| $^{16}O+^{28}Si$ | 250 | 1.00 | -212 | 2.10 | 1.50 | Linhua *et al*. (1985) |

$$V(r) = V_{Ri}\exp(-r_i^2/a_{Ri}^2) + V_{Ai}\exp(-r_i^2/a_{Ai}^2); \quad i = 1,2 \qquad (7)$$

where, $V_{Ri}>0$ and $V_{Ai}<0$ are the strengths of the repulsive and attractive terms, respectively, while $a_{Ri}$ and $a_{Ai}$ are their decay factors. These parameters are shown in Table 2.

The bound-state wavefunctions between the particles i and j in the initial and final channels are described by a harmonic- oscillator function (Linhua and Guozhu, 1988), which is given by:

$$\phi_{ij}(\vec{r}_{ij}) = \frac{1}{\sqrt{8}}\left(\pi a_i^2\right)^{-\frac{3}{4}}\left[\left(2-\sqrt{6}\right) + \sqrt{\frac{8}{3}}\left(r_i/a_i\right)^2\right]\exp-\left(r_i^2/2a_i^2\right); \quad i=1,2 \qquad (8)$$

where, $a_i$ is the oscillator length parameter.

The differential cross section for the stripping reaction with particle transfer is described by a clear form (Al-Farra, 2003) which is given by:

$$\frac{d\sigma}{d\Omega} = \frac{\mu_a\mu_b}{\left(2\pi\hbar^2\right)^2}\frac{K_a}{K_b}\frac{(2I_B+1)(2I_b+1)}{(2I_A+1)(2I_a+1)}\sum|T_{fi}|^2 \qquad (9)$$

where, the μ's and K's are the reduced masses and asymptotic wave numbers and $I_i$ is the total angular momentum of $i^{th}$ particle. The post-formulation DWBA transition amplitude has the form:

$$T_{fi} = \left\langle \psi_f^{(-)} \middle| V_{bA} + V_{cA} - \tilde{V}_{ab} \middle| \psi_i^{(+)} \right\rangle \qquad (10)$$

where, $\psi_i^{(+)}$ and $\psi_f^{(-)}$ are the distorted wave functions in the initial and final channels, respectively and $V_{ij}$ is the

