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Efficient Capacitance Matrix Computation of Large Conducting Bodies using the Characteristic Basis Function Method

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Abstract: Capacitance calculation of arbitrary-shaped conducting bodies is an important step in the prediction of electrostatic discharge and the quasi static analysis of various systems. In this study, the capacitances of arbitrary-shaped conducting surfaces are evaluated based on the Characteristic Basis Functions method (CBF) in conjunction with the Integral Equation Method (IEM). In this method, the surface of the conducting body is divided into a small number of blocks and the IEM with pulse basis function and point matching technique is employed to calculate the charge distribution on the surface of each block. The charge distributions on each block constitute the high-level basis functions of characteristic basis function method. The IEM is then applied to the conductors using the obtained CBF as a basis function for the blocks. The use of CBF results in a highly accurate solutions with significant savings in computation time and memory requirements.

Key words: Capacitance calculation, charge distribution, electrostatic, numerical method, Galerkin method

INTRODUCTION

There has been remarkable interest in the estimations of the charge distributions and the capacitance evaluation of different conducting structures such as rectangular plates, square plates, circular and annular discs, etc. located in free space. Capacitance matrix calculation represents an important step in the design and analysis of various structures such as very large integration system and the cylindrical capacitive sensor (Ardon *et al*., 2009; Al-Sabayleh, 2008; Azimi and Golnabi, 2009). Also, the electrostatic discharge in spacecraft model can be predicted using the equivalent circuit model (Ghosha and Chakrabarty, 2008).

The classical Integral Equation Method (IEM) has been widely used for the charge distribution and capacitance calculations of large conducting bodies due to its efficiency and simplicity. It is in well known that numerical methods based on the integral equation like Method of Moment, Boundary Element Method, Charge Simulation etc. can give accurate and efficient solutions whenever they can be applied. Their main advantage resides in neglecting discretization of the regions surrounding the active part of the problem keeping relatively compact dimensions of the numerical problem. However, the classical IEM using subsectional basis functions becomes highly inefficient for the analysis of large or complex conducting bodies (Ouda and Sebak, 1995). This is because the size of the associated matrix grows very rapidly as the shape of conductor becomes more complex, or a fine mesh is used to model a complex structure to guarantee a good solution accuracy. The direct solution requires memory storage of order N^2 and computational time of order N^3 for N number of unknowns (Harrington, 1985).

Various attempts had been made to reduce the memory storage and computational time requirements. However, these attempts are usually made for special geometries (Ghosha and Chakrabarty, 2008; Wu and Wu, 1988; Uchida *et al*., 2005) or use a complicated iterative solution (Nabors and White, 1991). Ouda and Sebak (1995) presented an approximate two stage method for the capacitance calculation; however the error is significant for conductors of close proximity with very high mutual couplings. In first stage the structure is divided into sections and IEM is used to obtain the charge distributions. The charge distribution obtained is then used in the second stage to calculate the change in the total charge stored on each conductor in the environment of the whole system. In this method, each conductor is taken as one element in the second stage which causes high errors for the adjacent sides of each conductor. Liu *et al*. (2000) proposed a technique based on the concept of Measured Equation of Invariance (MEI), to thin the MoM matrix numerically but this method produces high percentage of errors for some conductor configurations.

In this study, capacitances of conducting objects in free space are evaluated using the IEM in conjunction with the CBF method. The surface of the conducting body is divided into a small number of blocks and the IEM with pulse basis function and point matching technique is employed to calculate the charge distribution on the surface of each block. The rectangular patch shape is chosen for discretization because of its ability to conform easily to any geometrical surface or shape and at the same time maintaining the simplicity of approach compared to the triangular patch modeling. The obtained charge distributions on each block constitute the high-level basis functions of characteristic basis function method. The IEM is applied to the conductors using the obtained CBF method as a basis functions for the blocks. The use of CBF method results in a highly accurate solutions with significant savings in computation time and memory requirements.

The capacitance calculation of large-scale structure using the CBF method differs from other methods mainly since it includes the mutual coupling effects directly by using a new type of high-level basis function, referred to herein as primary and secondary CBFs, which are used to represent the unknown induced charges on the blocks and solved via the Galerkin method rather than using iterative refinements.

The accuracy of the CBF method and its advantages are illustrated by several examples and the computation times as well as the memory requirements are compared to those of conventional direct computation.

