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## Wavelet-based Pre-filtering for Low Cost Inertial Sensors

<sup>1</sup>A.M. Hasan, <sup>1</sup>K. Samsudin, <sup>2</sup>A.R. Ramli and <sup>3</sup>R.S. Azmir

Computer Systems Research Group, Department of Computer and Communication Systems,  
Universiti Putra Malaysia, 43400 UPM-Serdang, Malaysia

<sup>2</sup>Intelligent Systems and Robotics Laboratory, Institute of Advanced Technology,  
Universiti Putra Malaysia, 43400, UPM-Serdang, Malaysia

<sup>3</sup>Wireless and Photonic Networks (WiPNET) Research Center of Excellence,  
Universiti Putra Malaysia, 43400, UPM-Serdang, Malaysia

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**Abstract:** This study proposed to de-noise the IMU signal by effectively band-limiting the signal at the output of each inertial measurement sensor prior to its mechanization and further processing by the Strapdown INS (SDINS) algorithm. Wavelet Multi-Resolution Algorithm (WMRA) is utilized to improve the performance of the inertial sensors by removing their short term noise. The aim of this study is to reveal how WMRA is utilized to improve the performance of the inertial measurement unit systems and investigate how wavelet analysis can be used to analyse and de-noise output of the low-cost inertial sensors. The proposed multi-level decomposition was applied to real accelerometer and gyroscopes data obtained from MEMS IMU (MotionPak II). Different level of decomposition and thresholding filter was evaluated to obtain optimal results. Analysis of the results demonstrate reducing the INS position and velocity error for the specific IMU.

**Key words:** Vehicular navigation, strapdown inertial navigation system, inertial measurement unit, wavelet multi-resolution algorithm, global positioning system

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### INTRODUCTION

The Global Positioning System (GPS) has been at the head of current navigation technologies. However, the surveyors are still facing problems in places where the GPS signal gets lost due to different factors such as blockage by building, canopy and other natural and non-natural obstructions. Many of the studies are being carried out to address the issue of signal loss. Where some high sensitivity receivers can detect the reflected signals as well as direct signals but the accuracy degrades significantly if multipath is stronger than the direct signals (Syed *et al.*, 2006). However, Inertial Navigation System (INS) is one of the most popular mechanical navigation systems that can provide a navigation solution in case of GPS signal loss. It is worth to mention that the output given by the inertial sensors is in terms of accelerations and rotational velocity that need to be processed to get position and velocity information.

Inertial Navigation Systems (INSs) has been around since the mid of twentieth century and now gaining popularity due to technological advancements in micro-machined sensors that reduce the size of IMU as

well as the associated cost. The good thing about INS is its independent and jam proof navigation data, as compared to GPS that is dependent on satellite signal; however, INS accuracy degrades with respect to time making it a major drawback. Most of the errors in the INS are caused by sensor imperfections (instrumental errors); therefore, accuracy mostly depends on the type of sensors available. However, the cost of the INS is directly proportional to the accuracy, implying that high performance accurate sensors are still very expensive and limited to certain applications (Shaikh *et al.*, 2003; Miskam *et al.*, 2009).

An IMU is a black box housing inertial sensors that are mounted on three orthogonal axes. The combination of three accelerometers and three gyroscopes provide linear accelerations and angular velocity, respectively along three orthogonal axes. Both accelerometer and gyroscope operate on the inertial principles (Newton's Laws of Motion) that could be used to provide navigation solution (Mostafa, 2001). The measurements from the IMU are mathematically integrated to obtain position information and orientation (rotation about an axis). By tracking both the current linear accelerations and angular

velocity it is possible to determine the position of the body in fixed coordinate system (Mostafa, 2001; Schmidt and Barbour, 2001). The advantage of INS is that it could operate independently and provide reliable navigation data, as compared to GPS that is dependent on satellite signal. However, INS accuracy degrades with respect to time due to integration drift. Sensor imperfections coupled with instrumental errors also aggravate accuracy of the INS. The cost of high performance accurate sensors is still high and beyond the reach for certain civilian applications. Also, it is worth to mention that Tactical and navigation grade sensors are limited to commercial and military applications (Shaikh *et al.*, 2003; Skaloud and Schwarz, 1999).

The first INS was built and based on mechanical gyros with very complex and power consuming architecture. Later on strapdown solutions have been realized by using modern integrated electro-mechanical or electro-optical sensors (Shaikh *et al.*, 2003). These strapdown systems are mostly based on the MEMS (Micro Electro-Mechanical System) technology that is relatively inexpensive and compact. These cost-effective sensors, due to their short-term sustainability and opposite characteristics, are widely used in inertial navigation systems.

This study focuses on developing and implementing the strapdown INS algorithm by effectively band-limit the INS signal prior to its mechanization and further processing. The motivation behind this concept is schematically depicted in Fig. 1. Where Fig. 1a shows the main two types of errors in the inertial measurement unit are long and short term errors (Skaloud and Schwarz, 1999). Where multi-level decomposition is utilized to improve the performance of the inertial sensors

(gyroscopes and accelerometers) by removing their short term errors as shown in Fig. 1b. The separation of the high and low frequency inertial sensor noise components can be done by deionizing the inertial measurements before using them as input to the SDINS algorithm as done in this study. It must be mentioned that when using multi-level of decomposition the short term error was extensively reduced by wavelet deionizing (optimal low pass filtering) while the long term error still affect the INS performance for long time processing. In this study we can remove the effect of the short term error of the stand alone INS only, while the long term error can be removed by aiding the INS with another navigation devices such as GPS to eliminate or reduce the long-term error as shown in Fig. 1c.

### BACKGROUND ON INERTIAL NAVIGATION SYSTEMS

**Types of inertial navigation systems:** Inertial navigation can be classified into three basic categories:

**Geometric:** In this type, the navigation information was available in analog fashion directly from the gimbals angles. It is necessary to physically instrument two reference frames to provide this information and these two frames are an inertially non-rotating frame and a local navigation frame. In this kind of navigation system minimal computation capacity is required. At least five gimbals are necessary to provide the navigational quantities of interest, called latitude, longitude and vehicle roll, pitch and yaw (Lin, 1991).

