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Data-Oriented Model of Sine Based on Chebyshev Zeroes

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Abstract: This study presents a new method based on data-oriented theory for sine modeling. This model of sine made by an array of data based on Chebyshev zeroes. To compute sine by this model less mathematical operations are needed comparing to common methods. Hardware and software implementation of this model provides faster module.

Key words: Data-oriented model, data structure, sine, Chebyshev zeroes

INTRODUCTION

Data-oriented theory presents methods by which any concepts can be modeled in terms of data structures. Now a days large amount of data can be managed cheaply, accessed easily and fast by modern computing system because of advanced memory technology. Any concept can be modeled in this way with large amount of data to recognize and identify easily by modern computing systems. The main contribution of this study is to introduce data-oriented model for sine modeling.

Data-oriented modeling is a useful and applied method which is introduced and applied as follows:

- The basic structure of data-oriented modeling has been presented by Habibizad-Navin *et al*. (1999)
- Discrete structures like probability digraph, probabilistic language, complete tree walk and n-complete tree walk have been presented for many statistical concepts adaptation with computer. In other words requirement tools, definition and important mathematical theorems for these models have presented in by Habibizad-Navin *et al*. (2005a)
- The above methods and structures have been utilized for modeling and then using it for simulating uniform distribution (Habibizad-Navin *et al*., 2005b)
- Data-oriented models of population and sample named classified image have been presented and then provided an algorithm to estimate the distribution of a statistical population based on data-oriented model (Habibizad-Navin *et al*., 2005c)
- New data-oriented modeling of uniform random variable which is well-matched with computing systems has been presented by Habibizad-Navin *et al*. (2007a)
- A data oriented model of image has been presented by Habibizad-Navin *et al*. (2007b)
- Data-oriented modeling of fuzzy controller for controlling the Anti-lock braking system has been introduced by Habibizad-Navin *et al*. (2007c)
- A novel method for improving the uniformity of random number generator named uniformity improving method, or UIM in short, has been introduced and data-oriented model of uniform random variable named UDPD is simulated by this approach (Habibizad-Navin *et al*., 2007d)
- Digital probability hyper digraph for modeling random variable as the hierarchical data-oriented model has been introduced by Habibizad-Navin *et al.* (2007e)
- The basic mathematical discussions about data-oriented modeling of uniform random variable have been introduced by Habibizad-Navin *et al*. (2008)

The questions can be answered by data processing or by fewer amounts of mathematical operations by using these models. The methods to answer the questions by using large amount of data easily are called data-oriented methods. Because of less memory storage locations available and slowness of computing in the past, the problems were solved with fewer amounts of data using complex algorithms which are called function-oriented methods. Data-oriented methods require large amount of data with simple algorithms but function-oriented methods require less data with complex algorithms. In this study, data-oriented model of sine is presented.

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COMMON METHOD FOR COMPUTING SINE FUNCTION

The common method for computing sine function is reviewed here. This method computes sine function by using Maclaurin expansion as follows (Thomas and Finy, 1996):

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)}{(2n+1)} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots, \text{ for all x}$$

To obtain an approximation with a good accuracy more terms of the series are required to be considered. For example using 4 terms of this series to approximate sin(x) as:

$$\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!},$$

results an approximation with an error about

$$\frac{|x|^9}{9!}$$

(Thomas *et al*., 1996).

To obtain more accurate result more terms of this series, are required to be taken. Computing sine, by this series more operations including subtractions, which are due to cancellation error, are required.

Such methods are named function oriented method because they need more operations. The operation counts, errors and required memory locations in computing sine function using MATLAB software and common method with 7 terms of this series are shown in Table 1. The algorithm used to compute sine by MATLAB is presented in appendix A. MATLAB uses 6 memory location and 24 operations but common method uses 1 memory location and 36 operations.

