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Optimal Control of a Production Inventory System with Generalized Pareto Distributed Deterioration Items

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Abstract: This study is concerned with the optimal control of inventory-production system subject to generalized Pareto distributed deterioration items. We formulated the model as a linear optimal control problem. This study deals with continuous review policy to solve the optimal control model. We obtained an explicit solution using Pontryagin maximum principle. We derive explicit optimal policies for the inventory model. It is then numerical illustrated with sidered as the sinusoidal function of time then it does not show the convergence of the optimal level. Whenever, if we take the inventory goal level is fixed the help different demand patterns examples. Whenever the goal inventory level is con then it shows the convergence of the optimal inventory towards inventory goal level. The solution of the second-order differential equation is represented and shows the state of optimal inventory level is increasing but converges initially for the certain period of time. In this case, we found the optimal production level with time shows the slight variations with changing the shape of the demand functions. It is observed that the optimal production rates are not very sensitive to changes in the demand functions in case of generalized Pareto distribution.

Key words: Inventory-production systems, Pareto distributed deterioration, optimal control, continuous-review policy, optimality conditions, pontryagin maximum principle

INTRODUCTION

In general, in inventory models, two factors of the problem have been of growing interest to the researchers, one being the deterioration of items and the other being the variation in the demand rate with time. We are concerned in this study with dynamic problem that can be represented as an optimal control problem with one state variable (inventory level) and one control variable (rate of manufacturing) subject to time of deterioration. The novelty here is that the time of deterioration is a random variable (lifetime of the commodity) followed by generalized Pareto distribution and we consider the problem of controlling the production rate of a continuous review manufacturing system.

Many real problems and applications involve the control of dynamic systems, i.e., systems that evolve over time. Thus optimal control theory is a branch of mathematics developed to find optimal ways to control a dynamic systems either continuous-time systems or discrete-time systems. The optimal control theory has been applied to different inventory-production control problems where researchers are involved to analyze the effect of deterioration and the variations in the demand rate with time in logistics. Items deterioration is of great importance in inventory theory, as shown by the surveys of Raafat (1991), Shah and Shah (2000) and Goyal and Giri (2001). Inventory model with Weibull distribution for the lifetime of a commodity has been studied by many researchers Devi (2000), Wu *et al.* (2000), Chen and Lin (2003), Ghosh and Chaudhuri (2004), Al-khedhairi and Tadj (2007) and Baten and Kamil (2009). Two production systems with inventory-level-dependent demand are considered and Pontryagin maximum principle is used to determine the optimal control by Bounkhel and Tadj (2005). The optimal control of continuous-review models with deteriorating items has been addressed by Bounkhel and Tadj (2005), Tadj *et al.* (2006) and Benhadid *et al.* (2008). SrInivasa Rao *et al.* (2005, 2007) studied the inventory models with Pareto distribution deterioration rate to derive optimal order quantity with total cost minimized. But no attempt has been made to develop the inventory model as an optimal control problem and derive an explicit solution of an inventory model with generalized Pareto distribution deterioration using Pontryagin maximum principle. The continuous review policy of optimal control approach is to be novel in this framework. There seems to be no literature on the optimal control of continuous review manufacturing systems with generalized Pareto distribution deterioration items rate.

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We are especially interested in the application of optimal control theory to the production planning problem. Various authors attacked their research in the application of optimal control theory to the production planning problem. Some of them are: Sethi and Thompson (2000), Salama (2000), Riddalls and Bennett (2001), Zhang *et al.* (2001), Khemlnitsky and Gerchak (2002), Hedjar *et al.* (2004, 2007) and Bounkhel and Tadj (2005). Recently, El-Gohary *et al.* (2009) contributed to the application of optimal control theory with the assumptions of constant deterioration rate in production inventory systems. In the present study, we assume that the demand rate is time-dependent and the time of deterioration rate is assumed to follow a generalized Pareto distribution as well as a non-negative discount rate is considered for the inventory systems.