**Semi-analytic:** Semi-analytic systems physically instrument only one reference frame, either an inertially

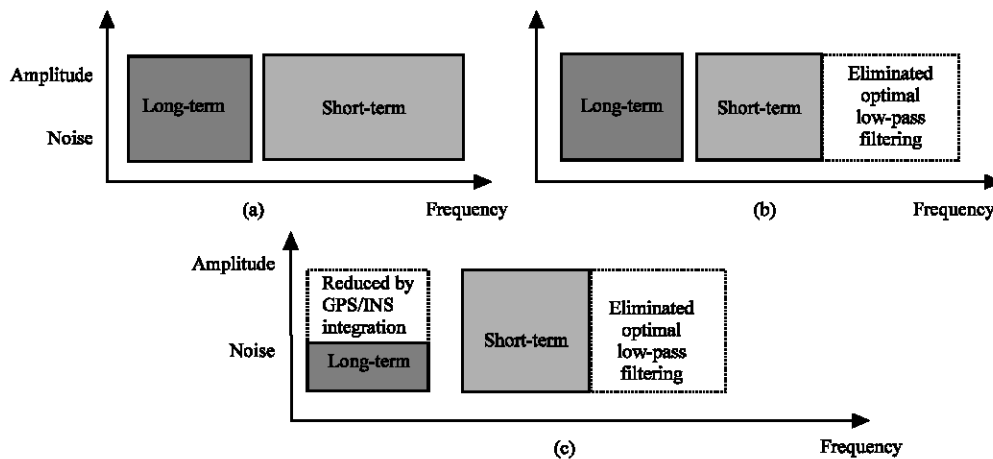


Fig. 1: A schematic plot of inertial noise in frequency domain: (a) before filtering, (b) after optimal low pass filtering only and (c) low-pass filtering with estimated INS/GPS error algorithm

non-rotating frame or a local navigational frame. Three gimbals are required to affect this coordinate navigational frame, but it is highly recommended to install four gimbals because there is a case of singularity in the first implementation for the computation of latitude and longitude being accomplished in a computer (Britting, 1971).

**Analytic:** Which is also called a strapdown inertial navigation system, which does not physically instrument a reference frame but rather use the gyroscopes outputs to calculate analytically the relative orientation between the system’s initial and present state (Lin, 1991).

It is important to make a distinction between the primary types of inertial navigation systems: Gimballed and Strapdown.

In inertial-platform gimbal mechanization, the gyroscopes mounted on a stable element measure angular rates and gimbal-drive systems use the angular rate information to null the angular motion sensed by the gyroscopes. In this manner, the gyroscopes and accelerometers on the stable element are inertially stabilized from the vehicle motion and the stable member physically represents an inertial reference frame. By double integrating the specific force taken from the accelerometers with a correction for gravity, position determination is possible (Bielas, 1994).

In strapdown inertial systems, sensors are mounted directly (or perhaps with vibration isolators) on the vehicle. Inertial sensors outputs now represent specific force and angular rate with respect to inertial space coordinatized in vehicle body axes. Therefore, to maintain an inertial reference frame, a computer-generated transformation matrix algorithm between body and inertial frame must be used to process the rate gyroscope outputs as the vehicle moves and its orientation changes. Then the accelerometer information must be transformed from the body frame to the inertial reference frame (Lin, 1991).

**Errors in INS:** Most of the error sources that distort the navigation solution are sensor errors or random disturbances. These are the residual errors exhibited by the installed gyros and accelerometers following calibration of the INS. The dominant error sources that affect the accuracy of the navigation solution obtained from INS such as alignment, scale factor, biasing, non-orthogonality and random noise as illustrated in Table 1 (Grejner-Brzezinska and Wang, 1998).

There are nine navigation errors caused by the accelerometers and gyroscopes. These errors in the

Table 1: Sensor generated errors in the INS (Grejner-Brzezinska and Wang, 1998)

Type of error	Description
Alignment errors	Roll, pitch and heading errors
Accelerometer bias or offset	A constant offset in the accelerometer output that changes randomly after each turn-on
Accelerometer scale factor error	Results in an acceleration error proportional to sensed acceleration
Nonorthogonality of gyros and accelerometers	The axes of accelerometer and gyro uncertainty and misalignment
Gyro drift or bias (due to temperature changes)	A constant gyro output without angular rate presence
Gyro scale factor error	Results in an angular rate error proportional to the sensed angular rate
Random noise	Random noise in measurement

Table 2: MotionPakII parameters specifications (Salychev *et al.*, 2000)

Performance	Rate Channels	Acceleration channels
Range	$\pm 100 \text{ deg sec}^{-1}$	5G
Bias	$< 2 \text{ deg sec}^{-1}$	$< 12.5 \text{ mG}$
Alignment to base resolution	$< 1$	$< 1$
	$< 14 \text{ deg h}^{-1}$	$< 10 \text{ G}$

accelerations and angular rates lead to steadily growing errors in position, velocity and attitude information. These navigation errors caused by the mathematical integration operation in the INS algorithm. The GPS navigation system can be used to aid the INS and prevent these time drift errors (Chiang *et al.*, 2008). In addition to these navigation errors the INS algorithm also suffers from acceleration and angular rates sensor reading inaccuracies caused by the earth gravity and rotation. These errors must be handled carefully especially in strapdown system rather than gimballed inertial sensors (Britting, 1971).

Knowledge of the error sources enables the system to cancel their effects as it navigates. In a strapdown system, however, only few of the sensor errors can be calibrated. Errors that cannot be calibrated will propagate into navigation errors when the system begins to navigate. These systems also require lengthy alignment time. If both of these necessities are not met, even the most accurate INS can become worthless (Noureldin *et al.*, 2004; El-Shiemy *et al.*, 2004). Figure 2 shows the effect of one-degree INS sensor errors on the position of the moving body noise, bias, scale factor, combined and initial condition accumulated error.

A raw IMU data collected from a low cost inertial sensor (MotionPakII) was used in this paper for analysis and de-noising the sensors outputs in order to improve the accuracy of the position and velocity components. Table 2 shows the specification of MotionPak II used in this study to provide real data for further manipulation and de-noising for both the accelerometer and gyroscope in terms of sensitivity range, bias, alignment and resolution obtained.

