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A Decentralized Stable Fuzzy Adaptive Controller for Large Scale Nonlinear Systems

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Abstract: A new method to design a decentralized Fuzzy Adaptive Controller (FAC) for a class of large scale nonaffine nonlinear systems is proposed in this study. It is assumed that functions of the subsystems and their interactions are unknown. To design controller, the lyapunov function is proposed for the system and then unknown parameters of controller and system are derived based on the stability theory. The robustness against uncertainty and external disturbance, the boundedness of the estimation errors, the convergence of the output error to zero and the lyapunov stability of the closed loop system are guaranteed. To use the knowledge of the experts in FAC is another advantage of controller. Robust adaptive control has been used to avoid chattering in adaptation laws. An illustrative example is given to show the promising performance of the proposed method.

Key words: Lyapunov stability, robust adaptive control, nonaffine nonlinear systems, large scale system, fuzzy system

INTRODUCTION

In the recent years, controller design for Large Scale Systems (LSS) and effort to extend it has attracted much attention. Research in controllers of LSS is motivated by many emerging applications that employ novel actuation devices for active control of industrial automation, cooperating robotic systems, power systems and aerospace processes. Centralized controller for the LSS is usually impractical due to the requirement of a large amount of information exchanges between subsystems and the lack of computing capacity (Karimi *et al*., 2007).

The tunable structure of the FAC and using the knowledge of experts in the FAC are reasons to attract many researchers to developed appropriate controllers for nonlinear systems especially for LSS (Ioannou and Sun, 1996).

In the recent year, FAC has been fully studied as follow:

In the first case, the TS fuzzy systems have been used to model nonlinear systems and then TS based controllers have been designed with guaranteed stability (Feng, 2002; Fenga *et al*., 2002). To model affine nonlinear system and to design stable TS based controllers have been employed by Hsu *et al.* (2003). Designing of the sliding mode fuzzy adaptive controller for a class of multivariable TS fuzzy systems are presented by Cheng and Chien (2006). The nonaffine nonlinear function are first approximated by the TS fuzzy systems and then stable TS fuzzy controller and observer are designed for the obtained model (Goleaa *et al*., 2003; Park and Park, 2004). In these studies, modeling and controller has been designed simply, but the systems must be linearizable around some operating points.

In the second case, the linguistic fuzzy systems have been used to design controllers for nonlinear systems.

Ying-Guo and Hua-Guang (1998), Jagannathan (1998), Tong *et al*. (1999), Tong *et al.* (2000) and Zhang and Bien (2000) have considered linguistic fuzzy systems to design stable adaptive controller for affine systems based on feedback linearization and furthermore, Tong *et al.* (2000) and Zhang and Bien (2000) has considered that the zero dynamic is stable. Stable FAC based on sliding mode is designed for affine systems by Labiod *et al*. (2005). Designing of the FAC for affine chaotic systems are presented by Tang *et al.* (1999) and Chen *et al.* (1999). To design stable FAC and linear observer for class of affine nonlinear systems are presented by Ho *et al.* (2005), Zhang (2006), Tong *et al.* (2004) and Shaocheng *et al.* (2005). Fuzzy adaptive sliding mode controller is presented for class of affine nonlinear time delay systems by Yu (2004), Chiang (2005) and Jianga *et al.* (2005). The output feedback FAC for class of affine nonlinear MIMO systems is suggested by Yiqian *et al*. (2004). The main incompetency of these studies is those restricted conditions on their nonlinear functions.

Labiod and Guerra (2007) and Tong *et al*. (2004) are involved stable FAC for class of nonaffine nonlinear systems. The deficiencies of these studies are bad performance of the controller when the controller has not

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been adjusted. Stable adaptive controller for class of linear LSS is proposed by Pagilla *et al*. (2007), Ioannou and Ponte (1988), Shi and Singh (1992) and Yousef and Simaan (1991). Chiang and Lu (2007) delt with designing FAC based on sliding mode for class of large scale affine nonlinear systems. (Zhang *et al*., 2002) presented decentralized sliding mode fuzzy adaptive tracking for a class of affine nonlinear systems in large scale systems. Wu (2002) designed FAC for a class of affine nonlinear time delayed systems. These studies have many restricted conditions on their nonlinear function.

FAC has been never applied to nonaffine nonlinear large scale systems. In this study, the stable decentralized robust adaptive controller has been designed based on fuzzy systems for a class of large scale nonaffine nonlinear systems. The controller is robust against uncertainties, external disturbances and approximation errors.

PROBLEM STATEMENT

Consider the following large scale nonaffine nonlinear system.

$$\begin{cases} \dot{x}_{i,l} = x_{i,l+1} \qquad l = 1,2,\ldots,n_i - 1 \\ \dot{x}_{1,n_i} = f_i(x_i, u_i) + m_i(x_1, x_2, \ldots, x_N) + d_i(t) \qquad i = 1,2,\ldots,N \\ y_i = x_{i,1} \end{cases} \tag{1}$$

where, $x_i = [x_{i,1}, \ldots, x_{i,n_i}]^T \in R^{n_i}$ is the state vector of the system which is assumed available for measurement, $u_i \in R$ is the control input, $y_i \in R$ is the system output, $f_i\ (x_i, u_i)$ is an unknown smooth nonlinear function, mi $(x_1, x_2, \ldots x_N)$ is an unknown interconnection term and $d_i(t)$ is bounded disturbance.

The control objective is to design an adaptive fuzzy controller for system (1) such that the system output $y_i(t)$ follows a desired trajectory $y_d(t)$ while all signals in the closed-loop system remain bounded.

In this study, the following assumptions have been considered concerning the system (1) and the desired trajectory $y_d(t)$.

Assumption 1: Without loss of generality, it is assumed that the nonzero function $f_u(x_i, u_i) = \partial f(x_i, u_i)/\partial u_i$ satisfies the following condition:

$$\begin{gathered} f_u(x_i, u_i) \geq f_{min} > 0 \text{ for all } (x_i, u_i) \in R^{n_i} \times R \\ \frac{df_u(x_i, u_i)}{dt} \geq f_{dm} \end{gathered} \tag{2}$$

where, $f_{dm} \in R$ is nonzero, known and constant.

Assumption 2: The desired trajectory and its time derivatives are all smooth and bounded.

