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Semigroup of EEG Signals during Epileptic Seizure

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Abstract: Fuzzy Topographic Topological Mapping (FTTM) is a mathematical model for solving neuromagnetic inverse problem where FTTM is a set consisting of elements with four components and three algorithms which link between the four components. In this study, we show that the first component of FTTM, namely magnetic contour plane which contains electroencephalography signals during epileptic seizure can be viewed as a semigroup of square matrices under matrix multiplication.

Key words: Electroencephalography, semigroup

INTRODUCTION

Electroencephalography (EEG) is a recording of electrical activity originating from the brain. It plays an important diagnostic role in epilepsy and provides supporting evidence of a seizure disorder as well as assisting with classification of seizures and epilepsy syndromes. The EEG had been used extensively to characterize the abnormal of brain activity. It is recorded on the surface of the scalp using electrodes, thus the signal is retrievable non-invasively. One of the major roles of EEG is as an aid to diagnose epilepsy.

The first serious attempt at seizure prediction was made by Viglione and Walsh (1975). An experiment based on seven seizures from five patients yielded 90% average correct separation between preseizure and non preseizure epochs of EEG in the training set. Initially, the system was not tested on data that had not been used in training. Further development of the project led to a patent for an electronic warning device for epilepsy. In 1972, The Terminal Man, a novel about an implanted brain-stimulating device to predict and stop seizures was published.

Two other groups of investigators submitted patents on systems to control epileptic seizures before onset in the 1970s, one using EEG features to trigger a warning to the patient and the other triggering a sustained biofeedback signal to abort seizures. Work on seizure prediction in the late 1970s and early 1980s consisted mainly of studies of visible features in the EEG, such as epileptic spikes and their relation to seizures (Lange *et al*., 1983). The discovery that abnormal activity in the epileptic and normal lobes became correlated about 20 min before seizure onset was corroborated by non linear techniques almost 15 years later.

Milton *et al*. (1987) postulated that the timing between seizures in a given patient occurred in a predictable pattern. Though they could not verify this idea, others later found varying degrees of predictability in temporal seizure patterns in human beings and animal models of epilepsy (Iasemidis *et al*., 1994).

The late 1980s and 1990s saw the application of nonlinear dynamics as a technique for predicting seizures. Transient drops in the principle Lyapunov exponent (PLE) were described by Iasemidis and colleagues as a route to seizures in temporal-lobe epilepsy (Iasemidis *et al*., 1990). In this study, the investigators proposed that the EEG became progressively less chaotic as seizures approached. This group later proposed that pre-ictal entrainment of the PLE in a critical mass of brain is necessary before seizure onset can occur. In 1994, a research group led by Elger and Lehnertz from Bonn, Germany, introduced application of the correlation dimension, another non-linear measure, for use in predicting seizures (Lehnertz *et al*., 1999).

Geva and Kerem (1998) applied intelligent systems, using fuzzy clustering in seizure predictions to analyze recordings from rodents induced to have generalized convulsive seizures by exposure to hyperbaric oxygen. In that study, wavelets (a way of identifying portions of the EEG with certain temporal and frequency characteristics) were used to calculate energy in the EEG signal. The investigators found a reliable increase in wavelet-derived energy an average of 4 min before electrical and clinical

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seizure onset of generalized seizures in two channels of EEG obtained from each of 25 rats.

In 1998, a group of investigators led by Baulac and Varela from the Hôpital de la Salpêtrière, in Paris, published evidence of seizure anticipation in preseizure segments (total of 6•3 h of data) using a measure called correlation density. This group has expanded the methods and volume of test data using a method called dynamical similarity (Le Van Quyen *et al*., 2001).