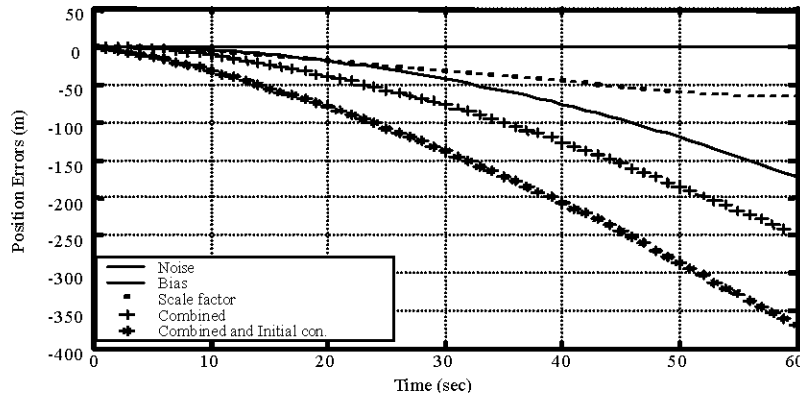


Fig. 2: Effect of one-degree INS sensor errors on the position of the moving body

**WAVELET MULTI-RESOLUTION ANALYSIS**

The concept of multi-resolution approximation of functions provides a powerful framework to understand wavelet decompositions. The basic idea is that of successive approximation, together with that of added details as one goes from one approximation to the next finer one (Donoho and Johnstone, 1995). The main advantage of wavelet analysis is that it allows the use of long-time wavelet intervals where more precise low-frequency information is needed and shorter intervals where high-frequency information is sought (Burrus *et al.*, 1998). Wavelet analysis is therefore capable of revealing aspects of data that other signal analysis techniques miss, such as trends, breakdown points and discontinuities in higher derivatives and self-similarity (Burrus *et al.*, 1998). Wavelets are also capable of compressing or de-noising a signal without appreciable degradation of the original signal. In general, the wavelet transformation of a time-domain signal is defined in terms of the projections of this signal into a family of functions that are all normalized dilations and translations of a wavelet function (Jaideva and Chan, 1999).

**Discrete Wavelet Transform (DWT):** Since dealing with discrete-time inertial sensor signals, the (DWT) is implemented instead of the Continuous Wavelet Transform (CWT). The DWT of a discrete time sequence  $x(n)$  is given as (Jaideva and Chan, 1999; Ahmed *et al.*, 2008; Burrus *et al.*, 1998):

$$C_{s,k} = \langle x(n), \Phi_{s,k}(n) \rangle = 2^{(s/2)} \sum_n x(n) \Phi_{s,k}(2^s n - k) \quad (1)$$

$$d_{s,k} = \langle x(n), \Psi_{s,k}(n) \rangle = 2^{(s/2)} \sum_n x(n) \Psi_{s,k}(2^s n - k) \quad (2)$$

where,  $\Phi_{s,k}$  is the scale function and  $\Psi_{s,k}$  is the wavelet function and  $2^{(s/2)} \Phi_{s,k}(2^s n - k)$ ,  $2^{(s/2)} \Psi_{s,k}(2^s n - k)$  are the scaled and shifted versions of  $\Phi_{s,k}$  and  $\Psi_{s,k}$ , respectively, based on the values of  $s$  (scaling coefficient) and  $k$  (shifting coefficient). The  $s$  and  $k$  coefficients acquire integer values for different scaling and shifted versions of  $\Phi_{s,k}(n)$ ,  $\Psi_{s,k}(n)$  and  $C_{s,k}$ ,  $d_{s,k}$ , respectively.

The original signal  $x(n)$  can be generated from the matching wavelet function using the following equation:

$$x(n) = \sum_k C_{s,k} \Phi_{s,k}(n) + \sum_s \sum_k d_{s,k} \Psi_{s,k}(n) \quad (3)$$

The wavelet function  $\Psi_{s,k}$  is not limited to exponential functions as in the case of Fourier Transform (FT) or Short Time Fourier Transform (STFT). The only restriction on  $\Psi_{s,k}$  is that it must be short and oscillatory (it must have zero average and decay quickly at both ends). This restriction ensures that the summation in the DWT transform equation is finite (Jaideva and Chan, 1999; Alnuaimy *et al.*, 2009; Putra *et al.*, 2010).

Since the low frequency fraction of the inertial measurement reading contain the majority of the inertial sensor dynamics during the static alignment phase, these inertial measurement readings can be de-noised using wavelet multi-level of decomposition to separate the low and high frequencies (Skaloud and Schwarz, 1999). Wavelet multi-level of decomposition separates each of the IMU reading (three for both the accelerometer and gyroscopes) into two parts. The first part is called approximation; this part is the output of low-pass filter of wavelet multi-level of decomposition, which includes the long-term noises, in addition to the earth gravity and rotation rate frequency components. Both of these two components exist together within very small frequency band at low frequency. The wavelet multi-level of decomposition are unable to separate earth gravity and

rotation rate from the IMU readings and thus it will propagate into the INS algorithm computation. The second part which is called the details obtained from the high pass filter of wavelet multi-level of decomposition includes the undesirable high frequency noise components of the SDINS and a lot of noise disturbances such as vehicle vibration.

Equations 1-3 are referred to as the analysis and synthesis equations. The wavelet transform offers advantages over its Fourier domain counterpart, where the basis function offers only a fixed frequency resolution and no localization in time (Burrus *et al.*, 1998; Chik *et al.*, 2009).

Theoretically, wavelet decomposition process can be continued for ever. Basically, the decomposition process can continue until the individual coefficients consist of a single frequency. On the other hand, an appropriate level of decomposition is elected based on the nature of the signal on a suitable criterion (Skaloud and Schwarz, 1999). In this study the data rate of the inertial sensors of MotionPak II is 32 Hz. Consequently, five levels of decomposition will limit the frequency band to 0.5 Hz. We conclude that five Level of Decomposition (LOD) are adequate to reduce the high frequency noise from the real inertial sensor measurement.

The proposed IMU de-noising procedure consists of (1) performing a wavelet analysis, using the analysis equations, (2) applying a thresholding of the wavelet coefficients and (3) recovering the de-noised signal using the synthesis equation. It is obvious that the choice of threshold in the second step above is crucial to the quality of the de-noising process and should be made carefully in addition to the selection of the type of wavelet function and its order.

**Selection of the appropriate filter:** The wavelet transform has a flexible feature of using a variety of filters that differ by their coefficients. After using all types of the wavelet filters such as (Daubechies, Coiflet, BiorSplines, Symlets). The deionizing result shows that Db3 wavelet filter is the best filter type used to remove the high frequency noise from the accelerometer and gyroscopes of the IMU which reduce the mean square error.

**Performance analysis of different thresholding algorithm:** Thresholding operations are applied on the coefficients of the wavelet and wavelet packet transforms and generally can be classified into Hard-thresholding and Soft-thresholding as described by Burrus *et al.* (1998).

The choice of threshold is crucial to the quality of the deionizing process and should be made carefully. In thresholding process coefficients smaller than threshold value (*Thrv*) are judged negligible, or noise other than signal (Rizzi *et al.*, 2009).