Assumption 3: the interconnection term satisfies the following:

$$|m_i(x_1, x_2, \ldots, x_N)| \leq \xi_{i0} + \xi_{i1}\|x_i\| \tag{3}$$

where, ξ_{ij} is an unknown time varying parameters.

Assumption 4: The disturbance in the above equation is bounded by:

$$|d_i(t)| \leq d_{max} \tag{4}$$

Define the tracking error vector as:

$$e_i = [e_{i,1}, e_{i,2}, \ldots, e_{i,n_i}]^T \in R^{n_i} \tag{5}$$

Where:

$$e_{i,1} = y_d - y_i \tag{6}$$

Taking the n_i^{th} derivative of both sides of the Eq. 6, the following equation can be derived.

$$e_{i,1}^{(n_i)} = y_d^{(n_i)} - y_i^{(n_i)} = y_d^{(n_i)} - f_i(x_i, u_i) - m_i(x_1, x_2, \ldots, x_N) - d_i(t) \tag{7}$$

Use Eq. 5 to rewrite the above equation as:

$$\dot{e}_i = A_{i0}e_i + b_i\{y_d^{(n_i)} - f_i(x_i, u_i) - m_i(x_1, x_2, \ldots, x_N) - d_i(t)\} \tag{8}$$

where, A_{i0} and b_i are defined below:

$$A_{i0} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n_i \times n_i} \text{ and } b_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \in R^{n_i} \tag{9}$$

Consider the vector $k_i = [k_{i,1}, k_{i,2}, \ldots, k_{i,n_i}]^T$ be coefficients of $L(s) = s^{n_i} + k_{i,n_i}s^{n_i-1} + \ldots + k_{i,1}$ and chosen so that the roots of this polynomial are located in the open left-half plane. This makes the matrix $A_i = A_{i0} - b_i k_i^T$ be Hurwitz. Thus, for any given positive definite symmetric matrix Q_i, there exists a unique positive definite symmetric solution P_i for the following Lyapunov equation:

$$A_i^T P_i + P_i A_i = -Q_i \tag{10}$$

Let v_i be defined as:

$$v_i = y_d^{(n_i)} + k_i^T e_i + v_i' \tag{11}$$

By adding and subtracting the term $k_i^T e_i + v_i'$ from the right-hand side of Eq. 8, the following equation is obtained.

$$\dot{e}_i = A_i e_i - b_i\{f_i(x_i, u_i) - v_i + m_i(x_1, x_2, ..., x_N) + d_i(t) + v_i'\} \tag{12}$$

Using assumption 1, Eq. 11 and the signal v_i which is not explicitly dependent on the control input u_i, the following inequality is satisfied:

$$\frac{\partial(f_i(x_i, u_i) - v_i)}{\partial u_i} = \frac{\partial f_i(x_i, u_i)}{\partial u_i} > 0 \tag{13}$$

Invoking the implicit function theorem, it is obvious that the nonlinear algebraic equation $f_i(x_i, u_i) - v_i = 0$ is locally soluble for the input u_i for an arbitrary (x_i, v_i). Thus, there exists some ideal controller $u_i^*(x_i, v_i)$ satisfying the following equality for a given $(x_i, v_i) \in R^{n_i} \times R$:

$$f_i(x_i, u_i^*) - v_i = 0 \tag{14}$$

As a result of the mean value theorem, there exists a constant λ in the range of $0<\lambda<1$, such that the nonlinear function $f_i(x_i, u_i)$ can be expressed around u_i^* as:

$$f_i(x_i, u_i) = f_i(x_i, u_i^*) + (u_i - u_i^*) f_{iu_\lambda} = f_i(x_i, u_i^*) + e_{u_i} f_{iu_\lambda} \tag{15}$$

Where:
$f_{iu_\lambda} = \partial f(x_i, u_i)/\partial u_i|_{u_i = u_{i\lambda}}$
$u_{i\lambda} = \lambda u_i + (1-\lambda) u_i^*$

Using Eq. 15 and 14 to rewrite Eq. 12 as follow:

$$\dot{e}_i = A_i e_i - b_i\{e_{u_i} f_{iu_\lambda} + m_i(x_1, x_2, ..., x_N) + d_i(t) + v_i'\} \tag{16}$$

However, the implicit function theory only guarantees the existence of the ideal controller $u_i^*(x_i, v_i)$ for Eq. 14 and does not recommend a technique for constructing solution even if the dynamics of the system are well known. In the following, a fuzzy system and classic controller will be used to obtain the unknown ideal controller.

FUZZY SYSTEMS

Figure 1 shows the basic configuration of the fuzzy systems considered in this study. Here, fuzzy systems can be considered as a multi-input, single-output: $x \in U \subset R^n \rightarrow y \in V \subset R$. Consider that a multi-output system can be separated into a group of single-output systems.

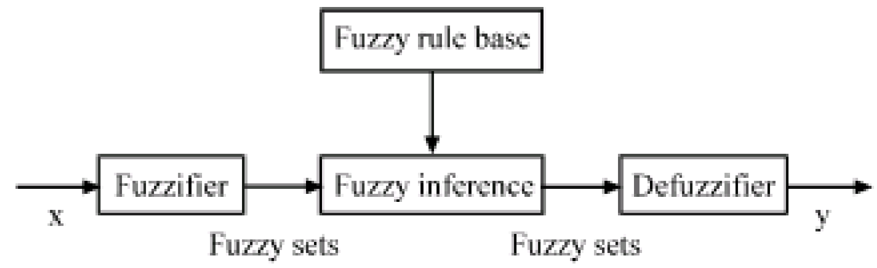

Fig. 1: Configuration of fuzzy system

The fuzzifier performs a mapping from a crisp input vector $x = [x_1, x_2, ..., x_n]^T$ to a fuzzy set, where the label of the fuzzy set are such as small, medium, large, etc.

The fuzzy rule base is consisted of a collection of fuzzy IF-THEN rules. Assume that there are M rules and the l^{th} rule is:

$$R^l(u):\ \text{if } (x_1 \text{ is } A_1^l \ ...\ x_n \text{ is } A_1^l) \text{ then } (y \text{ is } B^l) \qquad l = 1, 2, ..., M \tag{17}$$

where, $x = [x_1, x_2, ..., x_n]^T$ and y are the crisp input and output of the fuzzy system, respectively. A_j^l and B^l are fuzzy membership function in U_j and V, respectively.