Litt and Echauz (2002), applied intelligent systems techniques to seizure prediction. In that method, many quantitative features are extracted from the intracranial EEG, a subset is chosen that best enable seizure prediction in each individual patient and the features are focused in an attempt to predict optimally the probability of seizure onset in different time horizons (e.g., 10 min, 1 h, 1 day). They have also focused on analysis of standard electrophysiological measures associated with epilepsy and analysis of long-term recordings. They recently described a cascade of electrophysiological events, which appeared to take place over hours, leading to electrical seizure onset. Some of these changes include bursts of long term energy related to epileptiform activity and slowing, spatially-limited subclinical seizures and accumulation of energy in an increasing volume of tissue that leads to seizure onset (Litt *et al*., 2001).

During the past few years, seizure prediction work has branched out. There is awareness that single quantitative techniques are unlikely to predict seizures in all patients. New groups are contributing promising algorithms and processing tools (Protopopescu *et al*., 2001). The last few years have also kindled an interest in methods for predicting seizures from other physiological or non-physiological variables, though most are in early stages of development.

FUZZY TOPOGRAPHIC TOPOLOGICAL MAPPING

Fuzzy Topographic Topological Mapping (FTTM) is a novel method for solving neuromagnetic inverse problem to determine the current source, i.e., epileptic foci. FTTM Version 1 has been developed to present a 3-D view of an unbounded single current source (Ahmad *et al*., 2008; Ahmad, 1993; Li Yun and Ahmad, 2003) in one angle observation (upper of a head model). It consists of three algorithms, which link between four components of the model as shown in Fig. 1.

The four components are Magnetic Contour Plane (MC), Base Magnetic Plane (BM), Fuzzy Magnetic Field (FM) and Topographic Magnetic Field (TM) (Fig. 1). The MC is actually a magnetic field on a plane above a current

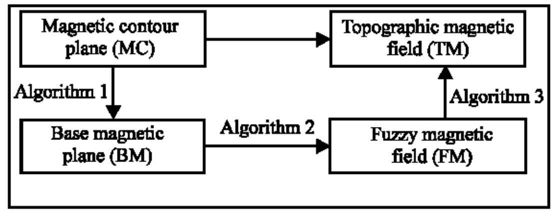

Fig. 1: Fuzzy topographic topological mapping (version1)

source with z = 0. The plane is lowered down to BM, which is a plane of the current source with z = -h. Then the entire BM is fuzzified into a fuzzy environment (FM), where, all the magnetic field readings are fuzzified. The final process is defuzzification of the fuzzified data to obtain a 3-D view of the current source (TM). FTTM Version 2 is another example of FTTM for more information see for example (Rahman, 2006).

MAGNETIC CONTOUR PLANE CONTAINS EEG SIGNALS

Zakaria and Ahmad (2007) has developed a new method for mapping high dimensional signal, namely EEG into a low dimensional space (MC). The whole processes of this novel model consisted three main parts. The first part was flattening the EEG where the transformation of three dimensional space into two dimensional space that involved location of sensor in patients head with EEG signal. The second part is the EEG signal was then processed by using Fuzzy c-Means clustering. The last part was to find the optimal number of cluster by using cluster validity analysis.

Zakaria's EEG coordinate system (Fig. 2a) is defined as:

$$C_{EEG} = \{((x, y, z), e_p): x, y, z, e_p \in R \text{ and } x^2 + y^2 + z^2 = r^2\}$$

where, r is the radius of a patient head. She modeled the human's head as a sphere.

Furthermore, the mapping of C_{EEG} to a plane (MC) is defined as follows:

S_t: $C_{EEG} \rightarrow MC$ (Fig. 2b) such that:

$$S_t\ ((x, y, z), e_p) = \left(\frac{rx + iry}{r+z}, e_p\right) = \left(\frac{rx}{r+z}, \frac{ry}{r+z}\right)_{e_p(x,y,z)}$$

Both C_{EEG} and MC were designed and proved by Ahmad (1993) and Li Yun *et al*. (2003) as 2-manifolds. Zakaria and Ahmad (2007) also had shown that S_t is a one to one function as well as being conformal. Details of proofs contain by Zakaria and Ahmad (2007).