In this study six methods are used to select the value of *Thrv*. the first method is based on estimating the standard deviation  $\sigma_x$  of the noise at each scale by dividing the noise power for the noisy signal over the standard deviation for the details coefficients as in  $Thrv = \sigma^2/\sigma_x$  (Ma *et al.*, 2002; Li and Zhao, 2009), another approach is used this relationship (Veterli *et al.*, 2000; Li and Zhao, 2009).

$$Thrv = \sqrt{\frac{(2 \sigma_x^2 \ln N)}{2}}$$

Where:

- $\sigma^2$  = Represents Noise Power for noisy signal
- $\sigma_x$  = Standard deviation for the detail coefficients
- N = Sequence length

Third method is stein's unbiased risk estimate (SURE) with MatLab code *rigsure*, selection using fixed form threshold with MatLab code *sqrtwolog*, selection using mixture of the last previous two selection rules with MatLab code *heursure* and the last selection rule use minimax principle with MatLab code *minimaxi* (Misite *et al.*, 2002).

Hard and soft threshold functions are widely used in practice, resulting in good effect. Hard thresholding function can preserve the accelerometer and gyroscopes output signals and characteristic but results in unsmooth accelerometer and gyroscope de-noised signal. However, soft threshold function can achieve smooth accelerometer and gyroscopes signal.

In this study soft thresholding was used to remove some of the noise of the details part of the signals with keeping the original features of the signal and improve the Signal to Noise Ratio (SNR). Where, Fig. 3 shows the Root Mean Square Error (RMSE) after applying soft thresholding using the six methods mentioned previously for the IMU accelerometers and gyroscopes, the lowest value for RMSE would have the highest value of SNR and the corresponding method is optimized to select the threshold value. Analysis shows that Steins Unbiased Risk Estimation (SURE) method is the best selection technique for the IMU output. An optimum selection rule is important to choose the threshold value for the wavelet analysis as it has a significant effect on position and velocity components and enhance the de-noising algorithm performance.

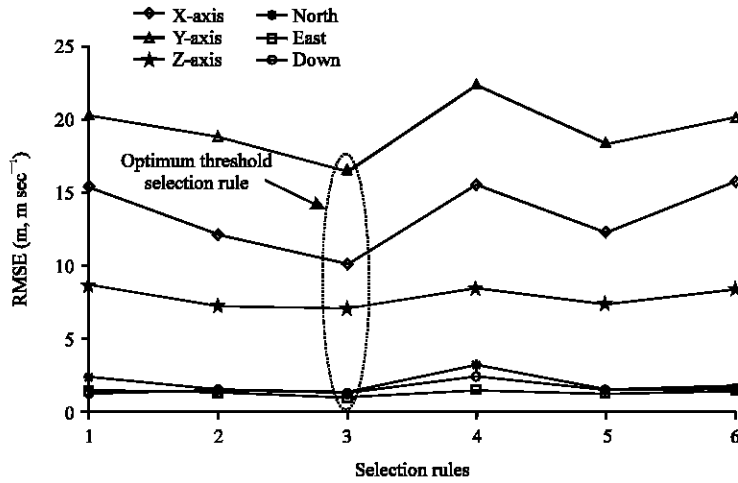


Fig. 3: Performance comparison after using six threshold selection rules

**PROPOSED IMU DEIONIZING FOR SDINS**

A strapdown INS (SDINS) algorithm has been implemented using Matlab for the wavelet multi-resolution algorithm to de-noise the IMU outputs and provide reliable navigation information. Wavelet deionizing analysis was conducted for kinematic inertial data over 2500 sec. Comparison has been made with relatively accurate GPS information as shown in Fig. 4 to compare the appropriate wavelet Level of Decomposition (LOD) required for removing the high frequency noise and disturbances from the IMU device.

**Terrestrial strapdown system dynamic equation:** The differential equation of the relative quaternion between body coordinate and geographic coordinate (Britting, 1971):

$$\dot{u} = \frac{1}{2}\Omega_{ib}^b \cdot u - \frac{1}{2}\Omega_{in}^b \cdot u \tag{4}$$

where, the angular velocity skew-symmetric matrix  $\Omega_{ib}^b$  and  $\Omega_{in}^b$  are given by:

$$\Omega_{in}^b = \begin{bmatrix} 0 & -w_D & w_E & w_N \\ w_D & 0 & -w_N & w_E \\ -w_E & w_N & 0 & w_D \\ -w_N & -w_E & -w_D & 0 \end{bmatrix} \tag{5}$$

$$\Omega_{ib}^b = \begin{bmatrix} 0 & w_Y & -w_P & w_R \\ -w_Y & 0 & w_R & w_P \\ w_P & -w_R & 0 & w_Y \\ -w_R & -w_P & -w_Y & 0 \end{bmatrix} \tag{6}$$

and

$$\begin{bmatrix} w_N \\ w_E \\ w_D \end{bmatrix} = \begin{bmatrix} (|w_{ie}| + 1)\cos L \\ -\dot{L} \\ -(|w_{ie}| + 1)\sin L \end{bmatrix} \tag{7}$$

where, [L, l, h]: are geodetic positions (latitude, longitude and height).  $w_R, w_P, w_Y$ : are the body angular velocities in the body coordinate (roll, pitch and yaw), respectively.

Body fixed coordinate to navigation coordinate ( $C_b^n$ ) can be described in terms of the quaternion parameters:

$$C_b^n = \begin{bmatrix} u_0^2 + u_1^2 - u_2^2 - u_3^2 & 2(u_1u_2 - u_0u_3) & 2(u_1u_3 + u_0u_2) \\ 2(u_0u_3 + u_1u_2) & u_0^2 - u_1^2 + u_2^2 - u_3^2 & 2(u_2u_3 - u_0u_1) \\ 2(u_1u_3 - u_0u_2) & 2(u_0u_1 + u_2u_3) & u_0^2 - u_1^2 - u_2^2 + u_3^2 \end{bmatrix} \tag{8}$$

The differential equations of the vehicle position in terms of latitude, longitude and heading can be arranged in matrix form:

$$\begin{bmatrix} \dot{L} \\ \dot{l} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} 1/(R_N + h) & 0 & 0 \\ 0 & 1/((R_N + h)\cos L) & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} \tag{9}$$

where,  $[V_N \ V_E \ V_D] = V^n$ : Geodetic velocity vector (North, East and down).  $R_N$  and  $R_E$ : are the radii of curvature in the north and east direction and given by:

$$R_N = \frac{r_e}{(1 - e^2 \sin^2(L))^{1.5}} \tag{10}$$

$$R_E = \frac{r_e}{\sqrt{1 - e^2 \sin^2(L)}} \tag{11}$$

and e : eccentricity (= 0.0818)