The fuzzy inference performs a mapping from fuzzy sets in U to fuzzy sets in V, based on the fuzzy IF-THEN rules in the fuzzy rule base.

The defuzzifier maps fuzzy sets in V to a crisp value in V. The configuration of Fig. 1 shows a general framework of fuzzy systems, because many different choices are allowed for each block in Fig. 1 and various combinations of these choices will construct different fuzzy systems (Wang, 1997). Here, the sum-product inference and the center-average defuzzifier are used for fuzzy system. Therefore, the fuzzy system output can be expressed as:

$$y(x) = \frac{\sum_{l=1}^{M} y^l \prod_{i=1}^{n} \mu_{A_i^l}(x_i)}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{A_i^l}(x_i)} \tag{18}$$

where, $\mu_{A_i^l}(x_i)$ is the membership degree of the input x_i to fuzzy set A_j^l and y^l is the point at which the membership function of fuzzy set B^l achieves its maximum value.

The fuzzy systems in the form of Eq. 18 are proven by Wang and Mendel (1993) to be a universal approximator if their parameters are properly chosen.

Theorem 1: Suppose f(x) is a continuous function on a compact set U (Wang, 1997). Then, for any $\varepsilon>0$, there exists a fuzzy system like Eq. 18 satisfying:

$$\sup_{x \in U} |f(x) - y(x)| \le \varepsilon \tag{19}$$

The output given by Eq. 18 can be rewritten in the following compact form:

$$y(x) = w(x)^T \theta \tag{20}$$

where, $\theta = [y^1\ y^2 ... y^M]$ is a vector grouping all consequent parameters and is $w(x) = [w_1(x)\ w_2(x) ... w_M(x)]^T$ a set of fuzzy basis functions defined as:

$$w_i(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^l}(x_i)}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{A_i^l}(x_i)} \tag{21}$$

The fuzzy system (Eq. 18) is assumed to be well defined so that $\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{A_i^l}(x_i) \neq 0$ for all $x \in U$.

FUZZY ADAPTIVE CONTROLLER DESIGN

Here, it has been shown how to develop a fuzzy system to adaptively approximate the unknown ideal controller.

The ideal controller can be represented as:

$$u_i^* = f_i(z) + u_{pid} + \varepsilon_{iu} \tag{22}$$

where, $f_i(z) = \theta_{i1}^* w_{i1}(z)$, θ_{i1}^* and $w_{i1}(z)$ are consequent parameters and a set of fuzzy basis functions, respectively. ε_{iu} is an approximation error that satisfies $|\varepsilon_{iu}| \leq \varepsilon_{max}$ and $\varepsilon_{max} > 0$. The u_{pid} is the primary controller that developed properly to initially control the underlying system and parameters θ_{i1}^* are determined through the following optimization.

$$\theta_{i1}^* = \arg\min_{\theta_{i1}} \left[\sup \left| \theta_{i1}^T w_{i1}(z) - f_i(z) \right| \right] \tag{23}$$

Denote the estimate of θ_{i1}^* as θ_{i1} and u_{irob} as a robust controller to compensate approximation error, uncertainties, disturbance and interconnection term to rewrite the controller given in Eq. 22 as:

$$u_i = \theta_{i1}^T w_{i1}(z) + u_{pid} + u_{irob} \tag{24}$$

In which u_{irob} is defined below.

$$u_{irob} = \frac{\left| b_i^T P_i e_i \right|}{f_{min} b_i^T P_i e_i} (\hat{\xi}_{i0} + \hat{\xi}_{i1} \|x_i\| + f_{min} u_{icom} + f_{min} u_{ir} + \hat{v}_i') \tag{25}$$

In Eq. 23, $\theta_{i1}^T w_{i1}(z)$ approximates the ideal controller, $\hat{\xi}_{i0} + \hat{\xi}_{i1} \|x_i\|$ tries to estimate the interconnection term, u_{icom} compensates for approximation errors and uncertainties, u_{ir} is designed to compensate for bounded external disturbances and $\hat{v}_i'$ is estimation of v_i'. Define error vector and use Eq. 24 and 25 to rewrite the error Eq. 16 as:

$$\dot{e}_i = A_i e_i - b_i \{ (\tilde{\theta}_{i1}^T w_{i1}(z) + u_{irob} - \varepsilon_{iu}) f_{iu_2} + m_i(x_1, x_2, ..., x_N) + d_i(t) + v_i' \} \tag{26}$$

Consider the following update laws.

$$\begin{aligned} \dot{\theta}_{i1} &= \Gamma_1 b_i^T P_i e_i w_{i1}(z_i) \\ \dot{\hat{\xi}}_{i0} &= \frac{\gamma_{\xi_{i0}}}{f_{min}} \left| b_i^T P_i e_i \right| \\ \dot{\hat{\xi}}_{i1} &= \frac{\gamma_{\xi_{i1}}}{f_{min}} \left| b_i^T P_i e_i \right| \|x_i\| \\ \dot{u}_{ir} &= \gamma_{u_{ir}} \left| b_i^T P_i e_i \right| \\ \dot{u}_{icom} &= \gamma_{u_{icom}} \left| b_i^T P_i e_i \right| \\ \dot{\hat{v}}_i' &= \gamma_{\hat{v}_i'} \left| b_i^T P_i e_i \right| \end{aligned} \tag{27}$$

where, $\Gamma_1 = \Gamma_1^T > 0, \Gamma_2 = \Gamma_2^T > 0, \gamma_{u_{ir}} > 0, \gamma_{u_{icom}} > 0, \gamma_{\hat{v}_i'} > 0,\ \gamma_{\xi_{i1}}, \gamma_{\xi 0} > 0$ are constant parameters.

In following equation, λ_{min}, λ_{max} and $svd_{max}(.)$ are the minimum, maximum eigenvalue and maximum singular value decomposition, respectively.

