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Comparison between Modified Fully Vectorial Effective Index Method and Empirical Relations Method for Study of Photonic Crystal Fibers

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Abstract: To design photonic crystal fibers (PCFs) in a less computational cost and time consumption way, it is better to use empirical relations method instead of other effective index methods. In this study, we intend to investigate both empirical relations method and modified fully vectorial effective index one to compare them with an accurate and powerful method like as full-vector finite element method. We found that empirical relations method has less error than the method of modified fully vectorial effective index in calculating PCFs parameters such as n_{eff} and the second order dispersion. In this study, we also calculate the third order dispersion by these methods. Finally, we will introduce the suitable method for designing PCFs among methods of empirical relations, fully vectorial effective index, modified fully vectorial effective index and scalar effective index.

Key words: Photonic crystal fibers, dispersion, effective index method

INTRODUCTION

During recent years, lots of studies have been done about PCFs or holy fibers. This is due to their capabilities of these fibers in handling propagation modes through themselves (Saitoh and Koshiba, 2005). This aspect has introduced these devices as the most popular and applicable optical instruments such as channel allocation in the wavelength division multiplexing transmission systems and Pressure Sensor Applications (Kim, 2003).

PCFs are categorized as mono-material fibers which have a central light guiding area surrounded by rods in a triangular lattice (Li *et al*., 2004). These rods are filled by air and their diameters and hole pitches are almost the same as the amount of wavelength. This novel structure of PCF causes new properties such as wide single-mode wavelength range, unusual chromatic dispersion and high or low non-linearity (Saitoh and Koshiba, 2005). There are several methods to analyze these fibers including: Effective Index Method, (EIM), Localized Basis Function Method, Finite Element Method (FEM), Finite Difference Method (FDM), Plane Wave Expansion Method (PWM) and Multi-Pole Method (Saitoh and Koshiba, 2005; Li *et al*., 2006; Sinha and Varshney, 2003).

Numerical methods consume too long time consuming and need large amount of iterative computations (Saitoh and Koshiba, 2005). Usually these methods are too mighty and their broad capabilities are not required for studying of PCFs. Despite of limitations and inaccuracies, other analytic methods are introduced to replace these ones (Saitoh and Koshiba, 2005). In the present study, Modified Fully Vectorial Effective Index Method (MFVEIM) and Empirical Relations Method (ERM) are studied among them.

Here, via fully vectorial effective index method (FVEIM), the effective cladding index of a hexagonal unit cell which consists of a fiber rod, is calculated with respect to the rod diameter and pitch (Λ). Then the effective index of PCF is obtained by using the effective cladding index (Li *et al*., 2004). However, comparing with an accurate method like as full vector finite element method (FVFEM), the effective index obtained by FVEIM is not accurate for values of d/Λ. In order to correct this problem, Yong-Zhao *et al.* (2006) suggested a method so called Modified FVEIM, which efficiently improved FVEIM. In fact, MFVEIM applies an effective core radius (r_c) which changes by hole diameter and hole pitch; while FVEIM uses a constant r_c (Yong-Zhao *et al*., 2006). In Empirical Relations Method (ERM), empirical relations for parameters of V (Normalized Frequency) and W (Normalized Transverse Attenuation Constant) of PCFs with respect to the basic geometrical parameters (i.e., the air hole diameter and the hole pitch) are formed (Saitoh and Koshiba, 2005). Then V and W are computed and used to calculate PCF's basic parameters (Saitoh and Koshiba, 2005). Hereafter, the obtained results of these two methods are compared and we show that the accuracy of the methods changes by Λ and d/Λ.

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We will also present the calculations of the second and third order dispersions for chromatic dispersion in PCFs with well known properties. The Sellmeier relation has been used to calculate material dispersion.

MODIFIED FULLY VECTORIAL EFFECTIVE INDEX METHOD

Both the effective cladding index and the effective index of the guided mode of the PCF are calculated using fully vectorial equations (Li *et al*., 2004).

Solving Maxwell equations in the infinite two-dimensional photonic crystal structure, we will get the modal index of fundamental space filling mode (n_{FSM}) (Bjarklev *et al*., 2003; Birks *et al*., 1997).