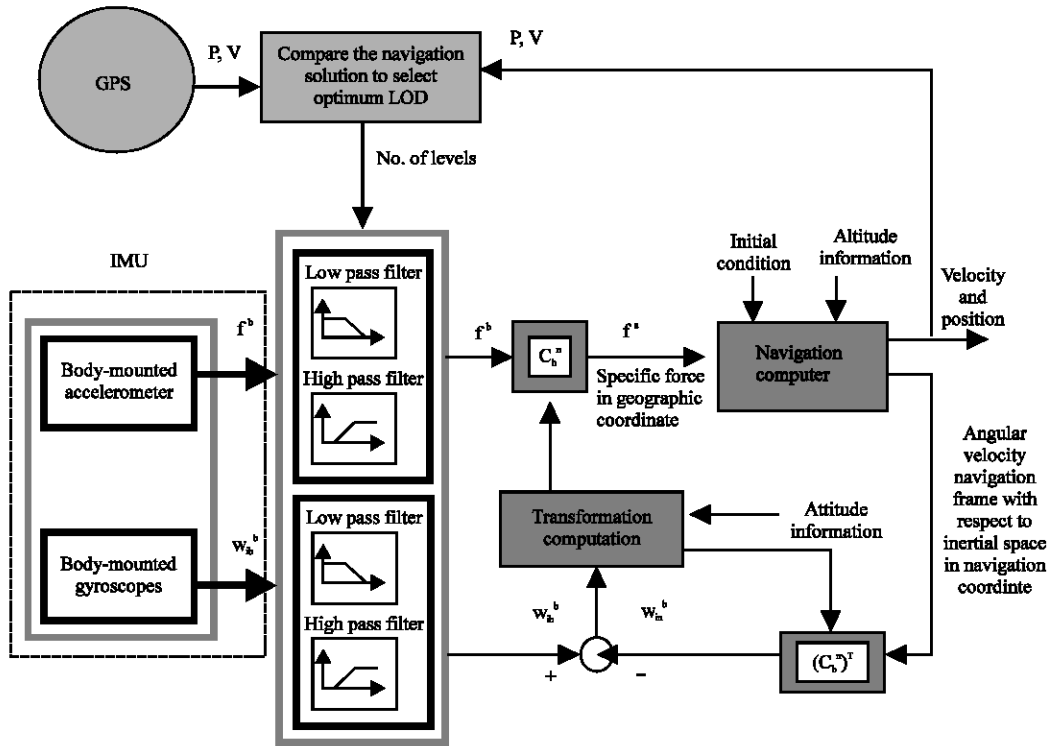


Fig. 4: Schematic diagram of development strapdown inertial navigation system

The differential equations relating the second derivative of the geodetic position and velocities can be derived as:

$$\begin{bmatrix} \ddot{V}_N \\ \ddot{V}_E \\ \ddot{V}_D \end{bmatrix} = C_b^n \cdot f^b + \begin{bmatrix} -\left[\frac{V_E}{(R_E + h) \cos L} + 2w_w\right] V_E \sin L + \frac{V_N V_D}{(R_N + h)} \\ \left[\frac{V_E}{(R_E + h) \cos L} + 2w_w\right] V_N \sin L + \frac{V_E V_D}{(R_E + h)} + 2w_w V_D \cos L \\ -\frac{V_E^2}{(R_E + h)} - \frac{V_N^2}{(R_N + h)} - 2w_w V_E \cos L + g_e \end{bmatrix} \quad (12)$$

where,  $f^b$  is specific force outputs in the body coordinate =  $[f_x, f_y, f_z]^T$ .  $g_e$  is gravity force applied on down direction.

Gravity force ( $g_e$ ) can be found from initial gravity  $g_0$ :

$$g_0 = 9.780327 [1 + 0.0053024 \sin^2(L) - 0.0000058 \sin^2(2L)] \quad (13)$$

and

$$g_e = g_0 - [3.0877 \times 10^{-6} - 0.0044 \times 10^{-6} \sin^2(L)] \cdot h + 0.072 \times 10^{-12} h^2 \quad (14)$$

Equation 4, 9 and 12 represent the mechanization equation for the terrestrial navigation system.

The raw data obtained from the inertial sensor contains substantial noise that needs to be filtered.

Vibration in the INS data can cause a lot of problems if not well taken care of. Vibration of the vehicle contributes to the noise in the data making it inaccurate; therefore, proper filtering techniques should be devised to get accurate and worth while results. As mentioned before, wavelet deionizing technique was used to filter the noise of INS data, which is widely used in filtering technique in the field of signal processing. It can be seen that wavelet-deionizing result is fairly smooth. It is found that five level of decomposition is adequate to reduce the short term error of the INS position and velocity components.

The anticipated de-noising procedure was applied to a real data collected from the Multi-Axis Inertial Sensing System (MotionPak II) MEMS-grade IMU. The MotionPakII consists of three orthogonally mounted micromachined quartz angular rate sensors and three silicon based accelerometers. The specifications of the MotionPak II IMU are given in Table 2.

The outputs of the inertial measurement unit were de-noised by applying five LOD to bound the output high frequency noise. As the MotionPak II measurements are supplied at a data rate of 32 Hz, the five decomposition levels bound the frequency band of the original signal from 16 to 0.5 Hz.



Table 3: The standard deviation and mean square error of the inertial sensors (before and after wavelet De-noising)

Level of decomposition	Position			Velocity		
	X-Axis	Y-Axis	Z-Axis	North	East	Down
<b>L 0</b>						
Mean	-3.2871e+003	9.8951e+003	1.8852e+004	167.0052	127.8663	-194.2489
SD	2.9163e+003	1.0207e+004	1.8087e+004	105.4729	91.1954	139.1683
<b>L 1</b>						
Mean	-2.2599e+003	1.4670e+004	1.6869e+004	123.0976	161.5681	-220.9847
SD	2.1907e+003	1.4308e+004	1.6443e+004	84.4170	112.1307	153.4484
<b>L 2</b>						
Mean	-389.7102	2.5430e+003	3.1665e+003	22.7750	27.0107	-38.5896
SD	368.3211	2.3750e+003	2.9519e+003	14.6322	18.2277	25.7651
<b>L 3</b>						
Mean	-152.9385	1.3169e+003	1.4857e+003	13.1436	17.3643	-24.5748
SD	207.0210	1.5515e+003	1.7709e+003	12.4476	16.2135	22.6270
<b>L 4</b>						
Mean	182.4194	-756.2387	-962.2448	-7.8726	-9.0770	11.8066
SD	184.3020	757.0310	969.6090	5.6305	6.4787	8.4720
<b>L 5</b>						
Mean	-112.7109	-302.1440	54.2811	2.5196	-1.6277	2.5545
SD	100.9315	268.8678	48.3019	1.4710	0.9225	1.4639

It must be mentioned that increasing the number of decomposition level could possibly lead to remove some of the useful frequency components such vehicle motion dynamic. It is clear that applying wavelet multi-resolution analysis to de-noise the inertial sensor outputs has proven its achievement in enhancing the output of the INS algorithm by reducing the estimated position and velocity errors as shown in Fig. 5a-f.