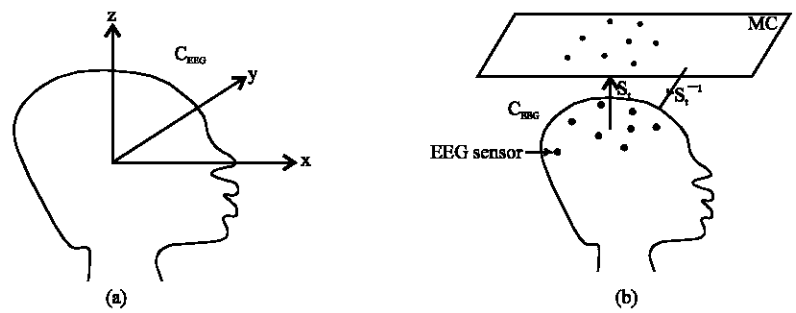

Fig. 2: (a) EEG coordinate system and (b) EEG projection

With the fact that S_t is conformal, therefore the mapping can preserve information, in particular angle and orientation of the surface and EEG signal recorded from the surface of high dimensional into a low dimensional spaces; i.e. mapping EEG signal into a plane.

Then, Zakaria and Ahmad (2007) implemented this technique followed by clustering on the real time EEG data obtained from patients who suffer from epileptic seizure. The signals were digitized at 256 samples sec^{-1} using Nicolet One EEG software. The average potential difference was calculated from the 256 samples of raw data at every second. Similarly to the position of electrodes, the EEG signal was also preserved during this new method. Subsequently, every single second of the particular average potential difference was stored into a file which contains the position of electrode on MC plane.

We rewrite the files in terms of square matrices. Therefore, every single second of the particular average potential difference was stored into a square matrix which contains the position of electrode on MC plane. Thus Magnetic Contour Plane became a set of (n×n) square matrices defined as:

$$MC_n = \left\{ \left[\beta_{ij}(z)_t \right]_{n\times n} : i, j \in \mathbb{Z}^+, \beta_{ij}(z)_t \in \mathbb{R} \right\}$$

where, $\beta_{ij}(z)_t$ is a potential difference reading of EEG signals from a particular ij sensor at time t.

SEMIGROUP OF MC_n

Here, we are going to show that the nonempty set of square matrices (EEG signals) satisfies all the axioms of a semigroup given (Whitelaw, 1978) under matrix multiplication. In other words, we are going to show that:

- $MC_n = \left\{ \left[\beta_{ij}(z)_t \right]_{n\times n} : i, j \in \mathbb{Z}^+, \beta_{ij}(z)_t \in \mathbb{R} \right\}$ is closed with respect to matrix multiplication and
- Matrix multiplication is associative on MC_n

Theorem 1: The set of (n×n) square matrices MC_n is a semigroup under matrix multiplication.

Proof: Firstly, let us show that MC_n is closed with respect to matrix multiplication. We pick:

$$A = \begin{pmatrix} \beta1_{1,1} & \cdots & \beta1_{1,n} \\ \vdots & \ddots & \vdots \\ \beta1_{n,1} & \cdots & \beta1_{n,n} \end{pmatrix} \text{ and } B = \begin{pmatrix} \beta2_{1,1} & \cdots & \beta2_{1,n} \\ \vdots & \ddots & \vdots \\ \beta2_{n,1} & \cdots & \beta2_{n,n} \end{pmatrix} \in MC_n. \text{ Then}$$

$$AB = \begin{pmatrix} \beta1_{1,1} & \cdots & \beta1_{1,n} \\ \vdots & \ddots & \vdots \\ \beta1_{n,1} & \cdots & \beta1_{n,n} \end{pmatrix} \begin{pmatrix} \beta2_{1,1} & \cdots & \beta2_{1,n} \\ \vdots & \ddots & \vdots \\ \beta2_{n,1} & \cdots & \beta2_{n,n} \end{pmatrix}$$

$$= \left(\sum_{j=1}^{n} \beta1_{i,j}\beta2_{j,k} \right)_{(m)}$$

Notice we go across the i-th row of the first matrix and down the k-th column of the second matrix to obtain the entry in position (i, k).