This study develops an optimal control model and utilizes Pontryagin maximum principle by Pontrygin *et al.* (1962) in case of continuous review policy to derive the necessary and sufficient optimality conditions for inventory systems where, the novelty we take into consideration in our research is that the time of deterioration is a random variable followed by the three-parameter generalized Pareto distribution. The probability density function of a generalized Pareto distribution having probability distribution of the form:

$$f(t) = \frac{1}{\sigma}(1+\xi t)^{-\left(\frac{1}{\xi}+1\right)}$$

Where,

$$t = \frac{x-\mu}{\sigma},\ x \ge \mu \text{ when } \xi \ge 0,\ \mu \le x \le \mu - \sigma/\xi \text{ when } \xi < 0 \text{ and } \mu \in (-\infty, \infty)$$

is the location parameter and $\sigma \in (-\infty, \infty)$ is the scale parameter and $\xi \in (-\infty, \infty)$ is the shape parameter. The probability distribution function is:

$$F(t) = 1-(1+\xi t)^{-1/\xi},\quad t > 0$$

The instantaneous rate of deterioration of the on-hand inventory is given by:

$$h(t) = \frac{f(t)}{1-F(t)} = \frac{1}{\sigma(1+\xi t)},\quad t > 0$$

This study deals with continuous review policy to solve the optimal control models by applying Pontryagin maximum principle. We derive explicit optimal policies for the inventory models where items are deteriorating with generalized Pareto distribution that can be used in the decision making process.

MODEL AND NOTATIONS

Consider a system where items are subject to generalized Pareto distributed deterioration. The fixed length of the planning horizon is T. The following notations will be used to describe the dynamics of the system:

- $x(t)$: Inventory level function at any instant of time $t \in [0, T]$
- $u(t)$: Production rate at any instant of time $t \in [0, T]$
- $y(t)$: Demand rate at any instant of time $t \in [0, T]$

The dynamics of the inventory level of the state equation which says that the inventory at time t is increased by the production rate u(t) and decreased by the demand rate y(t) and the rate of deterioration $1/(\sigma(1+\xi t))$ of generalized Pareto distribution can be written as according to:

$$dx(t) = [u(t) - y(t) - \frac{1}{\sigma(1+\xi t)}x(t)]dt \tag{1}$$

with initial condition $x(T) = 0$.

Now to build the objective function, we assume that an inventory goal level and a production goal rate are set and penalties are incurred when the inventory level, the production rate and the deterioration rate deviate from these goals. To explicitly write the objective function, we introduce the following additional notation:

- q : Inventory holding cost incurred for the inventory level to deviate from its goal
- r : Production unit cost incurred for the production rate to deviate from its goal.
- $\hat{x}(t)$: Inventory goal level
- $\hat{u}(t)$: Production goal rate
- $\rho \ge 0$: Constant non-negative discount rate

We want to keep the inventory x(t) as close as possible to its goal $\hat{x}(t)$ and also keep the production rate $u(t)$ as close to its goal level $\hat{u}(t)$. The quadratic terms $q[x(t)-\hat{x}(t)]^2$ and $r[u(t)-\hat{u}(t)]^2$ impose 'penalties' for having either x or u not being close to its corresponding goal level.

The objective function can be expressed as the quadratic form that we need to minimize:

$$\text{Minimize}\quad J(u,x,\hat{u}) = \frac{1}{2}\int_0^T e^{-\rho t}\left\{q[x(t)-\hat{x}(t)]^2 + r[u(t)-\hat{u}(t)]^2\right\}dt \qquad (2)$$

subject to Eq. 1 and the non-negativity constraint:

$$u(t) \geq 0, \quad \text{for all } t \in [0, T] \qquad (3)$$

Development of the optimal control model: To build our optimal control model, we consider that a firm can manufactures a certain product, selling some and stocking the rest in a warehouse. We assume that the demand rate varies with time and the firm has set an inventory goal level and production goal rate. We also assume that the firm has no shortage, the instantaneous rate of deterioration of the on-hand inventory follows the three parameters generalized Pareto distribution and the production is continuous.

