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Constraint Joint-Replenishment Inventory Control Problem with Fuzzy Rough Demand

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Abstract: An Uncertain Economic Order Quantity (UEOQ) model with payment in advance is developed to purchase high-price raw materials. A joint policy of replenishments and pre-payments is employed to supply the materials. The rate of demand is considered fuzzy rough variables, lead time is taken to be constant and it is assumed that shortage does not occur in the cycles. The cycle is divided into three parts; the first part is the time between the earlier replenishment-time to the next order-time (t_0), the second part is the period between t_0 to a payment-time (t_{1c}) and the third part is the period between t_{1c} to the next replenishment-time. At the start of the second part (t_0), α% of the purchasing cost is paid. The (1-α)% remaining purchasing cost is paid at the start of the third part (t_{1c}). The cost of the model is purchasing under incremental discount, clearance cost, fixed-order cost, transportation cost, holding and capital cost. Holding cost is for on-hand inventory and capital cost is for the capital that is paid for the next order. The constraints of the problem are space, budget and the number of orders per year. Also lead-time is considered less than a cycle time. We show that the model of this problem is a fuzzy rough mixed integer-nonlinear-programming type and in order to solve it, hybrid method of particle Swarm Optimization and fuzzy rough simulation is used. At the end, a numerical example is given to demonstrate the applicability of the proposed methodology in real world inventory control problems.

Key words: Joint replenishment, EOQ, particle swarm optimization, fuzzy rough programming

INTRODUCTION

Since its formulation in 1915, the square-root-formula for the Economic Order Quantity (EOQ) has been used in the inventory literature for a long time. This formula is based on the assumption of a constant demand. The discrete case of the dynamic version of EOQ was first discussed by Wagner and Whitin (1958). Regarding the continuous-time dynamic EOQ models, Silver and Meal (1969) were the first to suggested a simple modification of the classical square-root-formula in the case of time-varying demand.

The question of inventory shortages and backlogging were not considered at all by the aforementioned researchers. EOQ models for deteriorating items with a trended demand were considered by Xu and Wang (1990), Chung and Ting (1993, 1994), Hariga (1996), Benkherouf (1995), Jalan *et al.* (1996), Jalan and Chaudhuri (1999), Giri and Chaudhuri (1997) and Lin *et al.* (2000) etc. Numerous studies have been carried out to address the problems of imperfect quality EMQ model with rework (Hayek and Salameh, 2001; Chiu, 2003, 2007; Chiu *et al.*, 2004; Jamal *et al.*, 2004). Chiu and Chiu (2006) studied optimal replenishment policy for an imperfect quality EMQ model with backlogging and failure in repair using conventional approach. That is to derive the optimal lot size by utilizing differential calculus on the expected cost function with the need to prove optimality first. A few researches have then been carried out using the same method (Cardenas-Barron, 2001; Wee and Chung, 2007). In these researches that extend the algebraic approach to an imperfect quality EMQ model examined by Chiu and Chiu (2006), the use of differential calculus is replaced with an algebraic method and the optimal lot size solution is obtained under the expected cost minimization.

In this study, the EOQ model is extended to purchase expensive imported raw material in a joint replenishment policy. In addition, holding costs (including capital, storage and insurance cost for on hand inventory), purchasing cost under incremental discount, clearance

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and fixed order costs, transportation cost and capital costs for the next order are employed to the problem. All parameter of the problem are crisp and the quantity of the orders must be integer multiples of packets, each containing more than one product. Also the lead time is assumed less than the cycle length.

PROBLEM DEFINITION

Inventory holding as a tactical-level decision against non-secure situations to increase confidence level of responding to customer demands and also to resist against all kind of non-deterministic situations in receiving raw materials made inventory control and management an important concept in supply chain management. The model that is proposed in this research is an applied model developed based on the real constraints and environments of manufacturing companies. In these companies, the annual demands of different products are first estimated at the start of the year. Then, based on these estimates, the inventory and planning departments proceed to material planning. In some cases, improper material planning and control policies and loss of sufficient raw materials in proper times and quantities, result to customer complaints and loss of customer and market share. In some instances gaining the lost-market-shares needs more promotion and advertising costs and causes increased production cost. To confront with these instances and to minimize raw-material shortages, in this research a model for material planning and control is developed.

In a manufacturing company, the steps involved in the ordering process of the materials are:

- Forecast the number of finished products
- Forecast the required raw-materials
- Order the required materials that consist of:
- Releasing orders of the materials to a supplier.
- Paying a percentage (α%) of the purchasing cost at t_0.
- Paying the remaining payment (1-α)% at t_{lc}
- Transporting materials and receiving them at T

Figure 1 depicts the inventory control cycle of a material.

There are two payment methods in a material ordering process; (1) the credit transaction with a specific lead time and (2) cash method that has smaller lead time. In the cash method the purchasing cost of the total ordered material is paid to the supplier at the ordering time. However, in the credit transaction method α% of the material purchasing cost is paid at the order release time (t_0) and

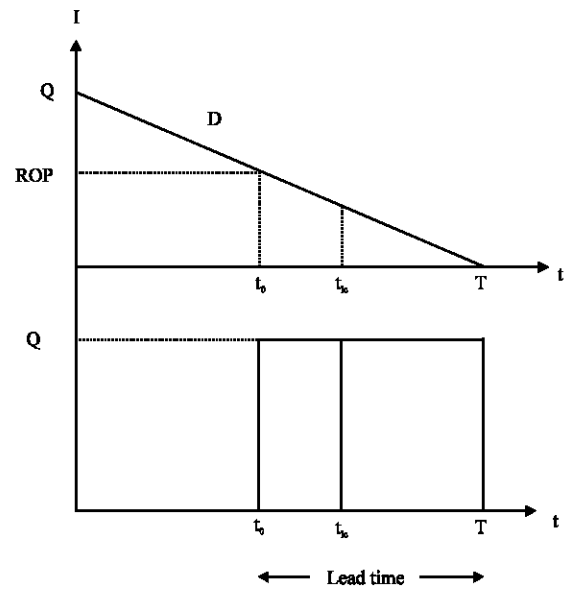

Fig. 1: Inventory control cycle

the remaining (1-α)% is paid at the starting time of the material transportation (t_{lc}). The ordered materials are received at T.

Let us assume that the credit transaction method is used for payments in which the lead-time is deterministic and constant. Materials ordering and their transportation are done in batch form and the amount of raw materials in each batch and the number of batches in each vehicle is deterministic. Based on the product groups' consistency, the ordered materials can be carried together by finite-capacity vehicles with maximum capacity of F (in volume).