$$\begin{array}{c} \\ i \end{array} \begin{pmatrix} & \vdots & \\ \beta1_{i,1} & \beta1_{i,2} \cdots & \beta1_{i,n} \\ & \vdots & \end{pmatrix} \begin{array}{c} k \\ \begin{pmatrix} & \beta2_{1,k} & \\ \cdots & \vdots & \cdots \\ & \beta2_{n,k} & \end{pmatrix} \end{array} = \begin{array}{c} \\ i \end{array} \begin{array}{c} k \\ \begin{pmatrix} & \vdots & \\ \cdots & \sum_{j=1}^{n} \beta1_{i,j}\beta2_{j,k} & \cdots \\ & \vdots & \end{pmatrix} \end{array}$$

Now $\beta1_{i,j}$, $\beta2_{j,k} \in \mathbb{R}$ for a particular time $t \in \mathbb{R}^+$ and without loss of generality, $\beta1_{i,j}\ \beta2_{j,k} \in \mathbb{R}$ for some time $t \in \mathbb{R}^+$, thus:

$$\sum_{j=1}^{n} \beta1_{i,j}\beta2_{j,k} \in \mathbb{R}$$

Since A, $B \in MC_n$ are arbitrary, therefore $AB \in MC_n$ and hence MC_n is closed with respect to matrix multiplication.

Secondly, let us show that matrix multiplication on MC_n is associative. Pick:

$A=\left[\beta 1_{i,j}\right]_{(nn)}, B=\left[\beta 2_{j,k}\right]_{(nn)}, C=\left[\beta 3_{k,l}\right]_{(nn)}$. Then

$$AB=\left(\sum_{j=1}^{n}\beta 1_{i,j}\beta 2_{j,k}\right)_{(nn)}$$

$$(AB)C=\left(\sum_{k=1}^{n}\left(\sum_{j=1}^{n}\beta 1_{i,j}\beta 2_{j,k}\right)\beta 3_{k,l}\right)_{(nn)}=\left(\sum_{k=1}^{n}\sum_{j=1}^{n}\beta 1_{i,j}\beta 2_{j,k}\beta 3_{k,l}\right)_{(nn)}$$

$$BC=\left(\sum_{k=1}^{n}\beta 2_{j,k}\beta 3_{k,l}\right)_{(nn)}$$

$$A(BC)=\left(\sum_{j=1}^{n}\beta 1_{i,j}\left(\sum_{k=1}^{n}\beta 2_{j,k}\beta 3_{k,l}\right)\right)_{(nn)}=\left(\sum_{j=1}^{n}\sum_{k=1}^{n}\beta 1_{i,j}\beta 2_{j,k}\beta 3_{k,l}\right)_{(nn)}.$$

Since $\sum_{k=1}^{n}\sum_{j=1}^{n}\beta 1_{i,j}\beta 2_{j,k}\beta 3_{k,l}=\sum_{j=1}^{n}\sum_{k=1}^{n}\beta 1_{i,j}\beta 2_{j,k}\beta 3_{k,l}$,

we have (AB)C = A (BC). The associativity of MC_n reveals that historical event is preserved in time (Nehaniv and Dautenhahn, 1998). It means that the property of time is actually embedded in MC_n.

We have shown that:

- $MC_n=\left\{\left[\beta_{ij}(z)_t\right]_{n\times n}: i,j\in\mathbb{Z}^+, \beta_{ij}(z)_t\in\mathbb{R}\right\}$ is closed with respect to matrix multiplication and
- Matrix multiplication on Mc_n is associative

In other words, magnetic contour plane (MC) is a semigroup of square matrices under matrix multiplication.

CONCLUSION

In this study, we have shown that the EEG signals during Epileptic Seizure can be viewed as a semigroup of square matrices under matrix multiplication. This work will enable us to proceed further in identifying characteristics of EEG signals during epileptic seizure.

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