By the virtue of Eq. 1 the instantaneous state of the inventory level x(t) at any time t is governed by the differential equation:

$$\frac{dx(t)}{dt} + \frac{1}{\sigma(1+\xi t)}x(t) = u(t) - y(t), \quad 0 \leq t \leq T, \quad x(T) = 0 \qquad (4)$$

This is a linear ordinary differential equation of first order and its integrating factor is:

$$= \exp\{\int \frac{1}{\sigma(1+\xi t)}dt\} = \sigma(1+\xi t)^{1/\sigma\xi}$$

Solving the differential equation the on-hand inventory at time t is obtained as:

$$x(t) = \frac{\sigma(u(t)-y(t))}{\sigma\xi+1}\left[(1+\xi T)^{1+1/\sigma\xi}(1+\xi t)^{-1/\sigma\xi} - (1+\xi t)\right], \quad 0 \leq t \leq T \qquad (5)$$

Assuming that x(0) = x is known and note that the production goal rate $\hat{u}(t)$ can be computed using the state Eq. 4 as:

$$\hat{u}(t) = y(t) + \frac{1}{\sigma(1+\xi t)}\hat{x}(t) \qquad (6)$$

We start by defining the variables z(t), k(t) and v(t) such that:

$$z(t) = x(t) - \hat{x}(t) \qquad (7)$$

$$k(t) = u(t) - \hat{u}(t) \qquad (8)$$

$$v(t) = \hat{u}(t) - y(t) - \frac{1}{\sigma(1+\xi t)}\hat{x}(t) \qquad (9)$$

Adding and subtracting the last term:

$$\frac{1}{\sigma(1+\xi t)}\hat{x}(t)$$

from the right hand side of Eq. 9 to the Eq. 1 and rearranging the terms we have:

$$d(x(t)-\hat{x}(t)) = [-\frac{1}{\sigma(1+\xi t)}(x(t)-\hat{x}(t)) + u(t) - y(t) - \frac{1}{\sigma(1+\xi t)}\hat{x}(t)]dt$$

Hence by Eq. 7:

$$dz(t) = [-\frac{1}{\sigma(1+\xi t)}z(t) + u(t) - y(t) - \frac{1}{\sigma(1+\xi t)}\hat{x}(t)]dt \qquad (10)$$

Now substituting Eq. 8 and 9 in Eq. 10 yields:

$$dz(t) = [-\frac{1}{\sigma(1+\xi t)}z(t) + k(t) + v(t)]dt \qquad (11)$$

The optimal control model (2) becomes:

$$\text{Minimize}\quad J(z,k) = \frac{1}{2}\int_0^T e^{-\rho t}\{q[z(t)^2] + r[k(t)^2]\}dt \qquad (12)$$

subject to an ordinary differential Eq. 11 and the non-negativity constraint k(t)≥0, for all t∈[0, T].

SOLUTION TO THE OPTIMAL CONTROL PROBLEM

Continuous-review policy: The following theorem is the main result of this section for the development control (Eq. 12) subject to revised state Eq. 11. We first assume the firm adopts a continuous-review policy. We adopt the widely used quadratic objective function of Holt *et al.* (1960).

Theorem: Necessary conditions for the pair (k, z) to be an optimal solution of Eq. 11 are:

$$\frac{d^2}{dt^2}z(t) + \left[\frac{\frac{d}{dt}r(t)}{r(t)} - \rho\right]\frac{d}{dt}z(t) + \left[\frac{d}{dt}\left(\frac{1}{\sigma(1+\xi t)}\right) - \frac{1}{\sigma(1+\xi t)}\left[\left(\frac{\frac{d}{dt}r(t)}{r(t)} - \rho\right) - \frac{1}{\sigma(1+\xi t)}\right] - \frac{q(t)}{r(t)}\right]z(t) = 0 \qquad \text{'(3.1)'}$$

and $z(0) = 0, \quad u(T) = \hat{u}(T), \quad u(t) \geq 0 \quad \text{for all} \quad t \in [0, T]$.

Proof By Pontryagin maximum principle, there exists adjoint function $\lambda(t)$ such that the Hamiltonian function:

$$H(t,z(t),k(t),\lambda(t)) = -\frac{1}{2}\int_0^T e^{-\rho t}\{q[z(t)^2]+r[k(t)^2]\}+\lambda(t)\left[-\frac{1}{\sigma(1+\xi t)}z(t)+k(t)+v(t)\right] \quad (13)$$