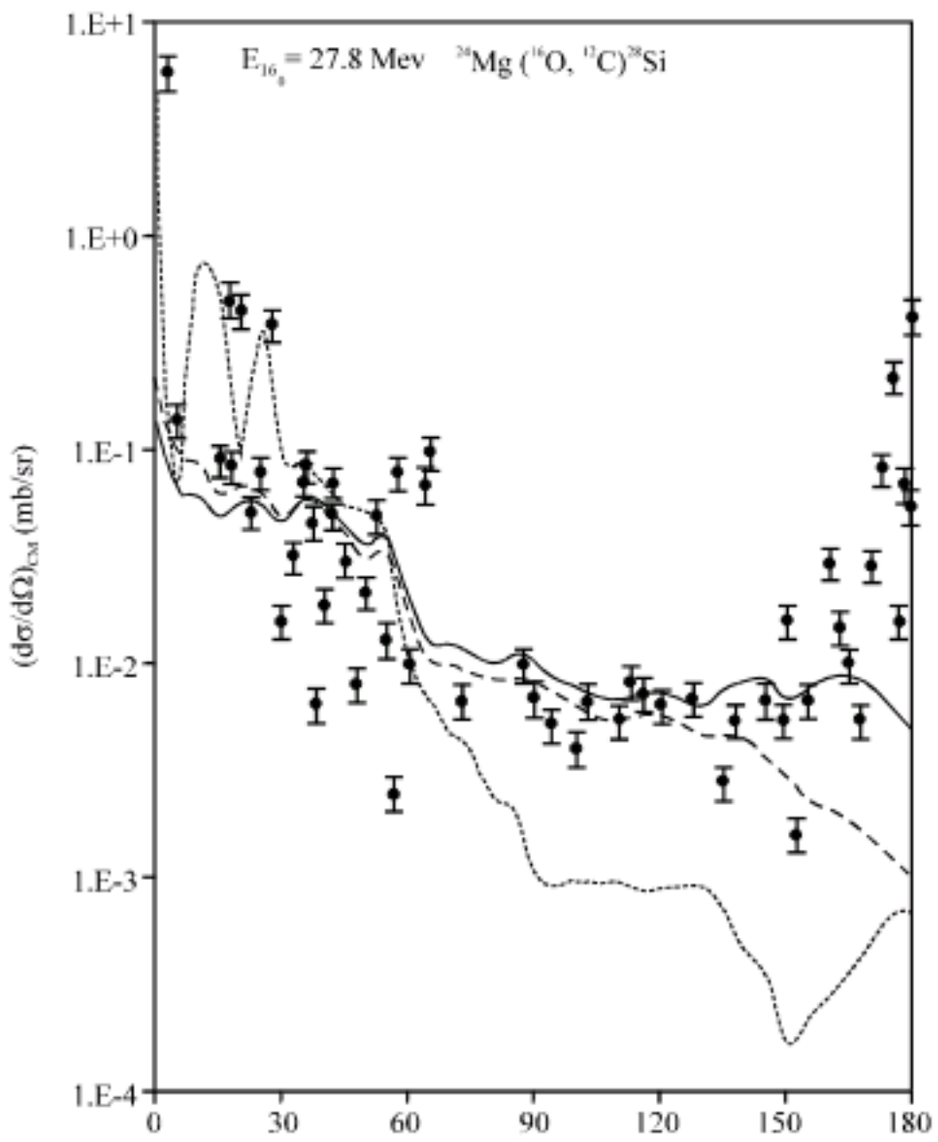


Fig. 1: The differential cross-section of the $^{24}Mg$ ($^{16}O$, $^{12}C$) $^{28}Si$ α-transfer reaction at 27.8 Mev incident energy leading to 0.0 $^{28}Si$ excited state. The dashed curve (folding model) is the present calculation. The solid curve is (WS+JD). The dotted line is the earlier study (WS+WS) and the dots are the experimental data taken from reference (Guozhu *et al*., 1984)