Lemma 1: The following inequality holds if $\lambda_{min}(Q_i) \geq -{}^{f_{dm}}\!/\!_{f_{min}} \lambda_{min}(P_i)$.

$$\frac{1}{f_{u_i}} e_i^T Q_i e_i + \frac{\dot{f}_{u_i}}{f_{u_i}^2} e_i^T P_i e_i \geq 0 \tag{28}$$

Proof: From assumption 1 and the lemma 1, it is obvious that:

$$\left(\lambda_{min}(Q_i) f_{min} + \lambda_{min}(P_i) f_{dm} \right) \geq 0 \tag{29}$$

This in turn leads to the following inequality.

$$\frac{1}{f_{u_i}^2} \left(\lambda_{min}(Q_i) f_{min} + \lambda_{min}(P_i) f_{dm} \right) \|e_i\|^2 \geq 0 \tag{30}$$

After some algebraic manipulations, the following inequality is obtained.

$$\frac{1}{f_{u_i}} e_i^T Q_i e_i + \frac{\dot{f}_{u_i}}{f_{u_i}^2} e_i^T P_i e_i \geq \frac{1}{f_{u_i}^2} \left(\lambda_{min}(Q_i) f_{min} \|e_i\|^2 + \lambda_{min}(P_i) f_{dm} \|e_i\|^2 \right) \tag{31}$$

Use Eq. 31 to have the following which completes the proof.

$$\frac{1}{f_{u_i}} e_i^T Q_i e_i + \frac{\dot{f}_{u_i}}{f_{u_i}^2} e_i^T P_i e_i \geq 0 \tag{32}$$

Lemma 2: based on lemma 1 and Eq. 10, the following inequality holds.

$$svd_{max}(A_i) \le -\frac{f_{dm}}{2f_{min}}\frac{\lambda_{min}(P_i)}{\lambda_{max}(P_i)} \tag{33}$$

Proof: Using Eq. 10 and after some algebraic manipulations, the following inequality is obtained.

$$\|Q_i\| \le \|A_i^T P_i\| + \|P_i A_i\| = 2\|P_i A_i\| \tag{34}$$

Using the Eq. 34, the following equation can be derived.

$$\|Q_i\| \le 2\|P_i\|\|A_i\| = 2\lambda_{max}(P_i) svd_{max}(A_i) \tag{35}$$

Use Eq. 29 and 35 to have the following which completes the proof.

$$svd_{max}(A_i) \le -\frac{f_{dm}}{2f_{min}}\frac{\lambda_{min}(P_i)}{\lambda_{max}(P_i)} \tag{36}$$

Theorem 2: Consider the error dynamical system given in Eq. 26 for the large scale system (1) satisfying assumption 1, interconnection term satisfying assumption 3, the external disturbances satisfying assumption 4 and a desired trajectory satisfying assumption 2, then the controller structure given in Eq. 24, 25 with adaptation laws Eq. 27 makes the tracking error converge asymptotically to a neighborhood of origin and all signals in the closed loop system be bounded.

Proof: Consider the following lyapunov function.

$$V = \sum_{i=1}^{N}\frac{1}{2}\left(\frac{1}{f_{u_i}}e_i^T P_i e_i + \tilde{\theta}_{i1}^T \Gamma_1^{-1}\tilde{\theta}_{i1} + \frac{\sum_{j=0}^{1}\tilde{\xi}_{ij}^2}{\gamma_{\xi_{ij}}} + \frac{\tilde{u}_{ir}^2}{\gamma_{u_{ir}}} + \frac{\tilde{u}_{icom}^2}{\gamma_{u_{icom}}} + \frac{\tilde{v}_i'^2}{\gamma_{\hat{v}_i'}}\right) \tag{37}$$

Where:

$$\begin{aligned}
\tilde{\theta}_{i1} &= \theta_{i1} - \theta_{i1}^* \\
\tilde{\xi}_{ij} &= \hat{\xi}_{ij} - \xi_{ij} \\
\tilde{u}_{ir} &= u_{ir} - d_{max}/f_{min} \\
\tilde{u}_{icom} &= u_{icom} - \varepsilon_{max} \\
\tilde{v}_i' &= \hat{v}_i' - |v_i'|
\end{aligned}$$

The time derivative of the lyapunov function becomes.

$$\begin{aligned}
\dot{V} = \sum_{i=1}^{N} & \frac{1}{2}\left(\frac{1}{f_{u_i}}\dot{e}_i^T P_i e_i + \frac{1}{f_{u_i}}e_i^T P_i \dot{e}_i + \frac{\dot{f}_{u_i}}{f_{u_i}^2}e_i^T P_i e_i\right) \\
& + \tilde{\theta}_{i1}^T \Gamma_1^{-1}\dot{\theta}_{i1} + \frac{\sum_{j=0}^{1}\tilde{\xi}_{ij}\dot{\hat{\xi}}_{ij}}{\gamma_{\xi_{ij}}} + \frac{\tilde{u}_{ir}\dot{u}_{ir}}{\gamma_{u_{ir}}} + \frac{\tilde{u}_{icom}\dot{u}_{icom}}{\gamma_{u_{icom}}} + \frac{\tilde{v}_i'\dot{\hat{v}}_i'}{\gamma_{\hat{v}_i'}}
\end{aligned} \tag{38}$$

Use Eq. 26, to rewrite above equation as:

$$\begin{aligned}
\dot{V} = \sum_{i=1}^{N} & \frac{1}{2}\left(\frac{1}{f_{u_i}}e_i^T (A_i^T P_i + P_i A_i) e_i + \frac{\dot{f}_{u_i}}{f_{u_i}^2}e_i^T P_i e_i\right) + \\
& \frac{1}{f_{u_i}}(b_i^T P_i e_i (v_i' + (\tilde{\theta}_{i1}^T w_{i1}(z) + u_{irob} - \varepsilon_{iu}) f_{u_i} + \\
& m_i(x_1, x_2, \dots, x_N) + d_i(t))) + \\
& \tilde{\theta}_{i1}^T \Gamma_1^{-1}\dot{\theta}_{i1} + \frac{\sum_{j=0}^{1}\tilde{\xi}_{ij}\dot{\hat{\xi}}_{ij}}{\gamma_{\xi_{ij}}} + \frac{\tilde{u}_{ir}\dot{u}_{ir}}{\gamma_{u_{ir}}} + \frac{\tilde{u}_{icom}\dot{u}_{icom}}{\gamma_{u_{icom}}} + \frac{\tilde{v}_i'\dot{\hat{v}}_i'}{\gamma_{\hat{v}_i'}}
\end{aligned} \tag{39}$$