Assume (k, z) is an optimal solution to the objective function (Eq. 11), then:

$$\frac{\partial}{\partial k(t)}H(t,z(t),k(t),\lambda(t)) = 0, \text{which is equivalent to } \lambda(t) = r(t)e^{-\rho t}k(t) \quad (14)$$

$$-\frac{\partial}{\partial t}\lambda(t) = \frac{\partial}{\partial z(t)}H(t,z(t),k(t),\lambda(t))$$

which is equivalent to:

$$\frac{\partial}{\partial t}\lambda(t) = q(t)e^{-\rho t}z(t) + \frac{\lambda(t)}{\sigma(1+\xi t)} \quad (15)$$

and

$$z(0) = 0, \quad \lambda(T) = 0 \quad (16)$$

Now, combining (Eq. 14 and 15) yields:

$$\frac{\partial}{\partial t}k(t) + \left[\frac{\frac{d}{dt}r(t)}{r(t)} - \rho - \frac{1}{\sigma(1+\xi t)}\right]k(t) = \frac{q(t)}{r(t)}z(t) \quad (17)$$

Since all the goal rates must satisfy the state equation, so we have:

$$\frac{d}{dt}\hat{x}(t) = \hat{u}(t) - y(t) - \frac{1}{\sigma(1+\xi t)}\hat{x}(t) \quad (18)$$

which yields with the state Eq. 1:

$$k(t) = \frac{d}{dt}z(t) + \frac{1}{\sigma(1+\xi t)}z(t) \quad (19)$$

Combining this Eq. 19 with the Eq. 17 yields the second order differential Eq. 12. From the Eq. 14 and 15 $z(0) = 0, \quad u(T) = \hat{u}(T)$ follows directly. So the proof is complete.

Now we solve the original objection function (2) subject to state Eq. 1 and derive the necessary optimality conditions using Pontryagin maximum principle (Sethi and Thompson, 2000).

Analytical solution of the original problem: The optimal control approach consists in determining the optimal control $\hat{u}(t)$ that minimizes the objective function (2) subject to the state Eq. 1. By the maximum principle of (Pontrygin, 1962), there exists adjoint function $\lambda(t)$ such that the Hamiltonian functional form as:

$$H(t,x(t),u(t),\hat{u}(t),\lambda(t)) = -\frac{1}{2}\int_0^T e^{-\rho t}\left\{q[x(t)-\hat{x}(t)]^2 + r[u(t)-\hat{u}(t)]^2\right\} + \lambda(t)\left[u(t) - y(t) - \frac{1}{\sigma(1+\xi t)}x(t)\right] \quad (20)$$

satisfies the control equation:

$$\frac{\partial}{\partial u(t)}H(t,x(t),u(t),\hat{u}(t),\lambda(t)) = 0 \quad (21)$$

the adjoint equation

$$\frac{\partial}{\partial x(t)}H(t,x(t),u(t),\hat{u}(t),\lambda(t)) = -\frac{d}{dt}\lambda(t), \quad \lambda(T) = 0 \quad (22)$$

and the state equation:

$$\frac{\partial}{\partial \lambda(t)}H(t,x(t),u(t),\hat{u}(t),\lambda(t)) = \frac{d}{dt}x(t), \quad x(0) = 0 \quad (23)$$

Then the control Eq. 21 is equivalent to:

$$u(t) = \hat{u}(t) + \frac{e^{\rho t}}{r}\lambda(t) \quad (24)$$

The adjoint Eq. 22 is equivalent to:

$$\frac{d}{dt}\lambda(t) = \frac{\lambda(t)}{\sigma(1+\xi t)} + qe^{-\rho t}[x(t) - \hat{x}(t)] \quad (25)$$

and the state Eq. 23 is similar to the Eq. 1.

Substitution expression (Eq. 24) into the state Eq. 1 yields:

$$\frac{d}{dt}x(t) = \hat{u}(t) + \frac{\lambda(t)e^{\rho t}}{r} - y(t) - \frac{1}{\sigma(1+\xi t)}x(t) \quad (26)$$

From (26) we have:

$$\frac{\lambda(t)e^{\rho t}}{r} = \frac{d}{dt}x(t) - \hat{u}(t) + y(t) + \frac{1}{\sigma(1+\xi t)}x(t) \quad (27)$$