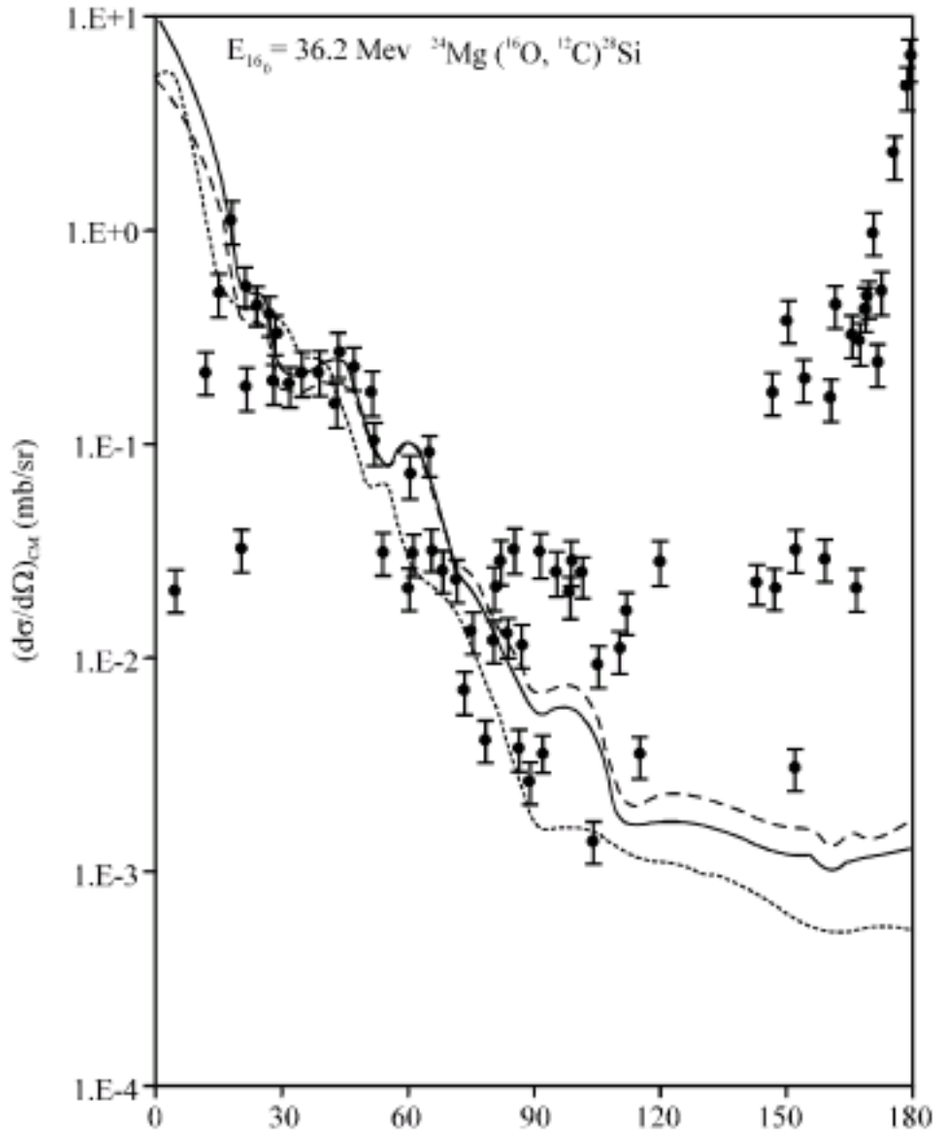


Fig. 2: The differential cross-section of the $^{24}Mg$ ($^{16}O$, $^{12}C$) $^{28}Si$ α-transfer reaction at 36.2 Mev incident energy leading to 0.0 $^{28}Si$ excited state. The dashed curve (folding model) is the present calculation. The solid curve is (WS+JD). The dotted line is the earlier study (WS+WS) and the dots are the experimental data taken from reference (Sanders *et al*., 1985)

interaction potential between the particle i and j, the index c refer to the transfer particle, while $\tilde{v}$ is the optical potential generating the distorted waves.

As could be done in such calculations, we have used the differential cross section without considering a spin-orbit coupling term (Satehler, 1964). Generally, the present spectroscopic factor are extracted from the relation:

$$S_{ij} = \frac{1}{N}\frac{(2j_i+1)}{(2j+1)}\frac{(d\sigma/d\Omega)_{exp.}}{(d\sigma/d\Omega)_{theor.}} \quad (11)$$

where, N is the normalization factor for the reaction (Al-Farra, 2003).

The parameters of the optical potential are taken as those used in the earlier calculations shown in Table 1. These parameters are found to reproduce the forward angles data reasonably well, but they don't fit the data at large angles. Therefore, the present optical potential (folding model) obtains the best fit to the data. The result obtained for the differential cross sections are shown in Fig. 1-4 by the dashed lines are compared with the previous calculations dotted lines employing, the solid curves (Farra, 2003) who used real and imaginary Wood Saxon and J-dependent, respectively (WS+JD) optical potentials, dotted line (Kurath, 1973), who used the (WS+WS) optical and experimental data points. Generally, the present calculations using folding model potential provide a substantially better description of the phase and magnitude of the angular distributions than the previous calculations.

The result obtained for the angular distribution of $^{24}Mg(^{16}O, ^{12}C)^{28}$ at incident energy 27.8 and 36.2 MeV is shown in Fig. 1 and 2, respectively, where the dashed curve is the present calculation (folding model) is compared with the previous (WS+JD) optical potential (solid curve), (WS+WS) optical potential (dotted line) and the experimental data dots (Guozhu *et al*., 1984) in Fig. 1 and (Chengqun *et al*., 1988) in Fig. 2.