Using assumption 1 yields and by assumptions 3 and 4, to rewrite Eq. 39 as follow:

$$\begin{aligned}
& \dot{V} \le \sum_{i=1}^{N} \frac{1}{2}\left(-\frac{1}{f_{u_i}}e_i^T Q_i e_i - \frac{\dot{f}_{u_i}}{f_{u_i}^2}e_i^T P_i e_i\right) + \frac{|b_i^T P_i e_i|}{f_{min}}|v_i'| - \\
& b_i^T P_i e_i \tilde{\theta}_{i1}^T w_{i1}(z) - \frac{|b_i^T P_i e_i|}{f_{min}}\left(\xi_{i0} + \xi_{i1}\|x_i\|\right) - |b_i^T P_i e_i| u_{icom} - |b_i^T P_i e_i| u_{ir} - \\
& \frac{|b_i^T P_i e_i|}{f_{min}}\hat{v}_i' + |b_i^T P_i e_i|\varepsilon_{max} + \frac{|b_i^T P_i e_i|}{f_{min}}\left(\hat{\xi}_{i0} + \hat{\xi}_{i1}\|x_i\|\right) + \frac{|b_i^T P_i e_i|}{f_{min}}d_{max} \\
& + \tilde{\theta}_{i1}^T \Gamma_1^{-1}\dot{\theta}_{i1} + \frac{\sum_{j=0}^{1}\tilde{\xi}_{ij}\dot{\hat{\xi}}_{ij}}{\gamma_{\xi_{ij}}} + \frac{\tilde{u}_{ir}\dot{u}_{ir}}{\gamma_{u_{ir}}} + \frac{\tilde{u}_{icom}\dot{u}_{icom}}{\gamma_{u_{icom}}} + \frac{\tilde{v}_i'\dot{\hat{v}}_i'}{\gamma_{\hat{v}_i'}}
\end{aligned} \tag{40}$$

Equation 40 can be rewritten as below:

$$\begin{aligned}
& \dot{V} \le \sum_{i=1}^{N} \frac{1}{2}\left(-\frac{1}{f_{u_i}}e_i^T Q_i e_i - \frac{\dot{f}_{u_i}}{f_{u_i}^2}e_i^T P_i e_i\right) + \\
& -\frac{|b_i^T P_i e_i|}{f_{min}}\underbrace{\left(\hat{v}_i' - |v_i'|\right)}_{\tilde{v}_i'} - b_i^T P_i e_i \tilde{\theta}_{i1}^T w_{i1}(z) - \frac{|b_i^T P_i e_i|}{f_{min}}\underbrace{\left(\hat{\xi}_{i1} - \xi_{i1}\right)}_{\tilde{\xi}_{i1}}\|x_i\| \\
& -\frac{|b_i^T P_i e_i|}{f_{min}}\underbrace{\left(\hat{\xi}_{i0} - \xi_{i0}\right)}_{\tilde{\xi}_{i0}} - |b_i^T P_i e_i|\underbrace{\left(u_{icom} - \varepsilon_{max}\right)}_{\tilde{u}_{icom}} - |b_i^T P_i e_i|\underbrace{\left(u_{ir} - d_{max}/f_{min}\right)}_{\tilde{u}_{ir}} \\
& + \tilde{\theta}_{i1}^T \Gamma_1^{-1}\dot{\theta}_{i1} + \frac{\sum_{j=0}^{1}\tilde{\xi}_{ij}\dot{\hat{\xi}}_{ij}}{\gamma_{\xi_{ij}}} + \frac{\tilde{u}_{ir}\dot{u}_{ir}}{\gamma_{u_{ir}}} + \frac{\tilde{u}_{icom}\dot{u}_{icom}}{\gamma_{u_{icom}}} + \frac{\tilde{v}_i'\dot{\hat{v}}_i'}{\gamma_{\hat{v}_i'}}
\end{aligned} \tag{41}$$

Using Eq. 27, the above inequality rewrites as:

$$\dot{V} \le \sum_{i=1}^{N} -\frac{1}{2}\left(\frac{1}{f_{u_i}}e_i^T Q_i e_i + \frac{\dot{f}_{u_i}}{f_{u_i}^2}e_i^T P_i e_i\right) \tag{42}$$

Use the lemma 1, $\dot{V} \le 0$ are satisfied. Using Barbalet's lemma, it is guaranteed the tracking error asymptotically to the neighborhood of the origin. Furthermore, the boundedness of the coefficient parameters is guaranteed. It completes the proof.

Remark 1: The term tanh(.) is a smooth approximation of the discontinuous term sign(.). The sign(.) function is not used in the study due to avoiding chattering in the response.

Remark 2: It is very important to select properly the controller parameters to gain a satisfactory performance. Here, at this stage, number of the rules and the input membership functions are obtained by trial and error.

Remark 3: To guarantee the boundedness of the parameters in the presence of the unavoidable approximation error, the proposed adaptive laws Eq. 27 is modified it by introducing a σ-modification term as follows:

$$\begin{aligned}
\dot{\theta}_{i1} &= \Gamma_1 b_i^T P_i e_i w_{i1}(z_i) - \sigma\Gamma_1\theta_{i1} \\
\dot{\hat{\xi}}_{i0} &= \frac{\gamma_{\xi_{i0}}}{f_{min}}\left|b_i^T P_i e_i\right| - \sigma\gamma_{\xi_{i0}}\hat{\xi}_{i0} \\
\dot{\hat{\xi}}_{i1} &= \frac{\gamma_{\xi_{i1}}}{f_{min}}\left|b_i^T P_i e_i\right|\left\|x_i\right\| - \sigma\gamma_{\xi_{i1}}\hat{\xi}_{i1} \\
\dot{u}_{ir} &= \gamma_{u_{ir}}\left|b_i^T P_i e_i\right| - \sigma\gamma_{u_{ir}} u_{ir} \\
\dot{u}_{icom} &= \gamma_{u_{icom}}\left|b_i^T P_i e_i\right| - \sigma\gamma_{u_{icom}} u_{icom} \\
\dot{\hat{v}}'_i &= \gamma_{\hat{v}'_i}\left|b_i^T P_i e_i\right| - \sigma\gamma_{\hat{v}'_i}\hat{v}'_i
\end{aligned} \tag{43}$$