By differentiating Eq. 26, we obtain:

$$\frac{d^2}{dt^2}x(t) = \frac{d}{dt}\hat{u}(t) - \frac{d}{dt}y(t) + \frac{1}{r}\left[e^{\rho t}\frac{d}{dt}\lambda(t) + \rho\lambda(t)e^{\rho t}\right] - \frac{1}{\sigma(1+\xi t)}\frac{d}{dt}x(t) + \frac{\xi x(t)}{\sigma(1+\xi t)^2} \quad (28)$$

Substitution expression (Eq. 25) into the Eq. 28 yields:

$$\frac{d^2}{dt^2}x(t) = \frac{d}{dt}\hat{u}(t) - \frac{d}{dt}y(t) + \frac{q}{r}[x(t) - \hat{x}(t)] + \frac{1}{\sigma(1+\xi t)}\left[\frac{\lambda(t)e^{\rho t}}{r} - \frac{d}{dt}x(t)\right] + \frac{\xi x(t)}{\sigma(1+\xi t)^2} \quad (29)$$

Finally, substituting expression (Eq. 27 into 29) to obtain:

$$\frac{d^2}{dt^2}x(t) - \left[\frac{q}{r} + \frac{\left(\xi + \frac{1}{\sigma}\right)}{\sigma(1+\xi t)^2}\right]x(t) = \frac{d}{dt}\hat{u}(t) - \frac{d}{dt}y(t) - \frac{q}{r}\hat{x}(t) + \frac{1}{\sigma(1+\xi t)}[y(t) - \hat{u}(t)] \quad (30)$$

Since a closed form solution is not possible, so this boundary value problem can be solved numerically together with initial condition x(0) = 0 and the terminal condition $\lambda(T) = 0$.

ILLUSTRATIVE EXAMPLES

In this section, we present some numerical examples. Numerical examples are given for three different cases of demand rates.

- Demand rate is constant: y(t) = y = 20
- Demand rate is linear function of time: y (t) = y_1 (t) t+y_2 (t) = t+15
- Demand is sinusoidal function of time: y(t) = 1+sint

In order to present illustrative examples of the results obtained we use the following parameters where the planning horizon has length T=12 months, $\rho = 0.001$, the inventory holding cost coefficient q = 5 the production cost coefficient r = 5 The goal inventory level is considered as $\hat{x}(t) = 1 + t + \sin(t)$ The shape and scale parameters of the generalized Pareto distribution rate are considered as $\sigma = 1$ and $\xi = 1$, respectively. Then the deterioration rate of Pareto distribution becomes:

$$h(t) = \frac{1}{1+t}, \quad t \in [0,T]$$

The inventory level x(t) in-terms of the first-order differential equation from (Eq. 4) and the second-order differential Eq. 20 considering the sinusoidal demand function are solved numerically using the version 6.5 of the mathematical package MATLAB displayed by Fig. 1 and 3, respectively. In particular, whenever the goal inventory level is considered as the sinusoidal function of time t i.e., $\hat{x}(t) = 1 + t + \sin(t)$ then Fig. 1 does not show the convergence of the optimal level. Whenever if we take the inventory goal level is as $\hat{x}(t) = 10$ then Fig. 2 shows the convergence of the optimal inventory towards inventory goal level. The solution of the second-order differential equation is represented by Fig. 2 and shows the state of optimal inventory level is increasing but converges initially for the certain period of time.

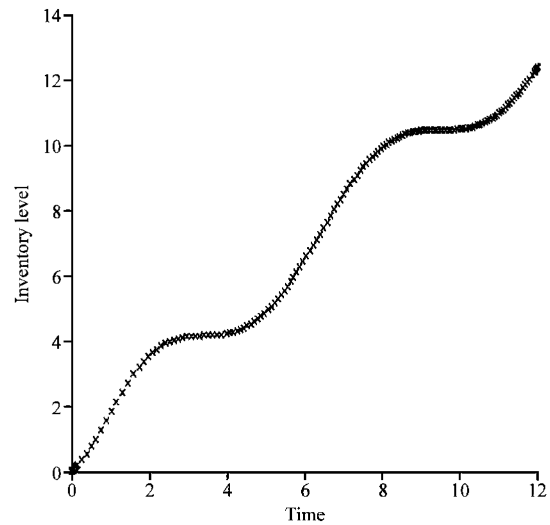

Fig. 1: The inventory level x(t) in-terms of the first-order differential equation. Optimal inventory level of the product with time. Source: Author compution