In this study, we plan to determine the optimal order quantity of each material in a joint replenishment policy such that the total cost is minimized. The costs are: (1) purchasing cost under incremental discount for each order, (2) holding cost for on hand inventory (including capital, warehouse and insurance costs), (3) capital cost of the next order, (4) transportation cost, (5) clearance cost and (6) fixed-order costs. Furthermore, we assume that the lead times are less than the cycle times of the materials.

SOME DEFINITIONS IN FUZZY THEORY

We adopt the concepts of the credibility, possibility and necessity theory, as well as credibility of fuzzy event and the expected value of a fuzzy variable. So, the definitions of these concepts are presented in the following based on Liu (2004).

Definition 1: A Fuzzy number is of LR-Type, if there exist reference functions L (for the left), R (for the right) and scalars $\alpha>0$, $\beta>0$ with

$$\mu(\tilde{\xi})=\begin{cases}1 & \tilde{\xi}\in[m,n]\\ L(\frac{m-\tilde{\xi}}{\alpha}) & \tilde{\xi}\le m\\ R(\frac{\tilde{\xi}-n}{\beta}) & \tilde{\xi}\ge n\end{cases} \quad (1)$$

where, $\tilde{\xi}$ is defined by $\tilde{\xi}=(m, n, \alpha, \beta)_{L\text{-}R}$. The triangular and trapezoidal fuzzy variable are specific kind of LR-Type.

Definition 2: Let $\tilde{\xi}$ be a fuzzy number with the membership function $\mu(\tilde{\xi})$. Then the possibility, necessity and credibility measure of the fuzzy event $\tilde{\xi}\ge r$ can be represented, respectively, by:

$$\mathrm{Pos}\{\tilde{\xi}>r\}=\sup_{\tilde{\xi}\ge r}\mu(\tilde{\xi}) \quad (2)$$

$$\mathrm{Nec}\{\tilde{\xi}\ge r\}=1-\sup_{\tilde{\xi}<r}\mu(\tilde{\xi}) \quad (3)$$

$$\mathrm{Cr}\{\tilde{\xi}\ge r\}=\frac{1}{2}\left[\mathrm{Pos}\{\tilde{\xi}\ge r\}+\mathrm{Nec}\{\tilde{\xi}\ge r\}\right] \quad (4)$$

Definition 3: The expected value of a fuzzy variable $\tilde{\xi}$ is defined as:

$$E\left[\tilde{\xi}\right]=\int_0^{\infty}\mathrm{Cr}\{\tilde{\xi}\ge r\}dr-\int_{-\infty}^{0}\mathrm{Cr}\{\tilde{\xi}\le r\}dr \quad (5)$$

For example the expected value of a triangular fuzzy variable $\tilde{\xi}=(\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3)$ is:

$$E[\tilde{\xi}]=\frac{1}{4}(\xi_1+2\xi_2+\xi_3) \quad (6)$$

Definition 4: Let $\tilde{\xi}$ be a fuzzy variable. Then the optimistic function of α is defined as:

$$\tilde{\xi}_{sup}(\alpha)=\sup\left[r\left|\mathrm{Cr}\{\tilde{\xi}\ge r\}\ge\alpha\right.\right],\quad \alpha\in(0,1] \quad (7)$$

Definition 5: Assume $C_1C_2...C_k$ are real constants and $G_1(\tilde{\xi})$, $G_2(\tilde{\xi})$,..., $G_k(\tilde{\xi})$ are functions of fuzzy variable, then:

$$E\left[\sum_{k=1}^{K}C_kG_k(\tilde{\xi})\right]=\sum_{k=1}^{K}C_kE(G_k(\tilde{\xi})) \quad (8)$$

Definition 6: Assume K is positive real constant and $\tilde{\xi}=(\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3)$ is triangular fuzzy variable. Then $K.\tilde{\xi}=(K\tilde{\xi}_1, K\tilde{\xi}_2, K\tilde{\xi}_3)$.

In this study, we consider an inventory problem in which the demand follows a LR-fuzzy rough number. In this problem, service rates, warehouse capacity and budget are considered constraints and the incremental discount policies are used to purchase the items. Furthermore, the quantity of the orders must be integer multiples of packets, each containing more than one product.

ROUGH THEORY

In this part of the study some concepts in rough environment as well as rough space and rough variable are introduced. The descriptions of these concepts are presented in the following based on Liu (2004).

Definition 7: Let Λ be a nonempty set, A be a σ-Algebra of subset of Λ, Δ be an element in A and π be a nonnegative, real valued, additive set function, then $(\Lambda, \Delta, A, \pi)$ is called rough space. Also, a rough variable is defined as a measurable function from a rough space to the real line.

Definition 8: A rough variable δ on the rough space $(\Lambda, \Delta, A, \pi)$ is a function from Λ to the real line $\Re$ such that for every Borel set O of $\Re$, we have $\{\vartheta\in\Lambda|\delta(\vartheta)\in O\}\in A$. The lower and upper approximation of the rough variable δ are then, respectively defined as follows:

$$\underline{\delta}=\{\delta(\vartheta)|\vartheta\in\Delta\},\ \overline{\delta}=\{\delta(\vartheta)|\vartheta\in\Lambda\} \quad (9)$$

For example a rough variable ([a, d], [c,d]) with $c\le a\le b\le d$ is a measurable function from a rough space $(\Lambda, \Delta, A, \pi)$ to the real line, where, $\Lambda=\{x|c\le x\le d\}$, $\Delta=\{x|a\le x\le b\}$ and $\delta(x)=x$ for all $x\in\Lambda$.

FUZZY ROUGH THEORY

In this part of the study some concepts in fuzzy rough environment. The descriptions of these concepts are presented in the following based on Liu (2004).

Definition 9: A fuzzy rough variable is a function δ from a rough space $(\Lambda, \Delta, A, \pi)$ to the set of fuzzy variables such that Poss $\{\delta(\lambda)\in B\}$ is a measurable function of λ for any Borel set B of $\Re$.

Definition 10: An n-dimensional fuzzy rough vector is a function δ from a rough space $(\Lambda, \Delta, A, \pi)$ to the set of

n-dimensional fuzzy vectors such that Poss$\{\delta(\lambda)\in B\}$ is a measurable function of λ for any Borel set B of $\Re^n$.

For example $\delta(\lambda) = (\delta-\lambda, \delta, \delta+\lambda)$is a triangular fuzzy rough variable which λ is the rough variable such as ([a, b], [c, d]) with $c\le a\le b\le d$.