In order to calculate n_{FSM} for the PCF, the hexagonal unit cell (Fig. 1) is approximated by a circular one of radius R. In calculating n_{FSM} using FVEIM, the boundary conditions at point P should be perfect electric and perfect magnetic conductor (Li *et al*., 2007). After applying the boundary condition to the characteristic equation, we will have the following equation:

$$\left(\frac{U_1'(Ta)}{TaU_1(Ta)}+\frac{I_1'(Ka)}{KaI_1(Ka)}\right)\left(\frac{n_{silica}^2U_1'(Ta)}{TaU_1(Ta)}+\frac{n_{air}^2I_1'(Ka)}{KaI_1(Ka)}\right)= l^2\left[\left(\frac{1}{Ta}\right)^2+\left(\frac{1}{Ka}\right)^2\right]^2\left(\frac{\beta\cdot c}{w}\right)^2 \quad (1)$$

where, $l = 1$,

$$U_1[T(w)\rho]=J_1[T(w)\rho]Y_1[T(w)R]-Y_1[T(w)\rho]J_1[T(w)R]$$

and I_1 is Bessel function. One should note that U_1 and I_1 should play a similar role as J_1 and K_1, respectively, in the characteristic equation of step index fiber (Li *et al*., 2004). K(w) and T(w) are defined as

$$K^2(w)=\beta^2(w)-n_{air}^2\left(\frac{w}{c}\right)^2 \text{ and } T^2(w)=\left(\frac{w}{c}\right)^2 n_{silica}^2(w)-\beta^2(w)$$

In all above equations, the derivatives are taken with respect to the function arguments. Note that the optimal radius for FVEIM is R = Λ/2 (Midrio *et al*., 2000) and we use the same radius for MFVEIM.

By solving Eq. 1 for β(w), we can calculate effective cladding index using $n_{FSM}(w) = \beta(w)c/w$.

Afterwards, $\beta_c(w)$ is achieved via solving Eq. 2:

$$\left(\frac{J_1'(\eta r_c)}{\eta r_cJ_1(\eta r_c)}+\frac{K_1'(\gamma r_c)}{\gamma r_cK_1(\gamma r_c)}\right)\left(\frac{n_c^2J_1'(\eta r_c)}{\eta r_cJ_1(\eta r_c)}+\frac{n_{eff}^2K_1'(\gamma r_c)}{\gamma r_cK_1(\gamma r_c)}\right)= l^2\left[\left(\frac{1}{\eta r_c}\right)^2+\left(\frac{1}{\gamma r_c}\right)^2\right]^2\left(\frac{\beta_c\cdot c}{w}\right)^2 \quad (2)$$

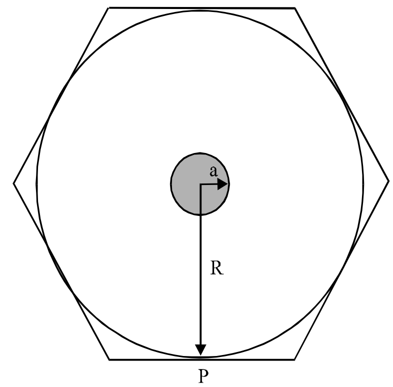

Fig. 1: The hexagonal unit cell and its circular equivalent

where, $l = 1$,

$$\eta^2(w)=\left(\frac{w}{c}\right)^2 n_c^2(w)\beta_c^2(w) \text{ and } \gamma^2(w)=\beta_c^2(w)-\left(\frac{w}{c}\right)^2 n_{eff}^2(w)$$

and, with $n_c(w)$ being the refractive index of the core material. Note that both $n_{silica}(w)$ in Eq. 1 and $n_c(w)$ in Eq. 2 are 1.45 as a fixed value in the current method.