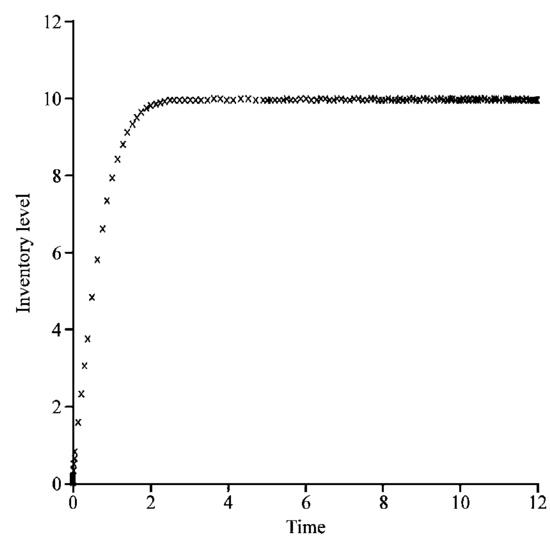

Fig. 2: The inventory goal level with time when it is fixed as $\hat{x}(t)$ 10. Optimal inventory level of the product with time t. Source: Author compution

However, in the subsections we present the model to measure the performance using different demand patterns. The production level with time t given $\hat{u}(t)$ from the Eq. 6 considering the mentioned above different demand rates

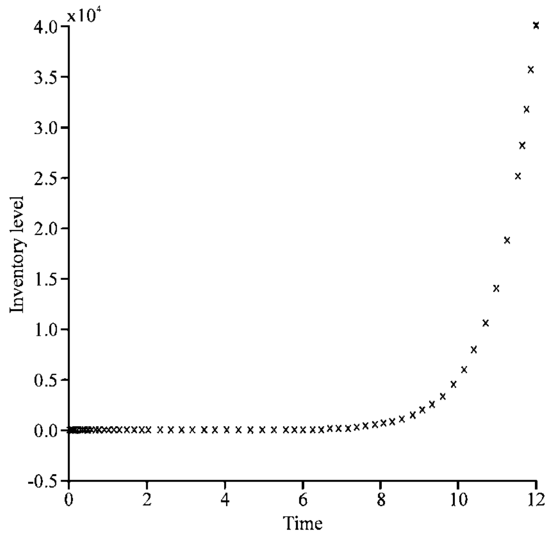

Fig. 3: The inventory level x(t) in-terms of the second-order differential equation. Source: Author compution

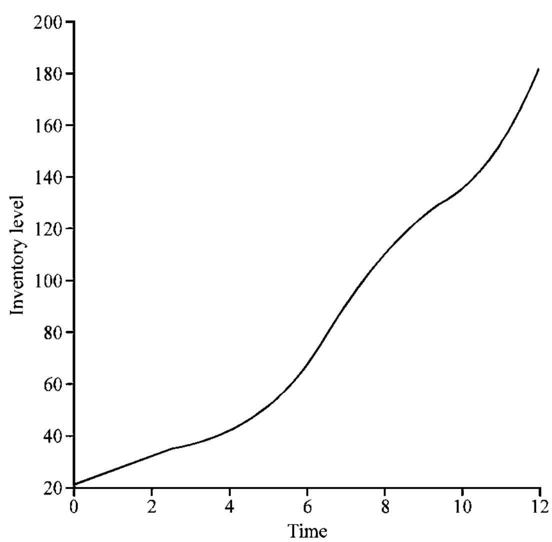

Fig. 4: Optimal production policy for constant demand rate with time. Source: Author compution

and keeping all other parameters unchanged yielded the figures represented by the Fig. 4-6. These Fig. 4-6 the slight variations of the optimal production level with time with changing the shape of the demand functions. It is observed that the optimal production rates are not very sensitive to changes in the demand functions in case of generalized Pareto distribution.