PROBLEM MODELING

The parameters and the variables: For i = 1, 2,.., n and j = 1, 2,.., P the parameters and the variables of the model are:

- P : The number of products
- D_j : The annual constant demand of the jth product
- $\tilde{D}_j$: The fuzzy rough annual constant demand of the jth product
- n_j : The number of items in the packets of the jth product
- h_{1j} : The holding cost per unit of on hand inventory of the jth product during a period
- h_{2j} : Is the capital cost per unit of the jth ordered product during the period before the payment of the remaining purchasing cost (10% of the total purchasing cost)
- h_{3j} : The capital cost per unit of the jth ordered product during the period after the payment of the remaining purchasing cost
- C^c_j : The clearance cost for each unit of the jth product
- C^p_{ij} : The constant purchasing cost of the jth product in the ith discount break point
- q_{ij} : The ith discount break point of the jth product
- ROP_j : The reorder point of the jth product
- f_j : The space required for each packet of the jth product
- L : The constant joint lead time for each order
- A : The fixed order cost per each order
- A_T : The fixed transportation cost per each shipment
- $\hat{f}$: The total available space in each trunk
- Q_i : The decision variable representing the order quantity of the jth product
- m_j : The decision variable representing the number of packets that have been ordered for the jth product
- TB : The total available budget
- T : The decision variable representing the joint cycle length
- N : The number of orders in each year (N = 1/T)
- N_T : The upper limit for number of orders
- C_H : The annual total holding cost of the products
- C_c : The annual total clearance cost
- C_p : The annual total purchasing cost of the products
- C_T : The annual total transportation cost of the products
- Z : The annual total costs

The objective function: The objective function of the model is to minimize the total cost of the joint replenishment problem given in Eq. 10.

$$Z = C_T+C_P+C_C+C_A+C_H \tag{10}$$

The terms in the right hand side of Eq. 10 are derived as follows:

Transportation Cost (C_T): The transportation cost is calculated based on Eq. 11, in which f_jm_j is the required space to ship the order of jth product from the supplier:

$$C_T = \begin{cases} A_T; & 0 < \sum_{j=1}^{p} f_jm_j \le \hat{f} \\ 2A_T; & \hat{f} < \sum_{j=1}^{p} f_jm_j \le 2\hat{f} \\ \vdots & \vdots \\ KA_T; & (K-1)\hat{f} < \sum_{j=1}^{p} f_jm_j \le K\hat{f} \end{cases} \tag{11}$$

By introducing the binary variables Y_k; k = 1, 2,..., k, the annual total transportation cost (occurring N times per year) can be incorporated with the mathematical model of the problem as:

$$\begin{aligned} C_T &= N\sum_{k=1}^{K} kA_TY_k = \frac{1}{T}\sum_{k=1}^{K} kA_TY_k \\ 0 &< \sum_{j=1}^{P} f_jm_j \le \hat{f}Y_1 \\ \hat{f}Y_2 &< \sum_{j=1}^{P} f_jm_j \le 2\hat{f}Y_2 \\ &\vdots \\ (K-1)\hat{f}Y_k &< \sum_{j=1}^{P} f_jm_j \le K\hat{f}Y_k \\ Y_1 + Y_2 + \cdots + Y_k &= 1 \\ Y_k &= 0,1 \quad \forall k = 1,2,\cdots,K \end{aligned} \tag{12}$$

Purchasing cost under incremental discount (C_P): The purchasing cost of the company for the jth product at each period and each order can be calculated using the incremental discount policy. It is necessary to indicate that, discount from supplier is considered for each order and not during a year. Let the incremental discount policy be:

$$C_j^P = \begin{cases} C_{1j}Q_j \quad ; \quad 0 < Q_j \le q_{1j} \\ C_{1j}q_{1j} + C_{2j}(Q_j - q_{2j}) \quad ; \quad q_{1_j} < Q_j \le q_{2_j} \\ \vdots \\ C_{1j}q_{1j} + C_{2j}(q_{2j} - q_{1j}) + \ldots + C_{nj}(Q_j - q_{n-1,j}); \quad Q_j \ge q_{nj} \end{cases} \tag{13}$$

where, q_{ij} and C_{ij} = 1, 2, ..., n are the discount points and the purchasing costs for each unit of the jth product that corresponds to its lth discount break point, respectively. In order to include the incremental discount policy in the inventory model, we use Eq. 14 to model the incremental discount policy.

$$\begin{gathered} C_j^P = C_{1j}Q_{1j} + C_{2j}Q_{2j} + \ldots + C_{nj}Q_{nj} \\ Q_j = Q_{1j} + Q_{2j} + \ldots + Q_{nj} \\ q_{1j}\lambda_{2j} \le Q_{1j} \le q_{1j}\lambda_{1j} \\ (q_{2j} - q_{1j})\lambda_{3j} \le Q_{2j} \le (q_{2j} - q_{1j})\lambda_{2j} \\ \vdots \\ 0 \le Q_{nj} \le M\lambda_{nj} \\ \lambda_{1_j} \ge \lambda_{2_j} \ge \cdots \ge \lambda_{n_j} \end{gathered} \tag{14}$$

λ_{ij} = 0, 1, ∀i, i = 1, 2,..., n is a very big number.

Finally the total annual purchasing cost will be (occurring N times per year):

$$\begin{gathered} C_P = N\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} = \frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} \\ Q_j = Q_{1j} + Q_{2j} + \ldots + Q_{nj} \\ q_{1j}\lambda_{2j} \le Q_{1j} \le q_{1j}\lambda_{1j} \\ (q_{2j} - q_{1j})\lambda_{3j} \le Q_{2j} \le (q_{2j} - q_{1j})\lambda_{2j} \\ \vdots \\ 0 \le Q_{nj} \le M\lambda_{nj} \\ \lambda_{1j} \ge \lambda_{2j} \ge \cdots \ge \lambda_{nj} \\ \lambda_{ij} = 0,1; \quad \forall j,\ j = 1,2,\cdots,P\ ,\forall i,\ i = 1,2,\cdots,m \end{gathered} \tag{15}$$