FORMULATIONS

The integral equation method: The potential of a perfectly conducting surface charged to a potential V is given by the Fredholm integral equation of the first kind for the unknown surface charge density σ.

$$V(r) = \frac{1}{4\pi\varepsilon_0} \int_{S_c} \frac{\sigma(r')}{|r-r'|} ds', \quad r \in S_c \qquad (1)$$

where, r and r' are the position vectors corresponding to observation and charge source points, respectively, ds' is an element of surface S_c and ε_0 is free space permittivity.

The charge distribution on general conductor geometries can be obtained by solving Eq. 1 using numerical method, where the arbitrary-shaped conductors are approximated by N planar rectangular patches. The classical IEM starts by approximating the unknown charges σ as a linear combination of a set of linearly independent expansion W_j (r) with the weights A_j:

$$\sigma(r) = \sum_{j=1}^{N} A_j W_j(r), \quad \text{where,} \quad W_j(r) \equiv \begin{cases} 1 & \text{r\in patch j;} \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

Applying the point matching technique produces a linear system for the unknowns A_j:

$$V(r_i) = \frac{1}{4\pi\varepsilon_0} \sum_{j=1}^{N} \frac{q_j}{a_j} \int_{S_c} \frac{1}{|r-r'|} ds', \quad r \in S_c \qquad (3)$$

where, A_j represents a constant charge density on the jth patch such that $A_j = q_j/a_j$, q_j and a_j are the charge and area of the jth patch, respectively. Equation 3 can also be written in matrix notation:

$$[P][q]=[V] \qquad (4)$$

where, [P] is an N×N matrix and [q] and [V] are column vectors of length N. The dense linear system of Eq. 4 can be solved for the surface charge distribution from a given set of patch potentials. To compute the jth column of the capacitance matrix, Eq. 4 must be solved for q_i given V vector with v_k = 1 if the kth patch belong to the jth conductor, else v_k = 0. The jth term of the capacitance matrix is computed by summing all the charges on the i^{th} conductor. Hence, the capacitance, C, of the conductor is obtained from the following equation:

$$C_{ij} = \sum_{i=1}^{N} q_{ik}, \quad k \in i \qquad (5)$$

Thus, using the classical IEM, an N×N system of equation must be solved to compute the capacitance matrix. The storage and computation time for this system are proportional to N^2 and N^3, respectively. Hence, attempts at using classical IEM to solve for complicated structures are usually abandoned.

The characteristic basis function method: The CBF method was proposed for large-scale periodic microstrip antenna arrays (Wan *et al*., 2005) and it is considered a general approach for dealing with the matrix equations of the form given in Eq. 4. For the system discretization, the conductors are divided into surfaces and each service is divided into blocks. Two types of CBFs are defined for each block, namely, the primary and the secondary basis functions. The primary CBFs are solutions for the charge distribution in the isolated blocks, whereas the secondary CBFs account for the field coupling between the blocks. Hence, the CBF method commences by segmenting the original surface into smaller blocks (Fig. 1) for M = 16.

Each block is extended by Δ in all directions and the extended block is discritized to N_i^e number of patches to construct a set of basis functions that are characteristics of that particular blocks. The IEM as described in the previous section is then employed to generating the CBFs $q_i^{(i)}$ for the block i by solving Eq. 6:

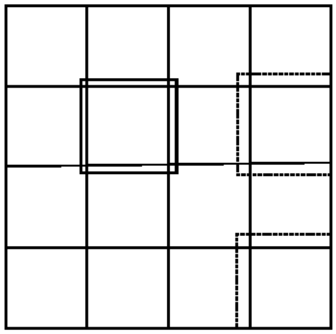

Fig. 1: Rectangular surface divided into blocks

$$\left[P_e^{(i)}\right]\left[q_i^{(i)}\right]=\left[V^{(i)}\right],\quad i=1,2,\cdots M \tag{6}$$

Notice that the matrix [P] size is $N_i^e \times N_i^e$ which is very small compare to the original problem. This process is repeated to generate the primary CBF method for all blocks.