SIMULATION RESULTS

Here, the proposed decentralized fuzzy model reference adaptive controller is applied to a two-inverted pendulum problem (Karimi *et al*., 2007) in which the pendulums are connected by a spring as shown in Fig. 2. Each pendulum may be positioned by a torque input u_i applied by a servomotor and its base. It is assumed that the angular position of pendulum and its angular rate are available and can be used as the controller inputs. The pendulums dynamics are described by the following nonlinear equations.

$$\begin{cases}
\dot{x}_{11} = x_{12} \\
\dot{x}_{12} = \left(\frac{m_1 gr}{j_1} - \frac{kr^2}{4j_1}\right)\sin(x11) + \frac{kr}{2j_1}(l-b) + \frac{\alpha}{j_1}\mathrm{sat}(u_1) + \frac{kr^2}{j_1}\sin(x_{21}) + d(t) \\
y_1 = x_{11}
\end{cases}$$
$$\begin{cases}
\dot{x}_{21} = x_{22} \\
\dot{x}_{22} = \left(\frac{m_2 gr}{j_2} - \frac{kr^2}{4j_2}\right)\sin(x_{21}) + \frac{kr}{2j_2}(l-b) + \frac{\alpha}{j_2}\mathrm{sat}(u_2) + \frac{kr^2}{j_2}\sin(x_{12}) + d(t) \\
y_2 = x_{21}
\end{cases} \tag{44}$$

where, y_1, y_2 are the angular displacements of the pendulums from vertical position. $m_1 = 2$ kg, $m_2 = 2.5$ kg are the pendulum end masses $j_1 = 0.5$ kg, $j_2 = 0.62$ kg are the moment of inertia, $k = 100$ N m^{-1} is spring constant, $r = 0.5$ m is the height of the pendulum, $g = 9.81$ m sec^{-2} shows the gravitational acceleration, $l = 00.5$ m is the natural length of spring, α_1, $\alpha_2 = 25$ are the control input gains and $b = 0.4$ m presents distance between the pendulum hinges.

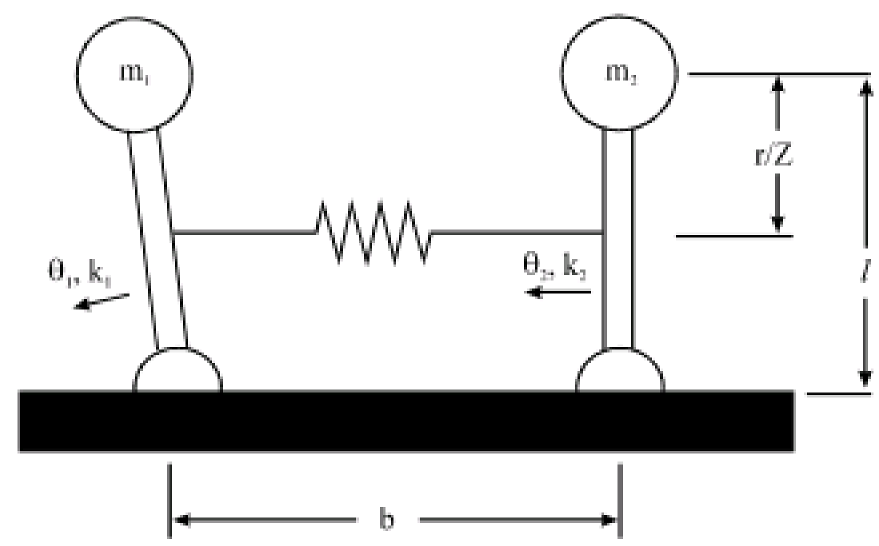

Fig. 2: Two inverted pendulum connected by a spring

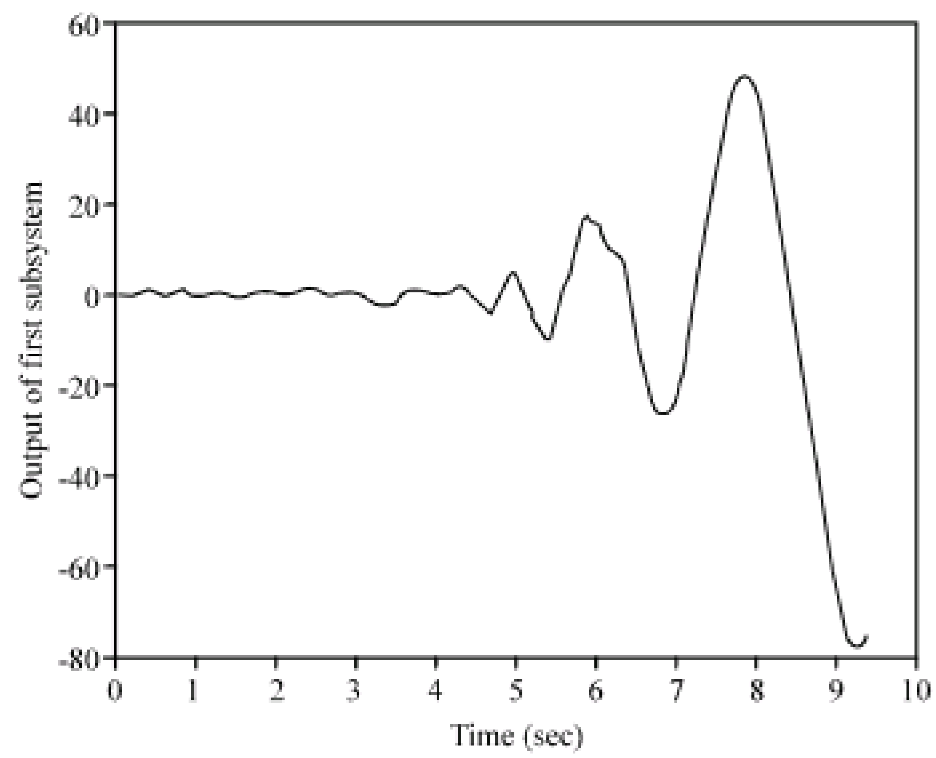

Fig. 3: Performance of the PID controller in first subsystem

The desired value of the outputs are zero ($y_{id} = 0$ for $i = 1, 2$). As discussed earlier, the following primary PI controller are obtained after some trials and errors.

$$u_{pid} = 40(e_i + \tfrac{1}{4}\int_0^t e_i d\tau) \tag{45}$$

Figure 3 and 4 show the outputs of the system where only the controller defined in Eq. 45 is applied to the system.