Clearance Cost (C_C): The clearance cost of the company for the jth product at each period is C^c_j, the order quantity is Q_j and the number of order per year is N. Hence, the total annual clearance cost of the company will be:

$$C_C = N\sum_{j=1}^{P} C_j^c Q_j = \sum_{j=1}^{P} C_j^c D_j \tag{16}$$

Fixed order cost (C_A): The fixed order cost of each order per period is A and the number of orders per year is N. Hence and the total annual fixed order cost in a disjoint ordering policy will be NA. However, since we are using the joint replenishment policy, the fixed order cost will change to:

$$C_A = \frac{A}{T} \tag{17}$$

Holding cost (C_H): According to Fig. 1, the holding cost during a cycle is:

$$\sum_{j=1}^{P} \frac{h_j^1 Q_j T}{2} \tag{18}$$

The first part of the capital cost occurs between the α% and (1-α)%-payment times and is derived as:

$$\sum_{j=1}^{P} h_j^2 Q_j (t_{1c} - t_0) \tag{19}$$

The other part of the capital cost occurs between (t_{1c}, T) and is calculated as:

$$\sum_{j=1}^{P} h_j^3 Q_j [L - (t_{1c} - t_0)] \tag{20}$$

so, the total holding cost during a year will be:

$$C_H = N\left[\sum_{j=1}^{P} \frac{h_j^1 Q_j T}{2} + \sum_{j=1}^{P} h_j^2 Q_j (t_{1c} - t_0) + \sum_{j=1}^{P} h_j^3 Q_j [L - (t_{1c} - t_0)]\right] \tag{21}$$

Also, according to Fig. 1 we have

$$Q_j = D_j T_j \tag{22}$$

Knowing that

$$N = \frac{1}{T} \rightarrow NT = 1 \tag{23}$$

Equation 12 can be written as (15) for a joint order case.

$$C_H = \sum_{j=1}^{P} \frac{h_j^1 D_j T}{2} + \sum_{j=1}^{P} h_j^2 D_j (t_{1c} - t_0) + \sum_{j=1}^{P} h_j^3 D_j [L - (t_{1c} - t_0)] \tag{24}$$

The constraints: As the total available warehouse space is F, the space required for each unit of the jth product is f_j and the order quantity of the jth product is Q_j, the space constraint will be:

$$\sum_{j=1}^{P} f_j Q_j \le F \tag{25}$$

Furthermore, since the total available budget is TB and the purchasing cost that is calculated is:

$$C_P = \frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij}$$

the budget constraint will be:

$$\frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} \le TB \tag{26}$$

For the sake of convenience in ordering, clearance, transportation and some other activities, we assume an upper limit for the number of orders per year. In other words:

$$N \le N_T \;\rightarrow \frac{1}{T} \le N_T \rightarrow T \ge \frac{1}{N_T} \tag{27}$$

Moreover, the quantities of the orders must be integer multiples of packets, each containing more than one product. That is

$$Q_j = n_j m_j \tag{28}$$

Knowing that

$$Q_j = D_j T \tag{29}$$

we have:

$$TD_j = n_j m_j \tag{30}$$

Finally the model of the problem becomes:

$$\begin{aligned}
\text{Min}: Z &= C_T + C_P + C_C + C_A + C_H \\
&= \frac{1}{T}\sum_{k=1}^{K} kA_T Y_k + \frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} + \sum_{j=1}^{P} C_j^c D_j + \frac{A}{T} + \sum_{j=1}^{P}\frac{h_j^1 D_j T}{2} \\
&+ \sum_{j=1}^{P} h_j^2 D_j (t_{1c} - t_0) + \sum_{j=1}^{P} h_j^3 D_j [L - (t_{1c} - t_0)] \\
&= \frac{1}{T}\left[A + \sum_{k=1}^{K} kA_T Y_k + \sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij}\right] + \left[\sum_{j=1}^{P}\frac{h_j^1 D_j}{2}\right]T \\
&+ \left[\sum_{j=1}^{P} C_j^c D_j + \sum_{j=1}^{P} h_j^2 D_j (t_{1c} - t_0) + \sum_{j=1}^{P} h_j^3 D_j [L - (t_{1c} - t_0)]\right]
\end{aligned}$$

s.t:

$$\frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} \le TB$$

$$\sum_{j=1}^{P} f_j Q_j \le F$$

$$T \ge \frac{1}{N_T}$$

$$0 < \sum_{j=1}^{P} f_j m_j \le \hat{f}Y_1$$

$$\hat{f}Y_2 < \sum_{j=1}^{P} f_j m_j \le 2\hat{f}Y_2$$

$$\vdots$$

$$(K-1)\hat{f}Y_k < \sum_{j=1}^{P} f_j m_j \le K\hat{f}Y_k$$

$$TD_j = n_j m_j$$

$$Q_j = Q_{1j} + Q_{2j} + \ldots + Q_{nj}$$

$$q_{1j}\lambda_{2j} \le Q_{1j} \le q_{1j}\lambda_{1j}$$

$$(q_{2j} - q_{1j})\lambda_{3j} \le Q_{2j} \le (q_{2j} - q_{1j})\lambda_{2j}$$

$$\vdots$$

$$0 \le Q_{nj} \le M\lambda_{nj}$$

$$Y_1 + Y_2 + \cdots + Y_k = 1$$

$$\lambda_{1j} \ge \lambda_{2j} \ge \cdots \ge \lambda_{nj}$$

$$\lambda_{ij} = 0,1; Y_k = 0,1 \;\; \forall k = 1,2,\cdots,K, \forall j, j = 1,2,\cdots,P, \forall i, i = 1,2,\cdots,m$$

$$\begin{aligned} &T \ge 0 \\ &m_j, Q_j \ge 0 \;\; \text{integer} \end{aligned} \tag{31}$$

According to definition (6) since:

$$\frac{h_j^1 T}{2}, h_j^2(t_{1c} - t_0), h_j^3[L - (t_{1c} - t_0)]$$

are positive constant, the model of the problem in uncertain state with fuzzy rough demand becomes:

$$
\begin{aligned}
\text{Min}: Z &= C_T + C_P + C_C + C_A + C_H \\
&= \frac{1}{T}\sum_{k=1}^{K} kA_T Y_k + \frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} + \sum_{j=1}^{P} C_j^c \tilde{D}_j + \frac{A}{T} + \sum_{j=1}^{P}\frac{h_j^1 \tilde{D}_j T}{2} \\
&+ \sum_{j=1}^{P} h_j^2 \tilde{D}_j (t_{1c} - t_0) + \sum_{j=1}^{P} h_j^3 \tilde{D}_j [L - (t_{1c} - t_0)] \\
&= \frac{1}{T}\left[A + \sum_{k=1}^{K} kA_T Y_k + \sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij}\right] + \left[\sum_{j=1}^{P}\frac{h_j^1 \tilde{D}_j}{2}\right] T \\
&+ \left[\sum_{j=1}^{P} C_j^c \tilde{D}_j + \sum_{j=1}^{P} h_j^2 \tilde{D}_j (t_{1c} - t_0) + \sum_{j=1}^{P} h_j^3 \tilde{D}_j [L - (t_{1c} - t_0)]\right]
\end{aligned}
$$