In FVEIM, the parameter of r_c takes a fixed value and different values are suggested for that in the references. But in MFVEIM, r_c changes when PCF has different relative hole diameters. In fact r_c is calculated by following formula:

$$\frac{r_c}{\Lambda}=b\left[a_3\left(\frac{d}{\Lambda}\right)^3+a_2\left(\frac{d}{\Lambda}\right)^2+a_1\left(\frac{d}{\Lambda}\right)^3+a_0\right]^{-1} \quad (3)$$

where, b = 0.6962, a_0 = 0.0236, a_2 = 0.0056 and a_3 = 0.1302 (Saitoh and Koshiba, 2005).

Afterwards, by using $n_{eff}(w) = \beta_c(w)c/w$, the effective index of PCF is obtained.

Now, it is possible to calculate the total dispersion using the following formula:

$$D\approx D_g+D_m=-\frac{\lambda}{c}\frac{d^2n_{eff}}{d\lambda^2}+D_m \quad (4)$$

where D_m is the material dispersion obtained from the Sellmeier relation.

EMPIRICAL RELATIONS METHOD

In this method, the refractive index of silica is considered constant as n_{core} = 1.45 and the effective core radius is defined as (Saitoh and Koshiba, 2005).

Recently, it has been claimed that the triangular PCFs can be well parameterized in terms of the V parameter (Koshiba and Saitoh, 2004) that is given by:

$$V = \frac{2\pi}{\lambda} a_{eff} (n_{core}^2 - n_{eff}^2)^{0.5} = (U^2 + W^2) \quad (5)$$

where,

$$U = \frac{2\pi}{\lambda} a_{eff} (n_{core}^2 - n_{eff}^2)^{0.5} \text{ and } U = \frac{2\pi}{\lambda} a_{eff} (n_{core}^2 - n_{eff}^2)^{0.5} \quad (6)$$

First, from the study by Saitoh and Koshiba (2005), we calculate V by using

$$V\left(\frac{\lambda}{\Lambda}, \frac{d}{\Lambda}\right) = A_1 + \frac{A_2}{1 + A_3 \exp\left(A_4 \lambda/\Lambda\right)}$$

(Saitoh and Koshiba, 2005), where,

$$A_i = a_{i0} + a_{i1}\left(\frac{d}{\Lambda}\right)^{b_{i1}} + a_{i2}\left(\frac{d}{\Lambda}\right)^{b_{i2}} + a_{i3}\left(\frac{d}{\Lambda}\right)^{b_{i3}}$$

Subsequently, the effective cladding index n_{FSM} is obtained from Eq. 5. Then referring to Table by Saitoh and Koshiba (2005) and from:

$$W\left(\frac{\lambda}{\Lambda}, \frac{d}{\Lambda}\right) = B_1 + \frac{B_2}{1 + B_3 \exp\left(B_4 \lambda/\Lambda\right)}$$

(Saitoh and Koshiba, 2005), where

$$B_i = c_{i0} + c_{i1}\left(\frac{d}{\Lambda}\right)^{d_{i1}} + c_{i2}\left(\frac{d}{\Lambda}\right)^{d_{i2}} + c_{i3}\left(\frac{d}{\Lambda}\right)^{d_{i3}}$$

we can calculate W. From Eq. 6 for given W and n_{FSM}, can be obtained and finally one can calculate the total dispersion using Eq. 4.

RESULTS

Figure 2 shows n_{FSM} calculated as a function of d/Λ by ERM, MFVEIM and FVFEM (Koshiba and Saitoh, 2002). The accuracy of our calculations is proved by Fig. 2.

Figure 3a shows that for d/Λ = 0.2, 0.3, 0.4, 0.7 and 0.8, the relative difference between by ERM and MFVEIM is almost high, while for d/Λ = 0.5 and 0.6 this difference is smooth and low. So, we can conclude that as d/Λ either increases or decreases more, two methods result in more different amounts for n_{eff}.