The secondary CBFs that account for the mutual coupling between various blocks are generated using Eq. 4, but with different excitations. For each block, there are M-1 secondary bases, which are obtained by solving the following equation:

$$\left[P_e^{(i)}\right]\left[q_k^{(i)}\right]=\left[P^{(i,k)}q_e^{(i)}\right],\quad k=1,2,\cdots i-1,i+1,\cdots,M \tag{7}$$

where, $q_k^{(i)}$ is the kth secondary basis functions for block t and $P^{(i,k)}$ is the excitation vector resulting from the mutual coupling between block i and block k. Even though the original blocks do not overlap with each other, Eq. 7 deals with an extended block and they do overlap. In view of this, two distinct cases are identified:

- There is no overlap (no common unknowns) between the extended block i and block k. In this case, the matrix $[P^{(i,k)}]$ size is $N_i^e \times N_k$
- In the second case, the extended block i shares some of the unknowns with the block k and we let $N_{i,k}^{(c)}$ be that number. We identify and eliminate these source locations from $[P^{(i,k)}]$ and $q_k^{(k)}$ thus making them $N_i^e \times (N_k - N_{i,k}^{(c)})$ and $(N_k - N_{i,k}^{(c)}) \times 1$, respectively. Note that the size of the forcing vector $V^{(i,k)}\, q_k^{(k)}$ remains N_i^e, in this case also

Then, the two CBFs types are employed as high-level basis and testing functions to generate a reduced matrix via the use of the Galerkin method. The solution to the entire problem is then expressed as a linear combination of the CBFs as follows:

$$[q]_{M^2 x1}=\sum_{k=1}^{M^2}\alpha_k[q_k^c] \tag{8}$$

where, q_k^c are the kth CBFs and are the unknown expansion coefficients of the kth block to be determined by using the reduced matrix. By inserting Eq. 8 into Eq. 4 and using the transpose of $[q^c]$ as the testing function, we obtain:

$$\left[q^c\right]^t[P]\left[q^c\right][\alpha]=\left[q^c\right]^t[V] \tag{9}$$

Or,

$$\left[P^c\right][\alpha]=\left[V^c\right] \tag{10}$$

where, $[q^c]$ is the matrix form of CBFs of dimension $N\times M^2$, given by:

$$\left[q^c\right]=\sum_{k=1}^{M}\begin{bmatrix} q_{k,1}\\ 0\\ \vdots\\ 0\end{bmatrix}+\sum_{k=1}^{M}\begin{bmatrix} 0\\ q_{k,2}\\ \vdots\\ 0\end{bmatrix}+\cdots+\sum_{k=1}^{M}\begin{bmatrix} 0\\ 0\\ \vdots\\ q_{k,M}\end{bmatrix} \tag{11}$$

where, $q_{k,M}$ is the kth CBFs of block, for k = 1, 2, …., M and $[\alpha]$ is the coefficient vector of dimension $M^2\times 1$ and $[P^C]$ is $M^2\times M^2$ matrix given by:

$$[P]=\begin{bmatrix} \langle q_{11}^t\ \ P_{11}\ \ q_{11}\rangle & \langle q_{11}^t\ \ P_{12}\ \ q_{22}\rangle & \cdots & \langle q_{11}^t\ \ P_{1M}\ \ q_{MM}\rangle\\ \langle q_{22}^t\ \ P_{21}\ \ q_{11}\rangle & \langle q_{22}^t\ \ P_{22}\ \ q_{22}\rangle & \cdots & \langle q_{11}^t\ \ P_{2M}\ \ q_{MM}\rangle\\ \vdots & \vdots & \ddots & \vdots\\ \langle q_{MM}^t\ \ P_{M1}\ \ q_{11}\rangle & \langle q_{MM}^t\ \ P_{M2}\ \ q_{22}\rangle & \cdots & \langle q_{MM}^t\ \ P_{MM}\ \ q_{MM}\rangle \end{bmatrix} \tag{12}$$

where, P_{ij} is the coupling matrix linking the original (unextended) blocks i and k. Note that each of the inner product entries in the above matrix results in a sub-matrix of size M×M.

The system of matrix Eq. 10 is typically quite small and thus can be solved directly and yet does not sacrifice the accuracy of the solution in the process. In addition, the use of CBF method does not result in a deterioration of the condition number of the matrix, as is often the case with other entire domain basis functions, which also serve to reduce the matrix size. Once the coefficients of the reduced matrix equation have been obtained, the solution for the original problem is readily recovered from the equation:

$$[q]=\left[q^c\right][\alpha] \tag{13}$$

The capacitance matrix can be easily computed using Eq. 5 once this solution is constructed.