s.t:

$$\frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} \le TB$$

$$\sum_{j=1}^{P} f_j Q_j \le F$$

$$T \ge \frac{1}{N_T}$$

$$0 < \sum_{j=1}^{P} f_j m_j \le \hat{f}Y_1$$

$$\hat{f}Y_2 < \sum_{j=1}^{P} f_j m_j \le 2\hat{f}Y_2$$

$$\vdots$$

$$(K-1)\hat{f}Y_k < \sum_{j=1}^{P} f_j m_j \le K\hat{f}Y_k$$

$$TD_j = n_j m_j$$

$$Q_j = Q_{1j} + Q_{2j} + \ldots + Q_{nj}$$

$$q_{1j}\lambda_{2j} \le Q_{1j} \le q_{1j}\lambda_{1j}$$

$$(q_{2j} - q_{1j})\lambda_{3j} \le Q_{2j} \le (q_{2j} - q_{1j})\lambda_{2j}$$

$$\vdots$$

$$0 \le Q_{nj} \le M\lambda_{nj}$$

$$Y_1 + Y_2 + \cdots + Y_k = 1$$

$$\lambda_{1j} \ge \lambda_{2j} \ge \cdots \ge \lambda_{nj}$$

$$\lambda_{ij} = 0,1; Y_k = 0,1 \quad \forall k = 1,2,\cdots,K, \forall j, j = 1,2,\cdots,P, \forall i, i = 1,2,\cdots,m$$

$$
\begin{aligned}
&T \ge 0 \\
&m_j, Q_j \ge 0 \ \text{integer}
\end{aligned}
\tag{32}
$$

A SOLUTION ALGORITHM

Since the model in (32) is mixed integer-nonlinear in nature, reaching an analytical solution (if any) to the problem is difficult (Gen and Cheng, 1997). Also efficient treatment of mixed integer nonlinear optimization is one of the most difficult problems in practical optimization (El-Sharkawi, 2008). Hence, we need to employ a meta-heuristic search algorithm to solve it. This algorithm is a hybrid method of Particle swarm optimization and fuzzy rough simulation described as follows.

Fuzzy rough simulation: In order to estimate uncertain demands of fuzzy rough variable, we employ a simulation technique. Fuzzy rough simulation plays an important role in fuzzy rough system. If $Z(\tilde{D},Q)$ is a real valued continues function and $\tilde{D}_j = (\delta_j - \lambda_j, \delta_j, \delta_j + \lambda_j)$ is a fuzzy rough variable and δ_j is a rough vector defined on rough space (Λ, Δ, A, π), in order to estimate the expected value $E[Z(\tilde{D},Q)]$, fuzzy rough simulation approach may be employed. In this approach by denoting $\vartheta_l = (\vartheta_{jl}, \vartheta_{j2}, \ldots, \vartheta_{jL}), l = 1,2,\ldots,L$ we generate $\underline{\vartheta}_l = (\underline{\vartheta}_{jl}, \underline{\vartheta}_{j2}, \cdots, \underline{\vartheta}_{jL})$ from Δ and $\overline{\vartheta}_l = (\overline{\vartheta}_{jl}, \overline{\vartheta}_{j2}, \cdots, \overline{\vartheta}_{jL})$ from Λ according to the measure π to estimate the rough variable. So, the objective function will be a function of $\delta(\underline{\vartheta})$ and $\delta(\overline{\vartheta})$. However, to evaluate the objective function we need to estimate $\delta(\underline{\vartheta})$ and $\delta(\overline{\vartheta})$.

After evaluating the rough variable in order to estimate the uncertain demands of the fuzzy model, we employ a simulation technique. Denoting $\tilde{D}_j$ by:

$$\tilde{D}_j = (\tilde{D}_1, \tilde{D}_2, \ldots, \tilde{D}_P)$$

μ as the membership function of $\tilde{D}$ and μ_j are the membership functions of $\tilde{D}_j$, we randomly generate $\tilde{D}_{jz}$ from the α-level sets of fuzzy variables $\tilde{D}_j$, j = 1, 2, ..., P and z = 1, 2, ..., N as $\xi_z = (\xi_{1z}, \xi_{2z}, \ldots, \xi_{pz})$ and $\mu(\xi_z) = \mu_1(\xi_z) \wedge \mu_2(\xi_z) \wedge, \ldots, \wedge \mu_p \xi_z)$, where, α is a sufficiently small positive number. Based on the definition in Eq. 11, the expected value of the fuzzy variable is

$$E\left[Z(\tilde{D},Q)\right] = \int_0^{+\infty} Cr\left\{Z(\tilde{D},Q) \ge r\right\} dr - \int_{-\infty}^{0} Cr\left\{Z(\tilde{D},Q) \le r\right\} dr \tag{33}$$

Then, provided N is sufficiently large, for any number $r \ge 0$, $Cr\{Z(\tilde{D}, Q) \ge r\}$ can be estimated by:

$$Cr\{Z(D_z,Q) \ge r\} = \frac{1}{2}\left(\underset{z=1,2,\ldots,Z}{\text{Max}}\{\mu_z | Z(D_z,Q) \ge r\} + 1 - \underset{z=1,2,\ldots,Z}{\text{Max}}\{\mu_z | Z(D_z,Q) < r\}\right) \tag{34}$$

And for any number $r<0$, $Cr\{Z(D_z, Q) \le r\}$ can be estimated by:

$$Cr\{Z(D_z,Q) \le r\} = \frac{1}{2}\left(\underset{z=1,2,\ldots,Z}{\text{Max}}\{\mu_z | Z(D_z,Q) \le r\} + 1 - \underset{z=1,2,\ldots,Z}{\text{Max}}\{\mu_z | Z(D_z,Q) > r\}\right) \tag{35}$$

However, the procedure of estimating E [Z ($\tilde{D}$, Q, $\hat{C}$)] by hybrid method of rough simulation and fuzzy simulation is shown in (Algorithm 1):

Step 1: Set E=0
Step 2: Randomly generate D_{jz} from α-level sets of fuzzy variables $\tilde{D}_j$ and set $\xi_z = (\xi_{1z}, \xi_{2z}, ..., \xi_{pz})$
Step 3: Set $a = Z(D_1, Q) \wedge Z(D_2, Q) \wedge ... \wedge Z(D_z, Q)$, $b = Z(D_1, Q) \vee Z(D_2, Q) \vee ... \vee Z(D_z, Q)$
Step 4: Randomly generate r from uniform [a, b]
Step 5: Generate $\underline{C}_{ijl}$ from Δ according to measure π
Step 6: Generate $\overline{C}_{ijl}$ from Δ according to measure π
Step 7: If r≥0, then

$$E \leftarrow E + Cr\left\{\frac{Z(\tilde{D},Q,\hat{C}(\overline{\delta}_l)) + Z(\tilde{D},Q,\hat{C}(\underline{\delta}_l))}{2L} \geq r\right\}$$