We found that five level is adequate to restrain most of the high frequency noise (short-term errors) existing in the inertial sensor measurement to keep away from removing part of the earth's rotation and gravity components. Figure 6a-c and 7a-c show the MotionPak II raw measurements for force and angular velocity measurements before and after five level of decomposition process, respectively. It is obvious that most of the high frequency noise components are suppressed after the fifth level and hence reducing the measurement uncertainty. Table 3 shows the mean values and standard deviation of the IMU output for five level of decomposition.

Noise was also observed in the INS data during static mode. Since the equipment is sensitive and logs data with a sampling rate of 32 Hz, even the minor variation in the environmental affects the data.

A combination of several filtering techniques can remove the INS noise to quite some extent. Figure 8a-f show the position and velocity of the INS algorithm after de-noising the accelerometers and gyroscopes output for five LOD compared with the reference GPS data. Also, from this figure we can observe that five LOD is adequate to remove the short term error existed in the accelerometers and gyroscopes reading from the IMU.

Figure 5 shows the resultant error in position and velocity after de-noising and indicates that five level of decomposition are suitable to remove the high frequency error of the IMU measurement. Increasing the level of decomposition results in undesired features of the navigation solution since the original features of the IMU data will be lost and from this results we can conclude that appropriate LOD can be optimized using an optimization technique such as genetic algorithm, particle swarm optimization and other optimization techniques without using reference GPS data for comparison to obtain accurate results.

### CONCLUSIONS

The intuition of filtering short-term noise from an IMU defined by the motion of the vehicle has been studied. A de-noising algorithm based on wavelet multi-resolution analysis has been introduced. In addition the results shoed that the proposed algorithm procedure could be performed and reduce the error for acceptable range of INS operating period and reduce the short term error to provides more accurate position and velocity if compared to the results obtained from non-denoised inertial data before the error will growth gradually.

Most of the current inertial de-noising methods suffer from the comparatively high noise levels of the inertial measurement unit. While, the anticipated technique is highly beneficial in providing fast and accurate navigation solution for several applications and improves the short term error of the low cost inertial measurement device. It was demonstrated that wavelet as a tool can be useful for analysis of the measurements. It also showed that wavelet

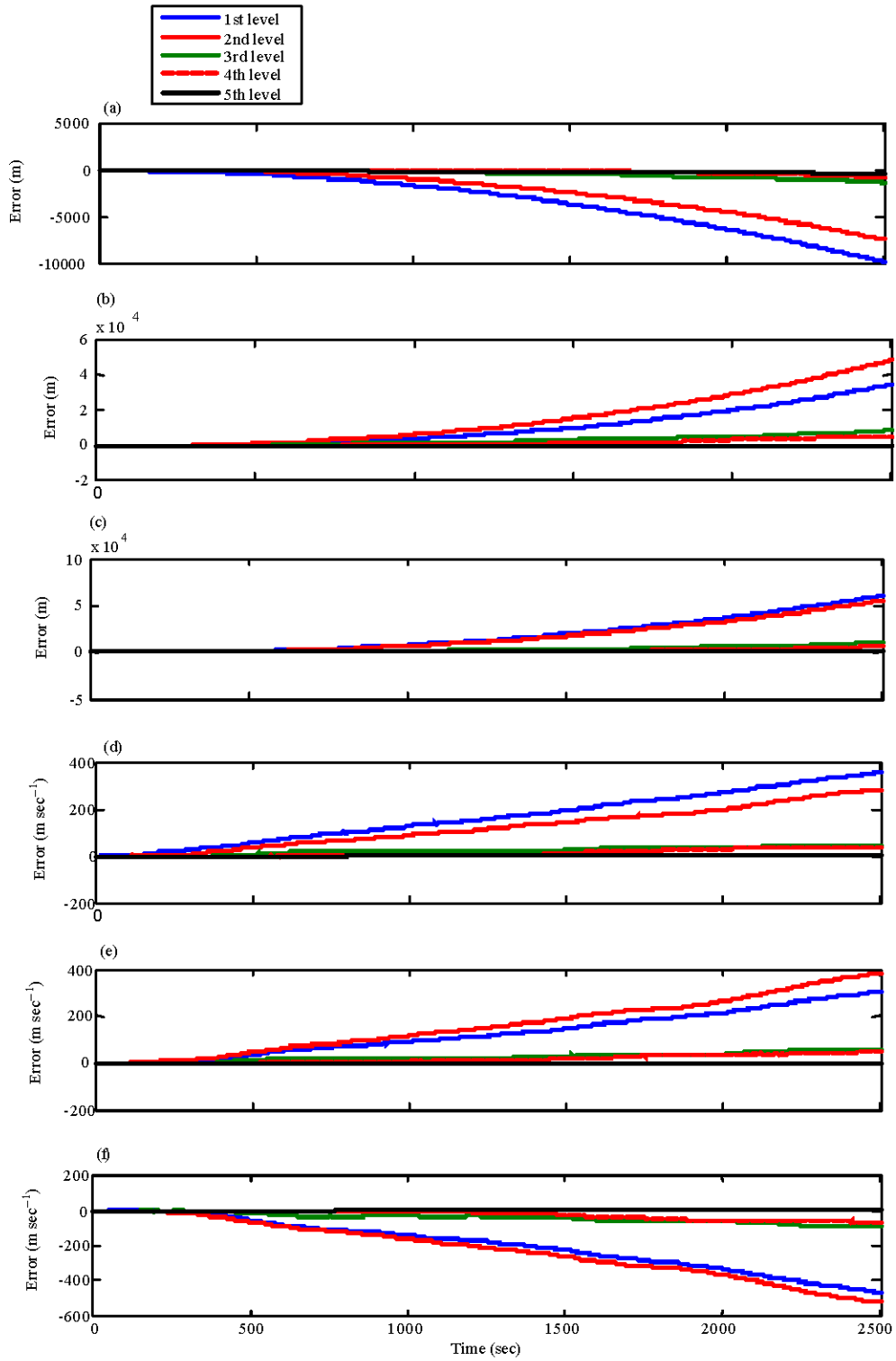


Fig. 5: Error in Position and velocity after using wavelet De-noising for 5-levels of Decomposition data. (a) X-axis, (b) Y-axis, (c) Z-axis, (d) North, (e) East and (f) Down