Obviously the primary controller by itself is not admissible. Now the proposed controller defined in Eq. 24, 25 has been applied to mentioned system. Initially the PID controller keeps the states of system x_{i1}, x_{i2} in the range of $[-1, 1]$, $[-5, 5]$. Let $x_i = [x_{i1}, x_{i2}]^T$, $z = [x_{i1}, x_{i2}, \nu_i]^T$ and ν_i are defined over $[-45, 45]$. For each fuzzy system input, it is defined 6 membership functions over the defined sets. Consider that all of the membership functions are defined by the Gaussian function:

$$\mu_j(\chi) = \exp((\chi - c)^2 / 2\delta^2)$$

where, c is center of the membership function and δ is its variance.

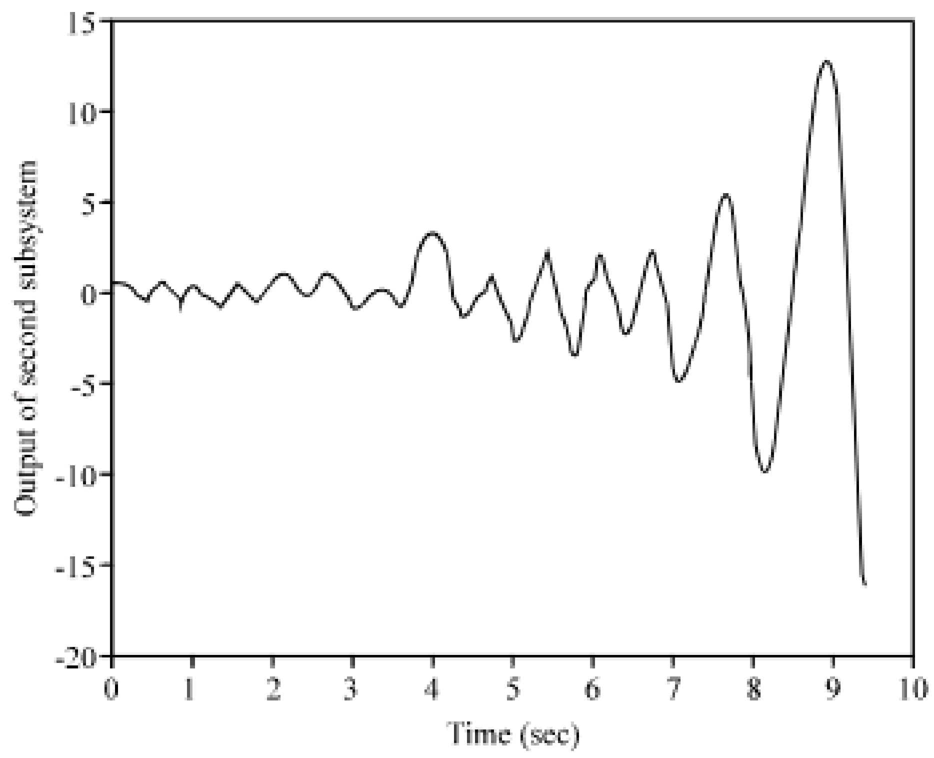

Fig. 4: Performance of the PID controller in second subsystem

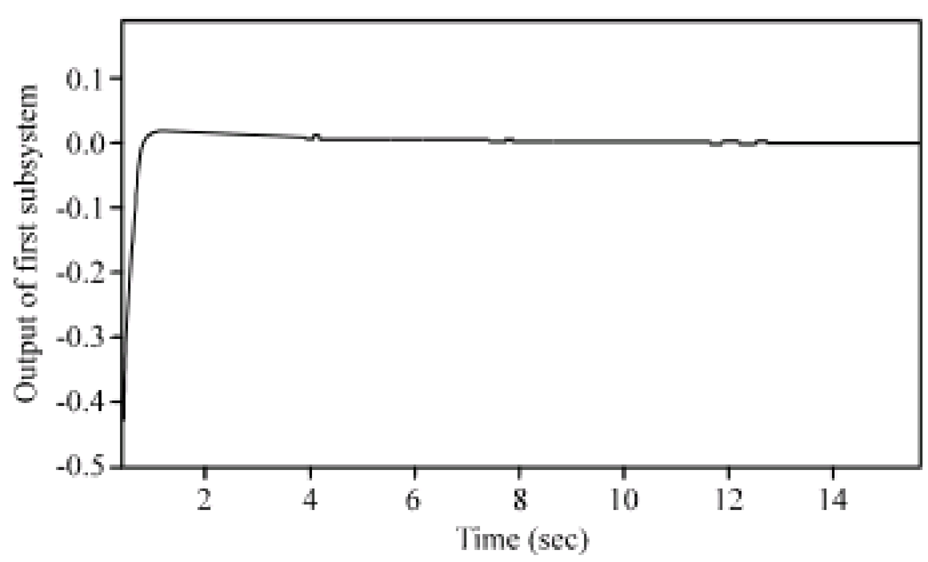

Fig. 5: Performance of the proposed controller in first subsystem

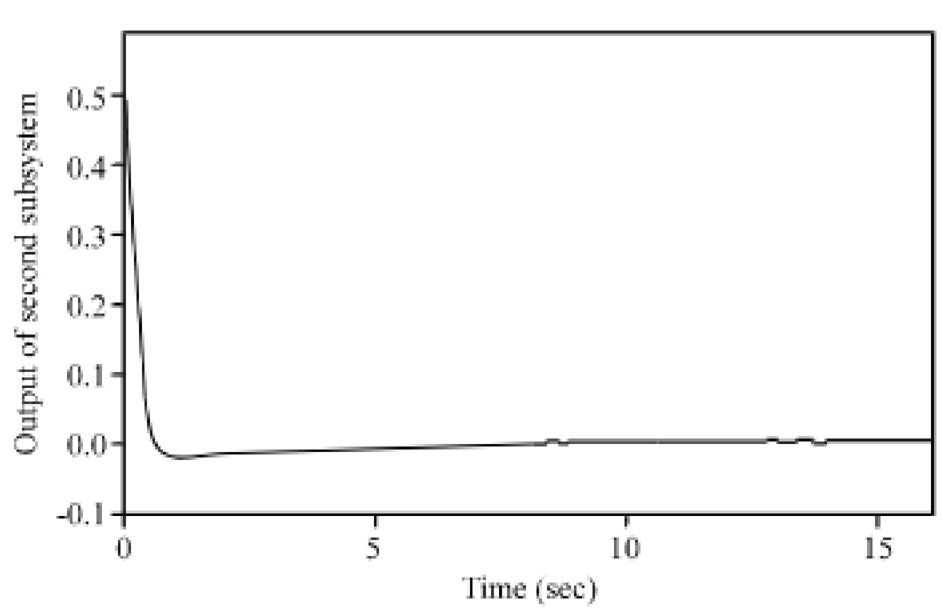

Fig. 6: Performance of the proposed controller in second subsystem

It has been assumed that the initial value of $\theta_{i1}(0)$, $\theta_{i2}(0)$, $u_{ir}(0)$, $u_{icom}(0)$ and $\hat{v}'_i(0)$ be zero. Furthermore, it has been assumed that $f_{min} = 1$, $\Gamma_1 = 10$, $\gamma_{\xi i0 \cdot} = 10$, $\gamma_{u_{com}} = 10$, $\gamma_{u_r} = 10$, $\gamma_{\hat{v}'_i} = 10$. In Eq. 43 and remark 1, $\sigma = 0.1$, $\varepsilon = 0.01$ has been considered. The parameters f_{dm},