By using the rule that by exceeding time complexity (need operation) memory complexity is decreased and vice versa the data-oriented theory has been introduced which models the concepts by data structures (large amount of data) for decreasing the time complexity of algorithms. Such a model for sine is mentioned further.

Table 1: Comparison of common method and MATLAB software method

Methods	Memory locations	No. of sums and subtractions	No. of multiplications and divisions	Errors
Common	1	6	30	2^{-58}
MATLAB	6	6	18	2^{-58}

NEW MODELING METHOD

Here, a new model of sine is presented by using an array of data. To make a model for sine in [a,b], n equidistant points, x_i, $a \le x_i \le b$, i, = 0, 1,....,n, are chosen and then compute:

$$\delta_i = \frac{\sin(x_{i+1}) - \sin(x_i)}{x_{i+1} - x_i} \text{ as slop and } \gamma_i = \sin(x_i), \text{ distance from origin.}$$

Next let

$$\sin(x) = \delta_i \times (x - x_i) + \gamma_i, x_i \le x \le x_{i+1}$$

which is a linear interpolation approximation for sine x in $[x_i, x_{i+1}]$. The interpolation error in $[x_i, x_{i+1}]$ is

$$e = (x - x_i)(x - x_{i+1}) \frac{\sin \alpha}{2!}$$

where, α is some point in $[x_i, x_{i+1}]$ (Thomas *et al*., 1996). This is easy to understand that its maximum absolute value is obtained for $x = (x_i, x_{i+1})/2$. To model sine in

$$\left[0, \frac{\pi}{2}\right], \text{ we let, } h = \frac{\pi}{360}, x_0 = 0, x_i = 0 + ih, i = 0,1,...,512$$

Then the interpolations corresponding to Chebyshev zeroes perform at the points:

$$x_i^{'} = (\frac{x_{i+1} - x_i}{2})t_0 + \frac{x_{i+1} + x_i}{2}$$

And

$$x_{i+1}^{'} = (\frac{x_{i+1} - x_i}{2})t_1 + \frac{x_{i+1} + x_i}{2}$$

Where:

$$t_0 = -\frac{\sqrt{2}}{2}, t_1 = \frac{\sqrt{2}}{2}$$

Now α_i and β_i are computed as

$$\alpha_i = \frac{\sin(x_{i+1}^{'}) - \sin(x_i^{'})}{x_{i+1}^{'} - x_i^{'}} \text{ and } \beta_i = \sin(x_i^{'})$$

and stored in fast memory. Then to find sin(x) for given x, first an interval $[x_i, x_{i+1}]$ containing x is defined from which $\sin(x) = \alpha_i \times (x - x_i)\beta_i, x_i, \le x \le x_{i+1}$ is computed with probably less errors.

Table 2: Obtained result of new method

Methods	Memory locations	No. of sums	No. of multiplications and divisions	Errors
New model	512	1	2	2^{-58}

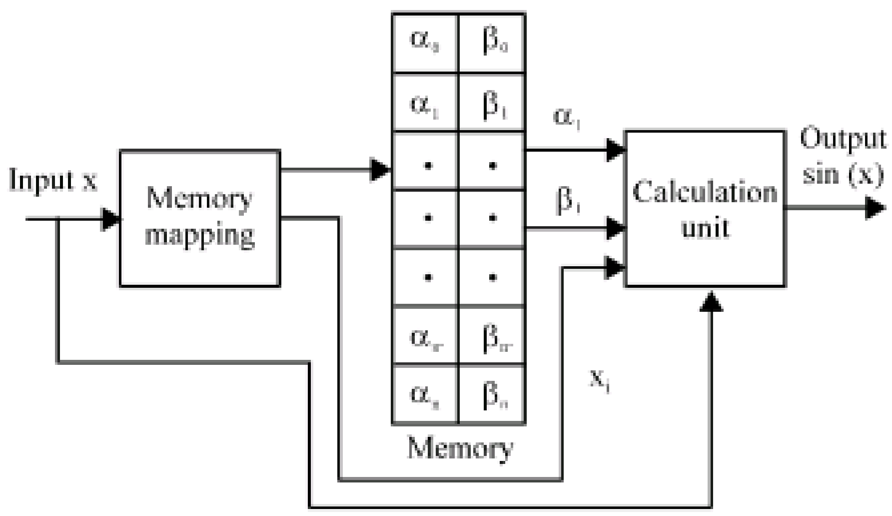

Fig. 1: Architecture of the model

Table 2 shows the simulation results of presented methods.