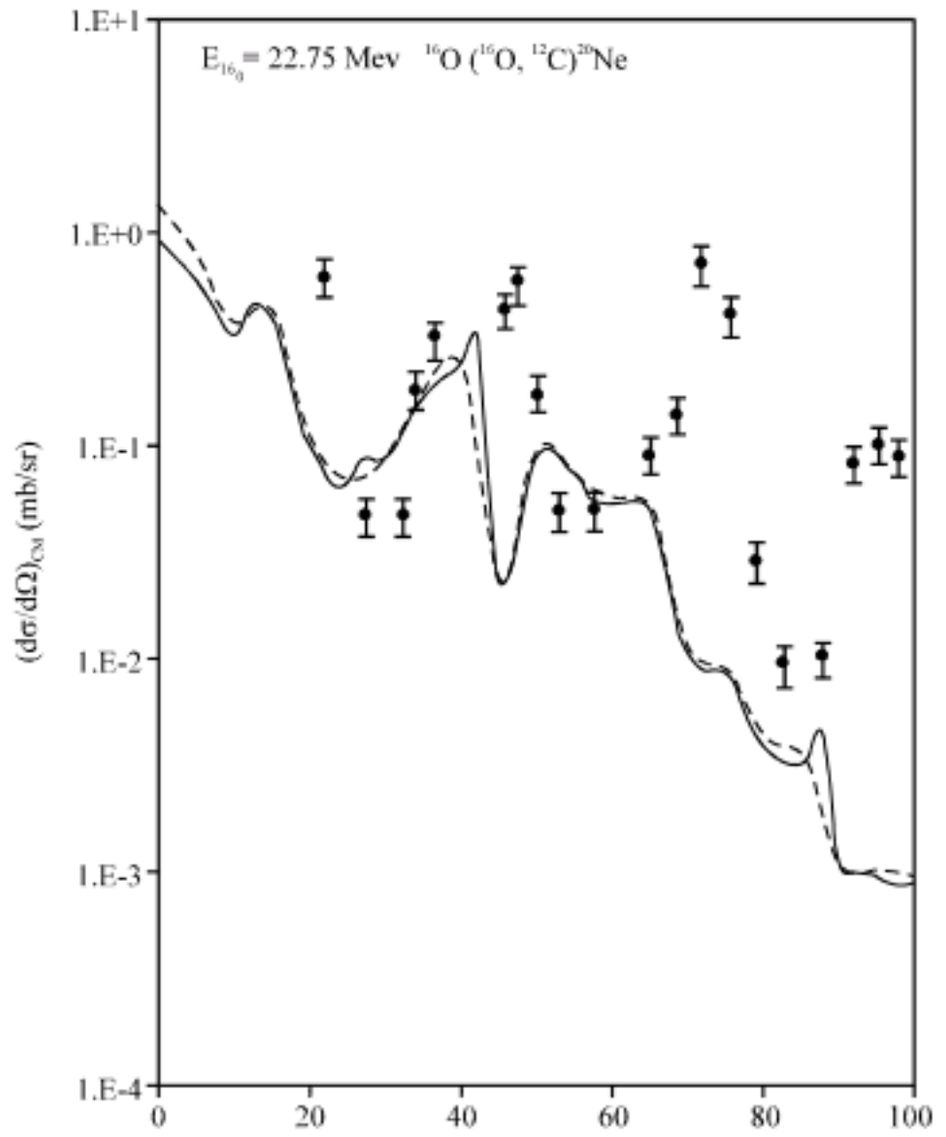


Fig. 3: The differential cross-section of the $^{16}O$($^{16}O$, $^{12}C$)$^{20}Ne$ α-transfer reaction at 22.75 Mev incident energy leading to 0.0 $^{20}Ne$ excited state. The dashed curve (folding model) is the present calculation. The solid curve is (WS+JD). The dots are the experimental data taken from reference (Chengqun *et al*., 1988)