Step 8: If r<0, then

$$E \leftarrow E - Cr\left\{\frac{Z(\tilde{D},Q,\hat{C}(\overline{\delta}_l)) + Z(\tilde{D},Q,\hat{C}(\underline{\delta}_l))}{2L} \leq r\right\}$$

Step 9: Repeat the fourth to eight steps for N times
Step 10: $E\left[Z(\tilde{D},Q,\hat{C})\right] = a \vee 0 + b \wedge 0 + E^* \frac{b-a}{N}$

Algorithm 1: Estimating E [Z($\tilde{D}$, Q, $\hat{C}$)]

Particle swarm optimization: New ways have been found to optimize problems for less than a century, but nature has used various ways of optimization for millions of million years. Recently scientists mimicked nature to solve different kinds of complex optimization problems. Most of these problems are so complicated and time consuming that we cannot use an exact algorithm to solve them. Thus, typically some non-precise algorithms are used to find a near optimum solution in a shorter period. We call these algorithms meta-heuristic (Dorigo and Stutzle, 2004). Many researchers have successfully used meta-heuristic methods to solve complicated optimization problems in different fields of scientific and engineering disciplines. Some of these meta-heuristic algorithms are simulating annealing (Aarts and Korst, 1989; Taleizadeh *et al*., 2008a), threshold accepting (Dueck and Scheuer, 1990), Tabu search (Joo and Bong, 1996), genetic algorithms (Al-Tabtabai and Alex, 1999; Taleizadeh *et al*., 2008b), neural networks (Gaiduk *et al*., 2002), ant colony optimization (Dorigo and Stutzle, 2004), fuzzy simulation (Taleizadeh *et al*., 2009), evolutionary algorithm (Laumanns *et al*., 2002) and harmony search (Lee and Geem 2004; Geem *et al*., 2001; Taleizadeh *et al*., 2008c).

Particle Swarm Optimization (PSO) was invented by Kennedy and Eberhart (1995) in the mid 1990s while attempting to simulate the choreographed, graceful motion of swarms of birds as part of a sociocognitive study investigating the notion of collective intelligence in biological populations. In PSO, a set of randomly generated solutions (initial swarm) propagates in the design space towards the optimal solution over a number of iterations (moves) based on large amount of information about the design space that is assimilated and shared by all members of the swarm. PSO is inspired by the ability of flocks of birds, schools of fish and herds of animals to adapt to their environment, find rich sources of food and avoid predators by implementing an information sharing approaches (Kennedy and Eberhart, 2001).

Also some researches about PSO have been done by Sha and Hsu (2008), Liu *et al*. (2008), Yapicioglu *et al*. (2007), Guo *et al*. (2008) and Rahimi-Vahed *et al*. (2007).

Basic PSO algorithm: The basic PSO algorithm consists of three steps; generating particle's positions and velocities, velocity update and position update.

Initializing particles' positions and velocities: A particle refers to a point in the designed space that changes its position from one move (iteration) to another, based on velocity updates. First, by using upper and lower bounds on the design variables values, X_{min} and X_{max}, the positions, X^i_k and velocities, V^i_k of the initial swarm of particles can be generated randomly. PSO is a population-based optimization algorithm. Each particle is an individual and the swarm is composed of particles. The relationship between swarm and particles in PSO is similar to the relationship between population and chromosomes in Genetic Algorithm and we will denote the swarm size by N. The positions and velocities are given in a vector format where, the superscript and subscript denoting the ith particle at time K. Rand is a uniformly distributed random variable that can take any value between 0 and 1. This initialization process allows the swarm particles can be generated randomly across the design space. Equation 36 and 37 are used to initialization particles which Δt is the constant time increment.

$$X^i_0 = X_{min} + Rand(X_{max} - X_{min}) \tag{36}$$

$$V^i_0 = \frac{X_{min} + Rand(X_{max} - X_{min})}{\Delta t} = \frac{Position}{time} \tag{37}$$

Update velocities: By using the fitness values which are functions of the particles current positions in the design space at time k, velocities of all particles at time k+1 are

updated. The fitness function value of a particle determines which particle has the best global value in the current swarm, P^g_k and also determines the best position of each particle over time, P^i in current and all earlier moves. The velocity update formula uses these two pieces of information for each particle in the swarm along with the effect of current motion, V^i_k to provide a search direction, V^i_{k+1} for the next iteration. To ensure good coverage of the design space and avoid entrapment in local optima.

The velocity update formula includes some random parameters, rand, represented by the uniformly distributed variables. The three values that affect the new search direction, namely, current motion, particle own memory and swarm influence, are incorporated via a summation approach as shown in Equation below with three weight factors, namely, inertia factor, w, self confidence factor, C_1 and swarm confidence factor, C_2, respectively:

$$\underbrace{V^i_{k+1}}_{\text{Velocity of particle i at time k+1}} = \overbrace{w}^{[0.4,1.4]} \underbrace{V^i_k}_{\text{Current motion}} + \overbrace{C_1}^{[1.5,2]} \underbrace{\text{Rand}\frac{(P^i - X^i_k)}{\Delta t}}_{\text{Particle memory influence}} + \overbrace{C_2}^{[2.2.5]} \underbrace{\text{Rand}\frac{(P^g_k - X^i_k)}{\Delta t}}_{\text{Swarm influence}} \quad (38)$$

Salman *et al*. (2003) used the values of 0.9, 2 and 2 for W, C_1 and C_2, respectively and suggests upper and lower bounds on these values as shown in Eq. above. Other combinations of values usually lead to much slower convergence or sometimes non-convergence at all. The tuning of the PSO algorithm weight factors is a topic that warrants proper investigation but is outside the scope of this study. According to some suggestion in some study we considered different values for each parameter of the algorithm and considered all possible combinations of them. Finally we chose the best combination of the parameters from all of considered values in Table 1.

Position update: Position update is the final step of each iterations. And it is done by using the current particle position and its own updated velocity vector shown in the Eq. below:

$$X^i_{K+1} = X^i_K + V^i_{K+1}\Delta t \quad (39)$$

In summery, the hybrid algorithm of PSO technique and fuzzy rough simulation will be:

- Repeat
- For each particle
- Update the velocities according to Eq. 38
- Update the positions according to Eq. 39
- Evaluate its fitness value according to the desired optimization fitness function according to algorithm 1
- Update P^g_k and P^i if necessary
- End for

Until a criterion is not met, usually a sufficient good fitness or a maximum number of iterations (As in genetic algorithm).