New model of sine uses 512 memory locations involve 3 operations. It is very fast and efficient in modern computing systems with fast memory. Figure 1 shows the architecture of such system that operates as follows:

- Memory mapping unit gets x as the input then provides i as the address of memory location
- By knowing i as the address of the memory, the interpolation parameters α_i and β_i are given
- Calculation unit provides sine(x) by interpolation with using α_i, β_i, x_i and x

CONCLUSION

Common method computes sine function by using Maclaurin expansion. To achieve good accuracy more terms of series are needed to compute the sine by this expansion. Such methods require more mathematical operations to obtain answer is named function oriented method. Against this method data-oriented modeling is a new theory that presents methods by which any concepts can be modeled in terms of data structures. Now a days large amount of data can be managed cheaply and also can be accessed easily and fast by modern computing system because of advanced memory technology. In this study data-oriented model of sine is presented by using an array of data.

To obtain sine in good accuracy of the computation more terms of the series are required by common methods. As the operation counts, errors and required memory locations in computing sine function using MATLAB software and common method with 7 terms of this series are shown in Table 3. MATLAB uses 6 memory locations and 24 operations but common method uses 1 memory location and 36 operations. While data-oriented model of sine uses 512 memory locations and computes it by 3 operations. It is very fast and efficient in modern computing systems with fast memory. As Table 3 shows, the new model is preferred in computational point of view.

Table 3: Obtained results of new method

Methods	Memory locations	No. of sums	No. of multiplications and divisions	Errors
Common	1	6	30	2^{-58}
MATLAB	6	6	18	2^{-58}
New model	512	2	1	2^{-58}

APPENDIX A

- Algorithm
- Since sin (-x) = -sin (x), we need only to consider positive x
- If $x<2^{-27}$(hx<0x3e400000, 0), return x with inexact if x! = 0
- sin (x) is approximated by a polynomial of degree 13 on $\left[0,\frac{\pi}{4}\right]$
- Where, sin (x) ≈ x+S1* x^3+...+S^6* x^{13}
- $\left|\frac{\sin(x)}{x} - (1 + S1 * x^2 + S2 * x^4 + S3 * x^6 + S4 * x^8 + S5 * x^{10} + S6 * x^{12})\right| \le 2^{-58}$
- $\sin(x+y) = \sin(x) + \sin'(x') * y \approx \sin(x) + (1 - x * \frac{x}{2}) * y$
- For better accuracy, let
- $r = x^3 * (S2 + x^2 * (S3 + x^2 * (S4 + x^2 * (S5 + x^2 * S6))))$
- Then $\sin(x) = x^3 + (S1 * x^2 + (x * (r - \frac{y}{2}) + y))$

S1 = -1.66666666666666324348e-01,/*0xBFC55555, 0x55555549*/

S2 = 8.33333333332248946124e-03,/*0x3F811111, 0x1110F8A6 /

S3 = -1.98412698298579493134e-04,/*0xBF2A01A0, 0x19C161D5*/

S4 = 2.75573137070700676789e-06,/*0x3EC71DE3, 0x57B1FE7D*/

S5 = -2.50507602534068634195e-08,/*0xBE5AE5E6, 0x8A2B9CEB*/

S6 = 1.58969099521155010221e-10;/*0x3DE5D93A, 0x5ACFD57C*/

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