NUMERICAL RESULTS AND DISCUSSION

Computer programs based on the CBF method and the classical IEM had been developed to determine the charge distribution and hence capacitance of general arbitrary shaped conducting structures. The programs were developed and tested in the electrical Engineering Department at the Islamic university of Gaza in the period from Dec. 2009 to March 2010. The capacitance matrix of a simple parallel plate, which is the most popular capacitor employed in the Electrical Engineering field, (Fig. 2) is obtained using the classical IEM and the CBF method. The capacitor consists of two square plates of side length equals 1 m and separated by a distance of 0.1 m. For the CBF method solution, each plate is divided into 4 blocks and each block is discretized to 16 patches. The CBF method results are compared with those obtained using the IEM where each plate is divided into 200 patches. An excellent agreement obtained for the capacitance matrix calculated using the classical IEM and the CBF method (Table 1). There is a significant storage requirements reduction using the CBF method, Matrix size is 64×64, in comparison to that of the classical IEM, matrix size is 400×400. Furthermore, the computational time using the CBF method is less than one sixth of that using the classical IEM.

The capacitances of annular circular disc, trapezoidal plate and annular triangular plate (Fig. 3), are obtained using the classical IEM, the CBF method and the method of rectangular subareas (Ghosha and Chakrabarty, 2008). There are excellent agreements between the capacitances obtained (Table 2) using the classical IEM and the CBF method where the error is within 1%. However, there up to 20% error in the capacitances obtained using the method of rectangular subareas in comparison to those obtained using the IEM.

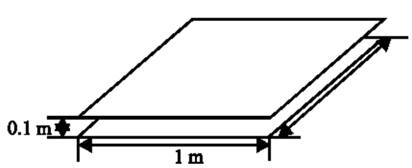

Fig. 2: Two parallel plate capacitor

Table 1: The capacitance matrix (pF) of the parallel plate capacitor

IEM	CBF method
$\begin{bmatrix} 127.5 & -105.5 \\ -105.5 & 127.5 \end{bmatrix}$	$\begin{bmatrix} 127.4 & -105.1 \\ -105.1 & 127.4 \end{bmatrix}$

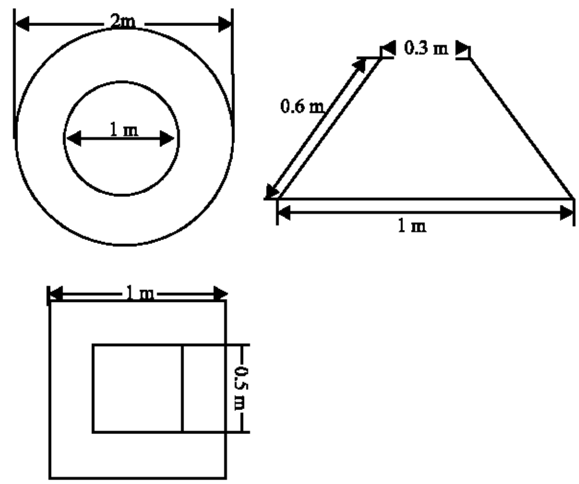

Fig. 3: Annular square plate, trapezoidal plate and Annular disk shapes

Table 2: The capacitance of annular square plate, trapezoidal plate and annular disk

Shapes	C (pF)	No. of patches
Annular circular disc		
Classical IEM	38.71	200
CBFM	38.53	9 blocks
Rectangular subareas	33.86	4
Trapezoidal plate		
Classical IEM	24.47	200
CBFM	24.35	6 blocks
Rectangular subareas	20.41	24
Annular triangular plate		
Classical IEM	39.69	200
CBFM	39.59	6 blocks
Rectangular subareas	36.66	24

CONCLUSIONS

The capacitances of arbitrary shaped conducting bodies are evaluated based on the Characteristic Basis Functions method in conjunction with the Integral equation method. For the CFB method solution, the surface of the conducting body is divided into a small number of blocks. The charge distribution on each block is obtained using the IEM with pulse basis function and point matching technique. The charge distributions constitute the high-level basis functions of CFB method which are employed for the capacitance matrix calculations. An excellent agreement obtained for the capacitance matrix calculated using the classical IEM and the CBF methods. However there up to 20% error in the capacitances obtained using the method of rectangular subareas in comparison to those obtained using the classical IEM. The accuracy of the method of rectangular subareas can be improved by increasing the number of sections which leads to significant increase of memory storage and computational time requirements. Furthermore, memory storage and computational time requirements of the CFB method is up to an order of magnate less than those of the classical IEM.

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