Hereby, the comparison between the accuracies of two above mentioned methods (ERM and MFVEIM) with respect to the method of Fully Vectorial Finite Element (FVFEM) will be made. Referring to (Saitoh and Koshiba,

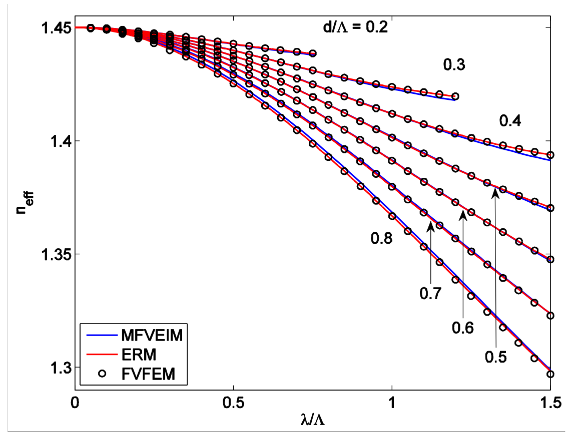

Fig. 2: n_{eff} as a function of λ/Λ, obtained by ERM, MFVEIM and FVFEM

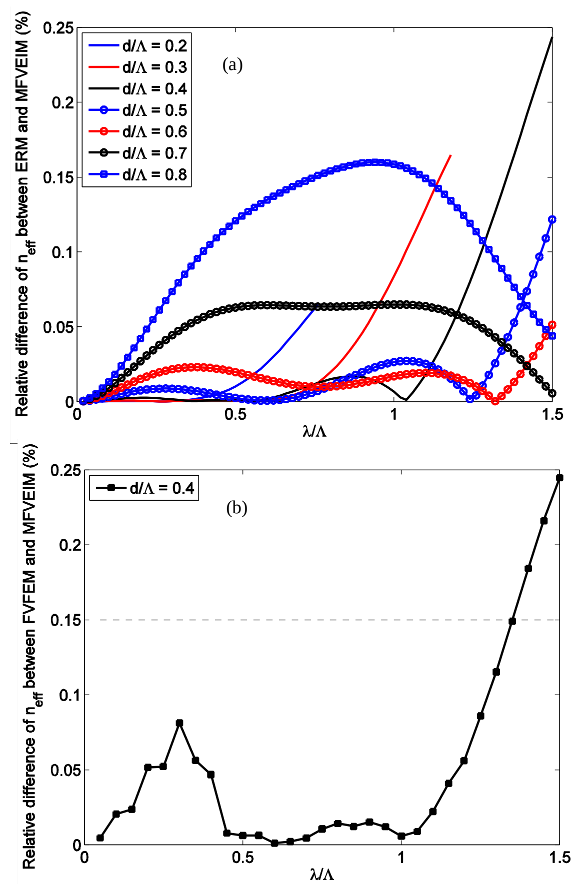

Fig. 3: Relative difference between n_{eff} obtained by different methods. (a) ERM and MFVEIM for several d/Λs. (b) FVFEM and MFVEIM for d/Λ=0.4

2005), it has been shown that achieved by ERM deviates less than 15% from that of FVFEM, while it is calculated in restricted range (Saitoh and Koshiba, 2005). Moreover,

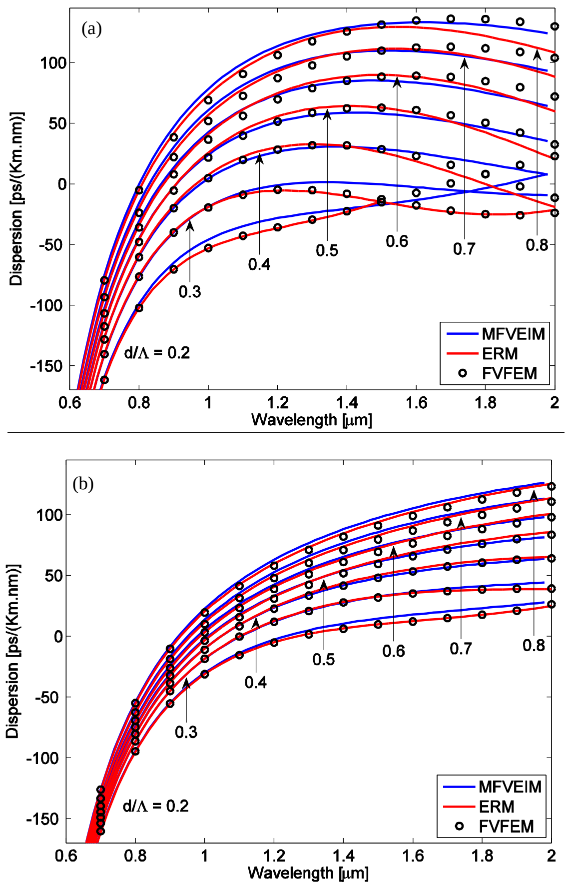

Fig. 4: The second order dispersion obtained by ERM, MFVEIM and FVFEM. (a) Λ = 2 μm (b) Λ = 3 μm