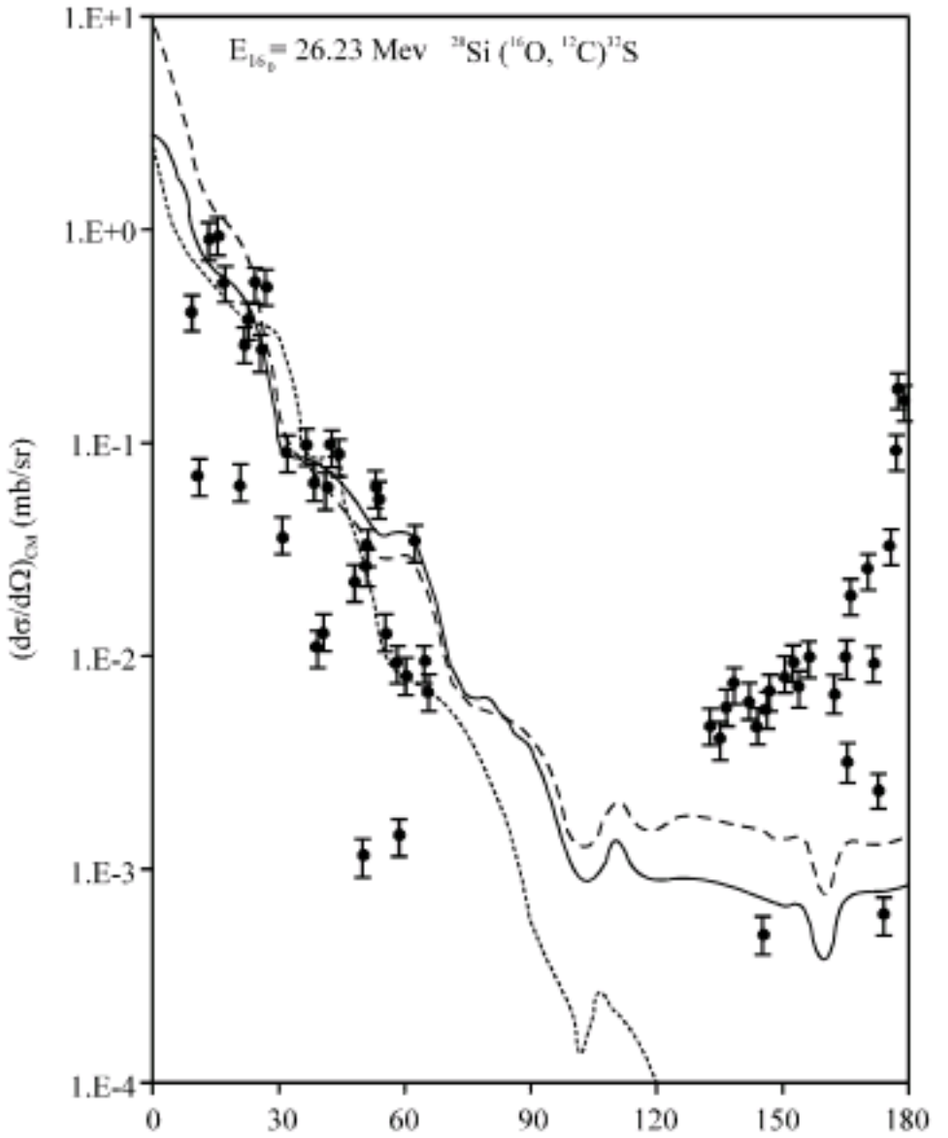


Fig. 4: The differential cross-section of the $^{28}Si$ ($^{16}O$, $^{12}C$) $^{32}S$ α-transfer reaction at 26.23 Mev incident energy leading to 0.0 $^{32}S$ excited state. The dashed curve (folding model) is the present calculation. The solid curve is (WS+JD). The dotted line is the earlier study (WS+WS) and the dots are the experimental data taken from reference (Linhua and Gouzhu, 1988)

## DISCUSSION

In this study, the differential cross sections of $^{24}Mg$ ($^{16}O$, $^{12}C$)$Si^{28}$, $^{16}O$($^{16}O$, $^{12}C$)$^{20}Ne$ and $^{28}Si$($^{16}O$, $^{12}C$)$^{32}S$ heavy ion reactions with α transfer have been estimated reasonably well using simple one-step DWBA calculations. The numerical calculations are carried out to find the angular distributions of this reaction. As shown in Fig. 1-4, it is clear that the present optical potential gives a better data fitting than the other optical potentials. In Fig. 1 and 2, it can be seen that the data of $^{24}Mg$($^{16}O$, $^{12}C$)$Si^{28}$, at incident 27.8 MeV the angular distribution, of our calculation gives better fit than the (WS+SW) optical potential and less fitting compared to (WS+JD). But at 36.2 MeV incident energy, the present optical potential is noticeably nearly good and significantly better than the previous work in both forward and backward angles. In Fig. 3, shows the data of $^{16}O$ ($^{16}O$, $^{12}C$) $^{20}Ne$ reaction at 22.75 Mev incident energy. It is clear that using the folding model gives the same as the earlier study, where the data are good at small angles and not fit at large angles. Finally, the $^{28}Si$ ($^{16}O$, $^{12}C$)$^{32}S$ reactions, shown in Fig. 4, using folding model behaves well for the forward angles as the same as the use of (WS+SW) and (WS+JD), but better at large angles.

Table 3: Extracted spectroscopic factors

| | | | | Spectroscopic factors | | |
|---|---|---|---|---|---|---|
| | | | | | Previous | |
| Reaction | Incident energy (MeV) | Excitation energy (MeV) | $J^{\pi}$ | Present | WS+JD | WS+WS |
| $^{16}O+^{24}Mg$ | 27.80 | 0.0 | $0^+$ | 0.80 | 0.81 | 0.72 |
| | 36.26 | 0.0 | $0^+$ | 0.89 | 0.87 | 0.79 |
| $^{16}O+^{16}O$ | 27.75 | 0.0 | $0^+$ | 0.78 | 0.78 | 0.76 |
| $^{16}O+^{28}Si$ | 26.23 | 0.0 | $0^+$ | 0.82 | 0.79 | 0.68 |

## CONCLUSION

In conclusion, the present study show that the one-step DWBA calculations using folding model are found to be appropriate to reproduce the cross-sections and capable of producing realistic predictions of the angular distribution at large angles region and better the earlier calculations. Finally, the spectroscopic factor is shown in Table 3.