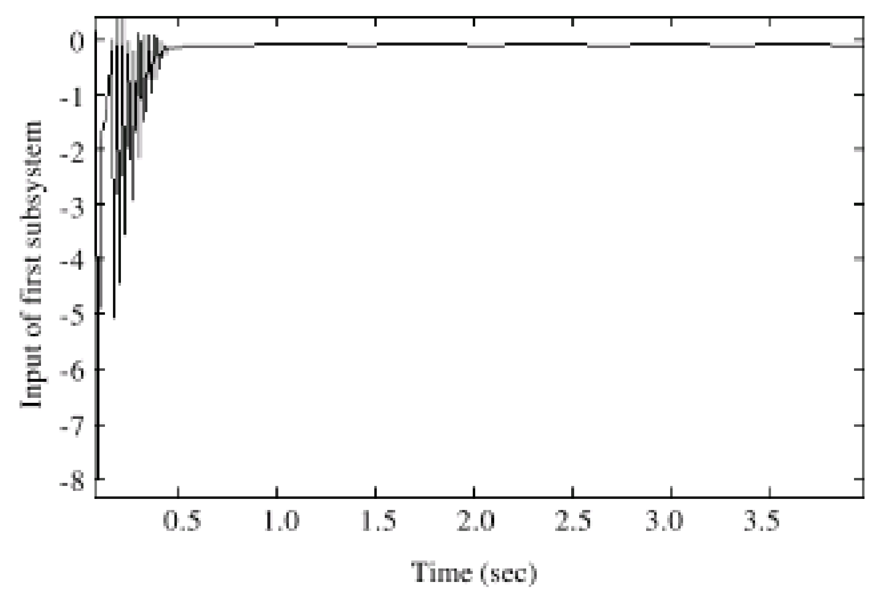

Fig. 7: Control input u_1

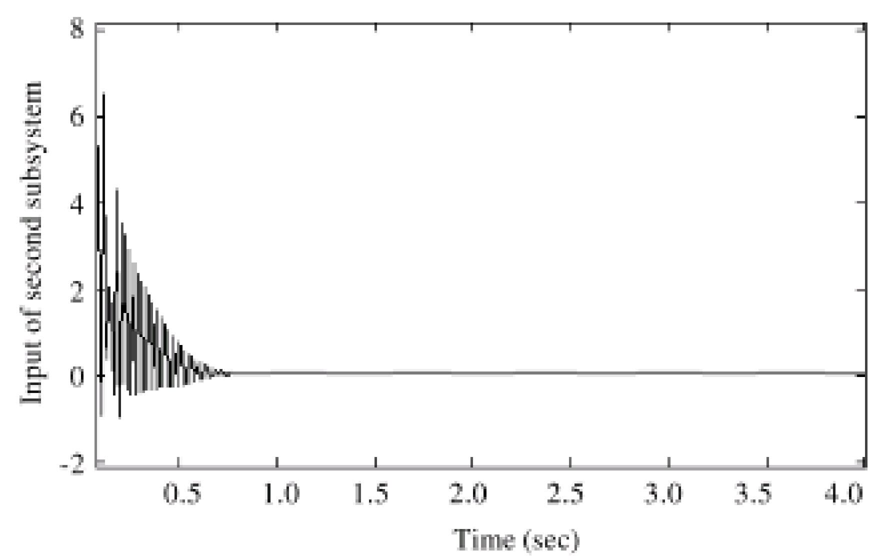

Fig. 8: Control input u_2

f_{min} and the vector $k_i = [k_{i1}, k_{i2},...,k_{i,ni}]^T$ have been chosen so that the lemma 2 holds.

As shown in Fig. 3-6, it is obvious that the performance of the proposed controller is promising. Based on these simulation results, the controller can stabilized the closed loop system. It can decrease error estimation and disturbances effect in the output of the subsystems. Figure 7 and 8 show the total input of each subsystem. It is shown that ripple in the input controller can decrease error estimation and disturbances attenuation.

CONCLUSION

A decentralized fuzzy model reference adaptive output tracking controller is proposed for a class of large scale nonaffine nonlinear systems in this study. Fuzzy systems used to approximate the knowledge of the experts in the controller design procedure. It has been shown that the derived adaptation laws guaranty the Lyapunov's stability of closed-loop system. Asymptotic convergence of the tracking error to zero is guarantied. Robustness against external disturbances and approximation errors, relaxing the conditions and using knowledge of experts are the merits of the proposed controller.

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