Numerical example: Consider a multi-product inventory control problem with ten products and general data given in Table 1, 2. The total available warehouse

Table 1: General data

Product	n_i	h^1_i	h^2_i	h^3_i	C^c_i	q_{1i}	q_{2i}	q_{3i}	f_i
1	10000	40	3	30	2	4000	7000	110000	4
2	5000	35	3	50	3	4000	7000	110000	5
3	1000	40	3	30	2	3500	6500	9500	6
4	500	35	3	50	3	3500	6500	9500	4
5	100	40	3	30	2	2500	4000	5500	5
6	10000	35	4	50	3	2500	4000	5500	6
7	5000	40	4	30	2	1500	3000	4500	4
8	1000	35	4	50	3	1500	3000	4500	5
9	500	40	4	30	2	500	1500	2500	6
10	100	35	4	50	3	500	1500	2500	4

Product	D_j	δ_j	C^p_{1j}	C^p_{2j}	C^p_{3j}	C^p_{4j}
1	(90000-δ_j, 90000, 90000+δ_j)	([2000, 3000][1000, 4000])	70	65	60	50
2	(85000-δ_j, 85000, 85000+δ_j)	([2000, 3000][1000, 4000])	70	65	60	50
3	(80000-δ_j, 80000, 80000+δ_j)	([2000, 3000][1000, 4000])	70	65	60	55
4	(75000-δ_j, 75000, 75000+δ_j)	([2000, 3000][1000, 4000])	70	65	60	50
5	(70000-δ_j, 70000, 70000+δ_j)	([2000, 3000][1000, 4000])	70	65	60	50
6	(65000-δ_j, 65000, 65000+δ_j)	([2000, 3000][1000, 4000])	95	90	85	75
7	(60000-δ_j, 60000, 60000+δ_j)	([2000, 3000][1000, 4000])	95	90	85	75
8	(55000-δ_j, 55000, 55000+δ_j)	([2000, 3000][1000, 4000])	95	90	85	75
9	(50000-δ_j, 50000, 50000+δ_j)	([2000, 3000][1000, 4000])	95	90	85	75
10	(45000-δ_j, 45000, 45000+δ_j)	([2000, 3000][1000, 4000])	95	90	85	75

Table 2: The parameters of the PSO algorithm

N	C_1	C_2	w
10	1.50	1.5	0.4
50	1.75	2.0	1.0
100	2.00	2.5	1.4

Table 3: The best result

Product	T	Z	m_j	Q_j
1	0.1678	7,942,100	2	20000
2			3	15000
3			14	14000
4			26	13000
5			118	11800
6			2	20000
7			3	15000
8			10	10000
9			17	8500
10			76	7600

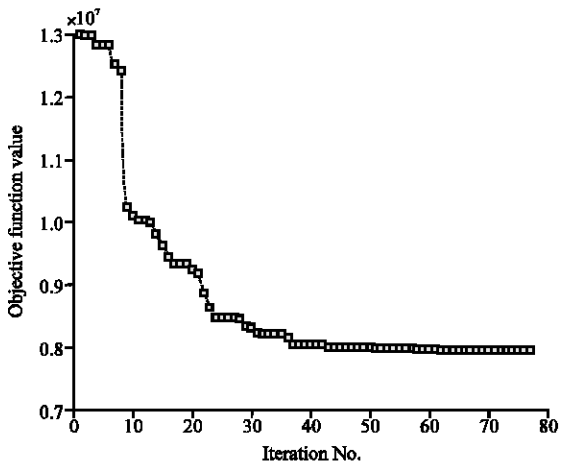

Fig. 2: The convergence graph

space, the total budget and the maximum number of order are F = 1300, TB = 8,000,000 and N_T = 6, respectively. Also, L =2.5 months, A = 2000, A_T = 300,000, $\hat{f}$ = 200, (t_0) = 0 and (t_{lc}) =1.5 Months and δ_j presents the rough variable of jth product in fuzzy rough demands. Table 2 shows different values of the PSO parameters used to obtain the solution. In this research, all of the possible combinations of the PSO parameters are employed and using the min (min) criterion the best combination of the parameters has been selected. The best combination of the PSO algorithm parameters is w = 1, C_1 = C_2 = 2, N = 100 and the best result is shown in Table 3. Furthermore, the convergence path of the best result of the objective function is shown in Fig. 2.

Conclusion and recommendations for future research: In this study, a joint replenishment multi-product multi constraint inventory model based on EOQ model with fuzzy rough demand to purchase high price raw materials was investigated. A mathematical modeling by incremental discount and transportation, clearance, fixed order, holding and shortage costs was introduced and shown to be mixed integer-nonlinear programming problems then, a hybrid meta-heuristic algorithm (Particle Swarm Optimization) and fuzzy rough simulation has been proposed to solve the fuzzy rough integer non-linear problem.

Some recommendations for future studies follow:

- Shortage can be included in the model
- Stochastic demands can be taken into account
- Stochastic or fuzzy lead-time can be considered

REFERENCES

Aarts, E.H.L. and J.H.M. Korst, 1989. Simulated Annealing and Boltzmann Machine. Stochastic Appro Computer. 1st Edn., John Wiley and Sons, Chichester, UK., ISBN: 0-471-92146-7.

Al-Tabtabai, H. and A.P. Alex, 1999. Using genetic algorithms to solve optimization problems in construction. Eng. Con. Arch. Man., 6: 121-132.

Benkherouf, L., 1995. On an inventory model with deteriorating items and decreasing time-varying demand and shortages. Eur. J. Operat. Res., 86: 293-299.

Cardenas-Barron, L.E., 2001. The economic production quantity (EPQ) with shortage derived algebraically. Int. J. Prod. Econ., 70: 289-292.

Chiu, S.W., D.C. Gong and H.M. Wee, 2004. The effects of the random defective rate and the imperfect rework process on the economic production quantity model. Jap. J. Ind. Appl. Math., 21: 375-389.

Chiu, Y.P., 2003. Determining the optimal lot size for the finite production model with random defective rate, the rework process and backlogging. Eng. Optimz., 35: 427-437.

Chiu, S.W. and Y.S.P. Chiu, 2006. Mathematical modeling for production system with backlogging and failure in repair. J. Sci. Ind. Res., 65: 499-506.