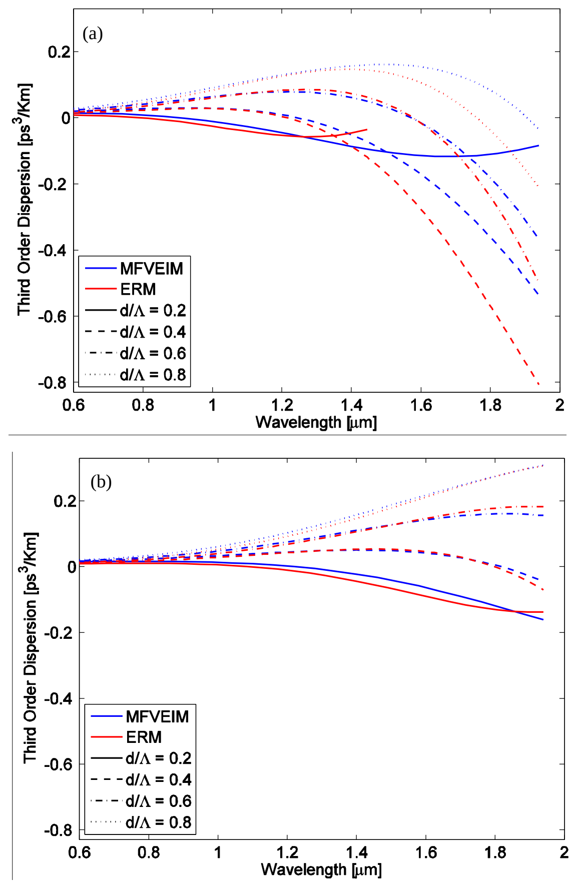

Fig. 5: The third order dispersion obtained by ERM, MFVEIM (a) Λ = 2 μm (b) Λ = 3 μm

Fig. 3b shows that the relative difference between n_{eff} obtained by MFVEIM and FVFEM can exceed 15%. So, it can be concluded that ERM is preferable from the accuracy view point.

Next, we show the accuracy of MFVEIM and ERM via comparing the results of second order dispersion obtained by them with the results of second order dispersion obtained by FVFEM. Figure 4a and b illustrate this comparison for Λ = 2 and 3 μm for same d/Λs.

The evaluation of n_{eff} via ERM causes the parameter of second order dispersion being closer to that was achieved by FVFEM.

It is interesting to observe that for Λ = μm, not only the does the second order dispersion from ERM and MFVEIM become closer to FVFEM, but also both methods agree better in results. And something else can be obtained, is that the MFVEIM and ERM can be used for big Λ. Because we have seen in our studying that the increasing the error in small pitch in both methods due to they can not be able to have good accuracy for calculating of effective interaction between mutual rods and between core and rods.

Figure 5a and b show the third order dispersion obtained by MFVEIM and ERM for Λ = 2 and 3 μm. Our previous claim for the second order dispersion is accurate for the third order dispersion too. It means that as pitch increases, both methods agree more.

DISCUSSION

We have seen that ERM has less error than MFVEIM in defined range. On the other hand, ERM is faster and simpler than MFVEIM. According to Li *et al*. (2004, 2006, 2007) show that FVEIM is more accurate than scalar effective index method (SEIM). Meanwhile, referring to (Yong-Zhao *et al*., 2006), one can see that MFVEIM is more accurate than FVEIM and it is shown that ERM is better than SEIM (Pourkazemi and Mansourabadi, 2008). As the result, so we can claim that ERM is more accurate, simpler and faster than three other methods (i.e., SEIM, FVEIM and MFVEIM) in its defined range.

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