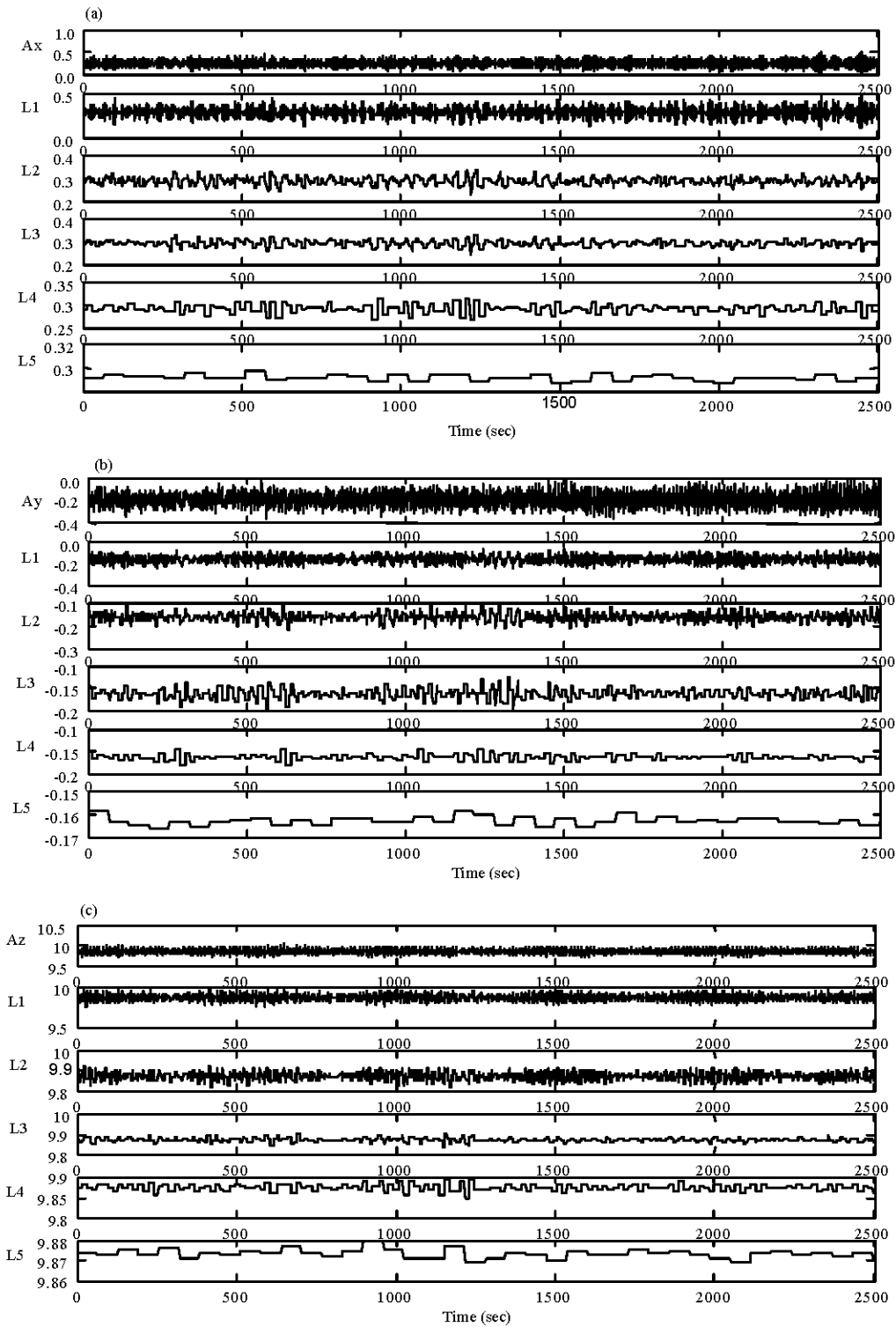


Fig. 6: Force measurements ( $\text{m sec}^{-2}$ ) before and after five level of decomposition Wavelet De-noising for (a) X-axis, (b) Y-axis, and (c) Z-axis respectively