Chiu, S.W., 2007. Optimal replenishment policy for imperfect quality EMQ model with rework and backlogging. Applied Stoch. Models Bus. Ind., 23: 165-178.

Chung, K.J. and P.S. Ting, 1993. A heuristic for replenishment of deteriorating items with a linear trend in demand. J. Operat. Res. Soc., 44: 1235-1241.

Chung, K.J. and P.S. Ting, 1994. On replenishment schedule for deteriorating items with time proportional demand. Prod. Plann. Contr., 5: 392-396.

Dorigo, M. and T. Stützle, 2004. Ant Colony Optimization. 1st Edn., MIT Press, Cambridge, Massachusetts, London, England, ISBN: 0-262-04219-3.

Dueck, G. and T. Scheuer, 1990. Threshold accepting: A general purpose optimization algorithm appearing superior to simulated annealing. J. Comput. Phys., 90: 161-175.

El-Sharkawi, L., 2008. Modern Heuristic Optimization Techniques. 1st Edn., Wiley Inter Science, New Jersey, USA., ISBN: 978-0471-45711-4.

Gaiduk, A.R., Y.A. Vershinin and M.J. West, 2002. Neural networks and optimization problems. Proc. IEEE 2002 Int. Conf. Control Applicat., 1: 37-41.

Geem, Z.W., J.H. Kim and G.V. Loganathan, 2001. A new heuristic optimization algorithm: Harmony search. Simulation, 76: 60-68.

Gen, M. and R. Cheng, 1997. Genetic Algorithm and Engineering Design. 1st Edn., John Wiley and Sons, New York, USA., ISBN: 0-471-12741-8.

Giri, B.C. and K.S. Chaudhuri, 1997. Heuristic models for deteriorating items with shortages and time-varying demand and costs. Int. J. Syst. Sci., 28: 153-159.

Guo, Y.W., W.D. Li, A.R. Mileham and G.W. Owen, 2008. Optimization of integrated process planning and scheduling using a particle swarm optimization approach. Int. J. Prod. Res., 40: 1-22.

Hariga, M., 1996. Optimal EOQ models for deteriorating items with time-varying demand. J. Operat. Res. Soc., 47: 1398-1405.

Hayek, P.A. and M.K. Salameh, 2001. Production lot sizing with the reworking of imperfect quality items produced. Prod. Plan Control, 12: 584-590.

Jalan, A.K., R.R. Giri and K.S. Chaudhuri, 1996. EOQ model for items with Weibull distribution deterioration, shortages and trended demand. Int. J. Syst. Sci., 27: 851-855.

Jalan, A.K. and K.S. Chaudhuri, 1999. Structural properties of an inventory system with deterioration and trended demand. Int. J. Syst. Sci., 30: 627-633.

Jamal, A.M.M., B.R. Sarker and S. Mondal, 2004. Optimal manufacturing batch size with rework process at a single-stage production system. Comput. Ind. Eng., 47: 77-89.

Joo, S.J. and J.Y. Bong, 1996. Construction of exact D-optimal designs by tabu search. Comput. Stat. Data Anal., 21: 181-191.

Kennedy, J. and R. Eberhart, 1995. Particle swarm optimization. Proceedings of the IEEE International Conference on Neural Networks. 27 November-December 1, Perth, Australia, pp: 1942-1948.

Kennedy, J. and R. Eberhart, 2001. *Swarm Intelligence*. 1st Edn., Academic Press, San Diego, CA., ISBN: 1-55860-595-9.

Laumanns, M., L. Thiele, K. Deb and E. Zitzler, 2002. Combining convergence and diversity in evolutionary multi-objective optimization. Evol. Comput., 10: 263-282.

Lee, K.S. and Z.W. Geem, 2004. A new structural optimization method based on the harmony search algorithm. Comput. Struct., 82: 781-798.

Lin, C., B. Tan and W.C. Lee, 2000. An EOQ model for deteriorating items with time-varying demand and shortages. Int. J. Syst. Sci., 31: 391-400.

Liu, B., 2004. Uncertainty Theory. 1st Edn., Springer-Verlag, Heidelberg, ISBN: 3-540-20584-5.

Liu, B., l. Wang and Y. Jin, 2008. An effective hybrid PSO-based algorithm for flow shop scheduling with limited buffers. Comput. Operat. Res., 35: 2791-2806.

Rahimi-Vahed, A.R., S.M. Mirghorbani and M. Rabbani, 2007. A hybrid multi-objective particle swarm algorithm for a mixed-model assembly line sequencing problem. Eng. Optimz., 39: 877-898.

Salman, A., I. Ahmad and S. Al-Madani, 2003. Particle swarm optimization for tast assignment problem. Microprocess. Microsyst., 26: 363-371.

Sha, D.Y. and C. Hsu, 2008. A new particle swarm optimization for the open shop scheduling problem. Comput. Operat. Res., 35: 3243-3261.

Silver, E.A. and H.C. Meal, 1969. A simple modification of the EOQ for the case of a varying demand rate. Prod. Inventory Manage., 10: 52-65.

Taleizadeh, A.A., M.B. Aryanezhad and S.T.A. Niaki, 2008a. Optimizing multi-product multi-constraint inventory control systems with stochastic replenishment. J. Applied Sci., 8: 1228-1234.

Taleizadeh, A.A., S.T.A. Niaki and V. Hosseini, 2008b. The multi-product multi-Constraint problem with Incremental Discount and Batch Order. Asian J. Applied Sci., 1: 110-112.

Taleizadeh, A.A., H. Moghadasi, S.T.A. Niaki and A. Eftekhari, 2008c. An EOQ-joint replenishment policy to supply expensive imported raw materials with payment in advance. J. Applied Sci., 2008: 1-11.

Taleizadeh, A.A., S.T.A. Niaki and M.B. Aryanezhad, 2009. Multi-product Multi-constraint inventory control systems with stochastic replenishment and discount under fuzzy purchasing price and holding costs. Am. J. Applied Sci., 6: 1-12.

Wagner, H.M. and T.M. Whitin, 1958. Dynamical version of the economic lot size model. Manage. Sci., 5: 89-96.

Wee, H.M. and C.J. Chung, 2007. A note on the economic lot size of the integrated vendor–buyer inventory system derived without derivatives. Eur. J. Operat. Res., 177: 1289-1293.

Xu, H. and H.P. Wang, 1990. An economic ordering policy model for deteriorating items with time proportional demand. Eur. J. Operat. Res., 46: 21-27.

Yapicioglu, H., A.E. Smitha and G. Dozierb, 2007. Solving the semi-desirable facility location problem using bi-objective particle swarm. Eur. J. Operat. Res., 177: 733-749.