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Variational Iteration Method for Solving Integral Equations

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Abstract: In this study, several integral equations are solved by He's variational iteration method. The method can solve various different non-linear equations as Volterra integral equations of the second kind, Ferdholm integral equations and mixed integral equations. Comparison with exact solution shows that the method is very effective and convenient for solving integral equations.

Key words: VIM, integral equation, approximate solution, exact solution, nonlinear differential equations

INTRODUCTION

Various kinds of analytical methods and numerical methods (Wazwaz *et al*., 1996) were used to solve integral equations. In this study, is applied He's variational iteration method (He, 1999) to solve integral equations. The method can solve various different non-linear equations (Bildik and Konuralp, 2006). To show the basic idea of this method, the following general nonlinear system is considered:

$$L\,[u(t)] + N[u(t)] = g(t) \tag{1}$$

where, L is a linear operator, N is a nonlinear operator and g(t) is a given continuous function. The basic character of the method is to construct a correction functional for the system, which reads

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(s)[Lu_n(s) + N\bar{u}_n(s) - g(t)]\,ds \tag{2}$$

where, λ is a Lagrange multiplier which can be identified optimally via variational theory, u_n is the n-th approximate solution and u_n denotes a restricted variation, i.e., $\delta\bar{u} = 0$.

VOLTERRA INTEGRAL EQUATIONS OF THE SECOND KIND

First, the Volterra integral equations of the second kind is considered, which read:

$$u\ (x) = f(x) + \int_a^x K(x,t)u(t)]\,dt \tag{3}$$

where, K(x, t) is the kernel of the integral equation.

Example 1:

$$u(x) = \sin ax + \frac{a}{2}\int_0^{\frac{\pi}{2a}} \cos ax\,u(t)\,dt \tag{1*}$$

Its iteration formula reads:

$$u_{n+1}(x) = \sin ax + \frac{a}{2}\int_0^{\frac{\pi}{2a}} \cos ax\,u_n(t)\,dt \tag{2*}$$

According to Eq. 1* it can assume an initial approximation

$$u_0(x) = \sin ax \tag{3*}$$

substituting Eq. 3* into Eq. 2* it has the following result

$$\begin{aligned}
u_1(x) &= \sin ax + \frac{a}{2}\int_0^{\frac{\pi}{2a}} \text{cas}\ ax\,u_0(t)dt \\
&= \sin ax + \frac{a\cos ax}{2}\int_0^{\frac{\pi}{2a}} \sin at\ dt \\
&= \sin ax + \frac{a\cos ax}{2}\left[\frac{-1}{a}\cos at\right]_0^{\frac{\pi}{2a}} \\
&= \sin ax + \frac{a\cos ax}{2}\left[0-(\frac{-1}{a})\right] = \sin ax + \frac{\cos ax}{2}
\end{aligned}$$

$$\begin{aligned}
u_2(x) &= \sin ax + \frac{a}{2}\int_0^{\frac{\pi}{2a}} \cos ax\ \ u_1(t)dt \\
&= \sin ax + \frac{a\cos ax}{2}\int_0^{\frac{\pi}{2a}}[\ \sin at + \frac{\cos at}{2}\ ]dt \\
&= \sin ax + \frac{a\cos ax}{2}\left[\frac{-1}{a}\cos at + \frac{\sin at}{2a}\right]_0^{\frac{\pi}{2a}} \\
&= \sin ax + \frac{a\cos ax}{2}\left[\frac{1}{a}\left[\frac{1}{2}+1\right]\right] \\
&= \sin ax + \frac{3}{4}\cos ax = \sin ax + \frac{2^2-1}{2^2}\cos ax
\end{aligned}$$

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$$u_3(x) = \sin ax + \frac{a}{2}\int_0^{\frac{\pi}{2a}} \cos ax \quad u_2(t)dt$$

$$= \sin ax + \frac{a\cos ax}{2}\int_0^{\frac{\pi}{2a}}[\ \sin at + \frac{3}{4}\cos at]dt$$

$$= \sin ax + \frac{a\cos ax}{2}\left[\frac{-1}{a}\cos at + \frac{\sin at}{2a}\right]_0^{\frac{\pi}{2a}}$$

$$= \sin ax + \frac{a\cos ax}{2}\left[\frac{1}{a}\left[\frac{3}{4}+1\right]\right]$$

$$= \sin ax + \frac{7}{8}\cos ax = \sin ax + \frac{2^3-1}{2^3}\cos ax$$

Continuous this process implies that:

$$u_n(x) = \sin ax + \frac{2^n-1}{2^n}\cos ax$$

Then

$$\lim_{n\to\infty} u_n(x) = \sin ax + \cos ax = u(x)$$

Example 2:

Consider the integral equation.

$$u(x) = \sqrt{x} + \int_0^1 xtu(t)dt \qquad (1^*)$$

Its iteration formula reads:

$$u_{n+1}(x) = \sqrt{x} + \int_0^1 xtu_n(t)dt \qquad (2^*)$$

According to Eq. 1* it can assume an initial approximation:

$$u_0(x) = \sqrt{x} \qquad (3^*)$$

substituting Eq. 3* into Eq. 2* implies the following result:

$$u_1(x) = \sqrt{x} + \int_0^1 xt\sqrt{t}dt$$

$$= \sqrt{x} + \frac{2}{5}x = \sqrt{x} + \frac{3^1-1}{5\times 3^0}$$

$$u_2(x) = \sqrt{x} + x\int_0^1 t(\ \sqrt{t} + \frac{2}{5}t\ )dt$$

$$= \sqrt{x} + \frac{8}{15}x = \sqrt{x} + \frac{3^2-1}{5\times 3^{2-1}}$$

$$u_3(x) = \sqrt{x} + x\int_0^1 t(\ \sqrt{t} + \frac{8}{15}t\ )dt$$

$$= \sqrt{x} + \frac{26}{45}x = \sqrt{x} + \frac{3^3-1}{5\times 3^{3-1}}$$

$$u_4(x) = \sqrt{x} + x\int_0^1 t(\ \sqrt{t} + \frac{26}{45}t\ )dt$$

$$= \sqrt{x} + \frac{80}{135}x$$

$$u_n(x) = \sqrt{x} + \frac{3^n-1}{5\times 3^{n-1}}x$$

$$\lim_{n\to\infty} u_n(x) = \sqrt{x} + 0.6x$$

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