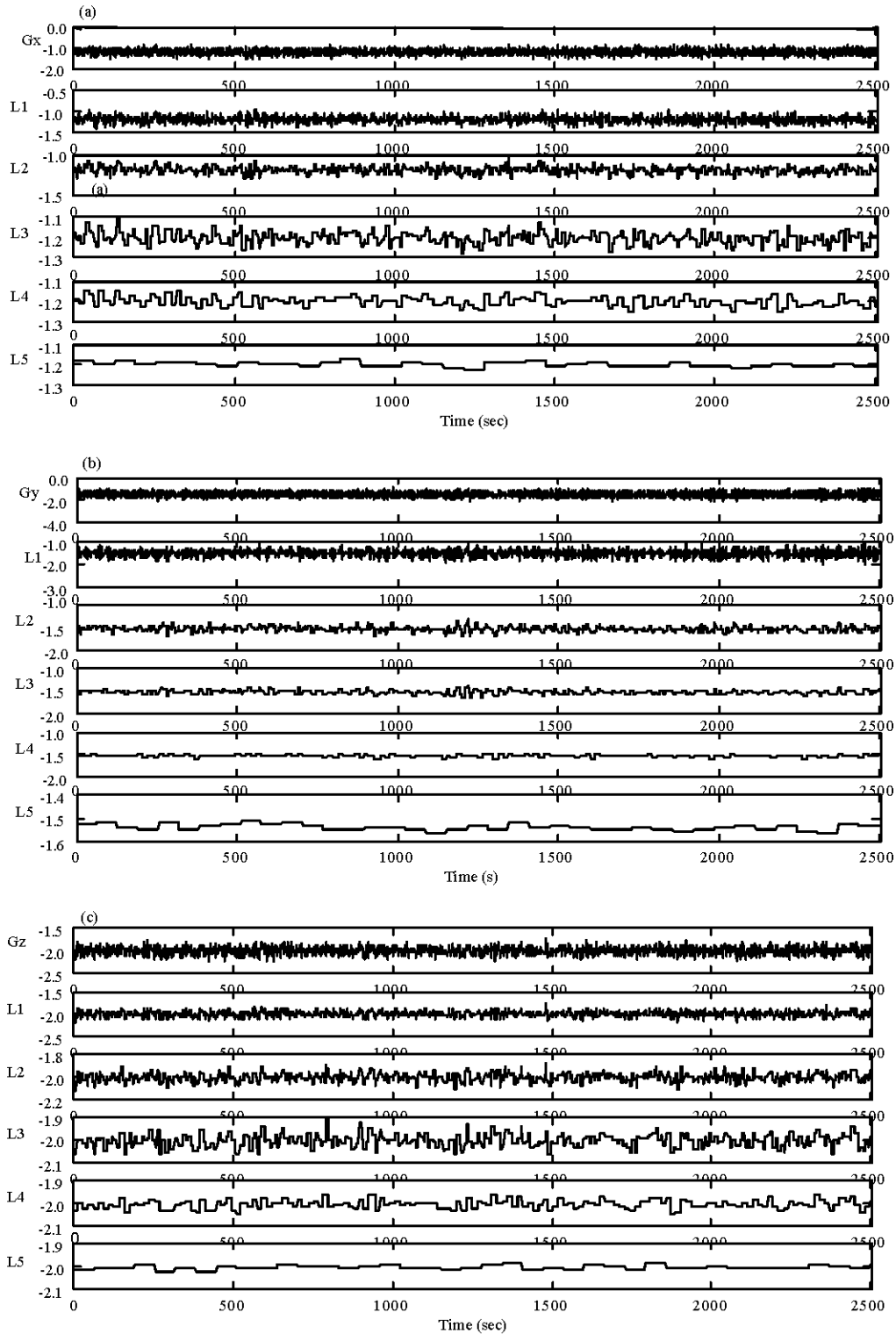


Fig. 7: Angular velocity Measurements ( $\text{deg sec}^{-1}$ ) before and after five level of decomposition Wavelet De-noising for (a) X-axis, (b) Y-axis and (c) Z-axis, respectively

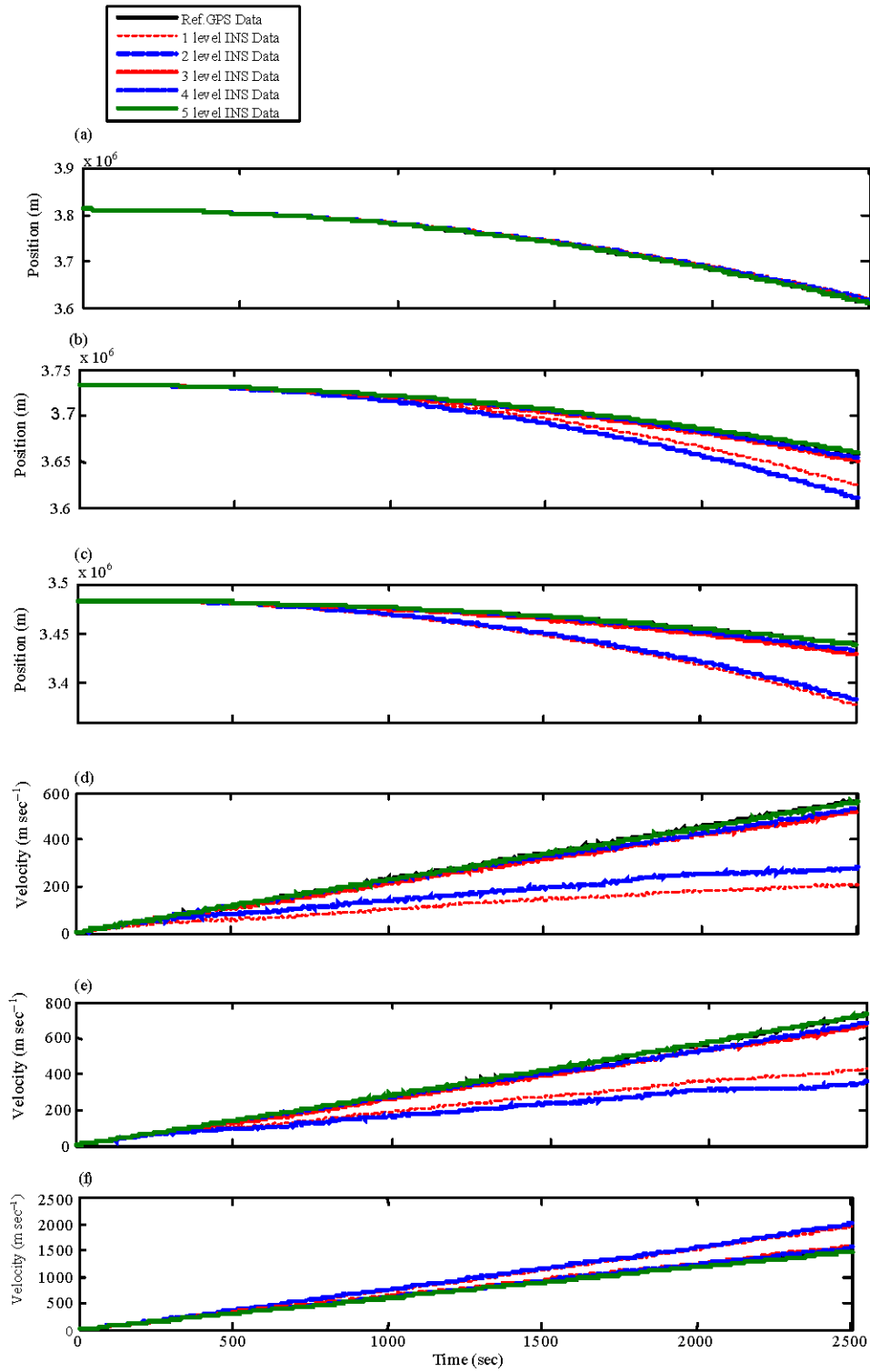


Fig. 8: Strapdown INS Position and velocity in ECEF-Frame after using wavelet de-noising with reference GPS data along (a) X-axis, (b) Y-axis, (c) Z-axis, (d) North, (e) East, and (f) Down, respectively

based de-noising can be used as excellent tool to remove the noise from the measurements reading of the IMU. Finally, experimental results obviously indicates the capability of the proposed pre-filtering approach to reduce the standard deviation of the estimated error and increase the SNR of the accelerometer and gyroscopes measurements as the RMSE reduced and provide an accurate navigation solution for several navigation application. The proposed method contribute positively in reducing the high frequency noise of the inertial sensors where GPS aiding can not provide the predictable reduction in the high frequency noise of the inertial sensors.

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