Constant demand function: In this subsection, we present the model with constant demand function. Substituting

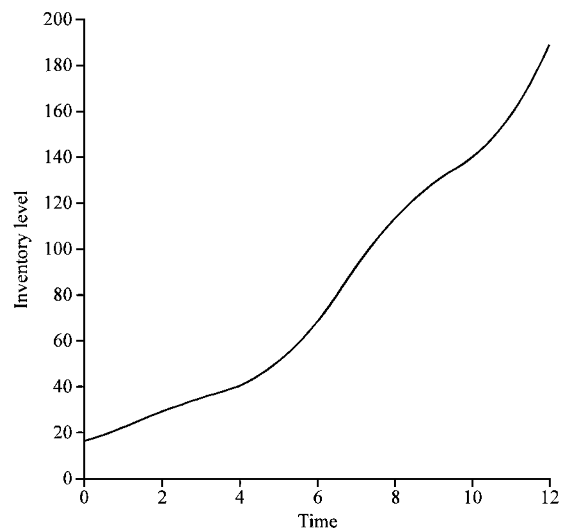

Fig. 5: The Optimal production policy for linear demand rate with time. Source: Author compution

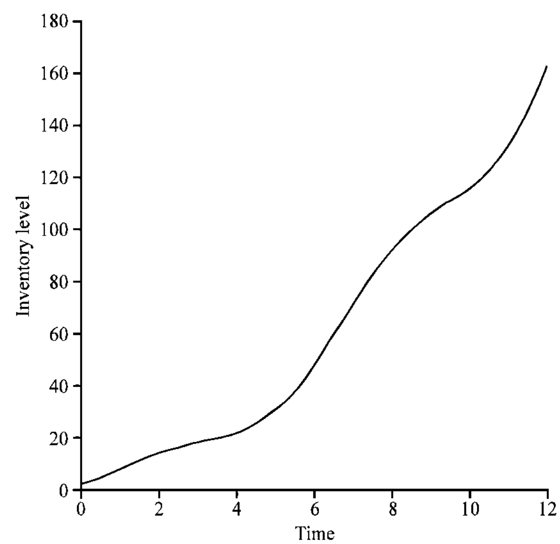

Fig. 6: The Optimal production policy for sinusoidal demand rate with time. Source: Author compution

$y_1(t) = y_1 = 20$ instead of y(t) in the controlled system (4) we have:

$$\frac{dx_1(t)}{dt} = u_1(t) - y_1(t) - \frac{1}{\sigma(1+\xi t)} x_1(t), \quad 0 \le t \le T, \quad x(T) = 0$$

from which the production goal rate $\hat{u}(t)$ can be computed (assuming x(0) = x) as:

$$\hat{u}_1(t) = y_1(t) + \frac{1}{\sigma(1+\xi t)} \hat{x}_1(t)$$

displayed by Fig. 4.

Linear demand function: In this subsection, we present the model with linear demand function. Substituting linear $y_2(t) = t+15$ instead of y(t) in the controlled system (4) we have:

$$\frac{dx_2(t)}{dt} = u_2(t) - y_2(t) - \frac{1}{\sigma(1+\xi t)} x_2(t), \quad 0 \le t \le T, \quad x(T) = 0$$

from which the production goal rate $\hat{u}(t)$ can be computed (assuming x(0) = x) as:

$$\hat{u}_2(t) = y_2(t) + \frac{1}{\sigma(1+\xi t)} \hat{x}_2(t)$$

displayed by Fig. 5.

Sinusoidal demand function: In this subsection, we present the model with sinusoidal demand function. Substituting $y_2(t) = 1+\sin(t)$ instead of y(t) in the controlled system (4) we have:

$$\frac{dx_3(t)}{dt} = u_3(t) - y_3(t) - \frac{1}{\sigma(1+\xi t)} x_3(t), \quad 0 \le t \le T, \quad x(T) = 0$$

from which the production goal rate $\hat{u}(t)$ can be computed (assuming x(0) = x) as:

$$\hat{u}_3(t) = y_3(t) + \frac{1}{\sigma(1+\xi t)} \hat{x}_3(t)$$

displayed by Fig. 6.

CONCLUSION

In this study, we developed an optimal control model in inventory-production system with generalized Pareto distribution deteriorating items. We derived the explicit solution of the optimal control models of an inventory-production system under a continuous review-policy using Pontryagin maximum principle. However, we gave numerical illustrative examples for this optimal control of a production-inventory system with Pareto distribution deteriorating items.

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