## REFERENCES


Al-Farra, A.K.A.R., 2003. A study of two-proton transfer reaction using optical potential model. Turk. J. Phys., 27: 241-246.

Ass'ad, A.I. and H.S. Ashour, 2007. Analysis of alpha transfer reactions using different optical potentials. J. Applied Sci., 7: 1001-1004.

Bilwes, B., R. Bilwes, L. Stuttge, F. Ballester and J. Diaz *et al*., 1987. Folding model analysis of $^{32}S + ^{32}S$ elastic scattering at 70 90 97.09 120 and 160 MeV. Nulc. Phys., A473: 353-364.

Chengqun, G., N. Pingzhi and H. Guozhu, 1988. On the reactions from cov-channel to ion-channel in LCNO theory. Nulc. Phys., 485: 282-290.

El-Azab, F.M., A.M.A. Nossair and A.A. Ibraheem, 2008. Folding model analysis of 6he + 12c elastic scattering. Int. J. Modern Phy. E, 17: 715-733.

Farra, A.A., 2003. Contribution of J-dependent potential in differential cross-sections of two-nucleon transfer reactions. Int. J. Theoret. Phys., 42: 881-881.

Farra, A.A. and A.I. Ass'ad, 2004. Contribution of the spin-orbit potential on the large angle cross-sections of alpha transfer reactions. Rajasthan Acad. Phy. Sci., 3: 257-268.

Filho, L., R. Lépine-Szily, A.C.C. Villari and O.P. Filho, 1989. Effect of a-transfer polarization potential in the $^{24}Mg + ^{16}O$ system. Phys. Rev., 39: 884-890.

Guozhu, H., G. Chengqun and N. Pingzhi, 1984. Alpha transfer process in $^{16}O+^{24}Mg$ elastic scattering. Phys. Rev., C30: 534-538.

Jain, A.K. and C.S. Shastry, 1979. Optical potential for nucleus-nucleus systems. Phys. Rev., C19: 848-855.

Kurath, D., 1973. Alpha-structure amplitudes for the 1p shell. Phys. Rev., C7: 1390-1395.

Linhua, X. and H.E. Guozhu, 1988. Molecular orbital theory for intermediate structure in the excitation function of heavy-ion reactions. Nulc. Phys., 485: 291-303.

Linhua, X., G. Cheng-Qun, N. Ping-Zhi and H. Guo-Zhu, 1985. Alpha transfer mechanism in heavy-ion reaction. Phys. Rev., 31: 2128-2132.

Mermaz, M.C., F. Auger and B. Fernandez, 1983. Diffractional model fits of alpha transfer reactions induced by $^{16}O$ projectiles on $^{16}O$ and $^{28}Si$ target nuclei. Phys. Rev., C28: 1587-1593.

Perez, S.M., 1973. Possible form for the 6li-6li optical potential. Phys. Rev., C8: 1606-1608.

Sanders, S.J., H. Ernst, W. Henning, C. Jachcinski, D.G. Kovar, J.P. Schiffer and J. Barrette, 1985. Energy dependence of the cross sections for the $^{24}Mg(^{16}O,^{12}C)^{28}Si(g.s.)$ reaction. Phys. Rev., C31: 1775-1782.

Satehler, G.R., 1964. The distorted-waves theory of direct nuclear reactions with spin-orbit effects. Nuci. Phys., 55: 1-33.

Yang, Y.X., Q.R. Li and W.Q. Zhao, 2002. Total reaction cross sections for the $^{12}C + ^{12}C$ system from a folding model analysis. J. Phys. G. Nucl. Part. Phys., 28: 2561-2566.

Yosio, K. and T. Taro, 1984. Resonant structures in the $^{16}O(^{16}O, ^{12}C)\ ^{20}Ne$ reaction. Phys. Rev., C30: 97-106.

Zhang, G.L., L. Hao and X.Y. Le, 2009. Nucleon–nucleon interactions in the double folding model for fusion reactions. Chinese Phys., B18: 136-141.