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Wavelet Coefficient Extraction Algorithm for Extracting Fatigue Features in Variable Amplitude Fatigue Loading

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Abstract: An extraction computational algorithm for fatigue feature editing is presented in this study. The magnitude of the time domain Morlet wavelet coefficient level was used as the parameter to set gate value for the eliminating process of the 60 sec original signal. It was important to maintain the signal statistical parameters and the total fatigue damage of the mission signal as close as to the original signal, with the retention of the original load sequences. At the end of the process, by using this approach, segments containing the higher Morlet wavelet coefficients that contribute to the more fatigue damaging events were retained and were then joined so produce the optimum mission signal length of 13.8 sec. This short signal gave a 77% reduction in length with only 8.7% reduction in the fatigue damage. In conclusion, the extraction of the fatigue features using the Morlet wavelet successfully created a new mission signal which retains the majority of the higher fatigue damaging events in the time history.

Key words: Fatigue strain signal, Morlet wavelet coefficient, extraction algorithm, mission signal

INTRODUCTION

In a fatigue life assessment, fatigue signal extraction is described as a method for fatigue data editing which lead to summarize a fatigue signal. The method is performed by segment identification and extraction that contributes to the more fatigue damaging events to a metallic material. On the other hand, segments containing lower amplitude cycles are omitted, since these data type theoretically gave minimal or no fatigue damage. The goal of the removal of those parts from the original signal is to generate a new shortened mission signal, for which this signal type can be used to reduce the testing time and cost for fatigue testing (Abdullah, 2005). Without editing the service load, both the things become prohibitive (Abdullah, 2007). Two key factors are suggested for achieving an efficient design and modification processes to ensure adequate fatigue life assessment i.e., the signal statistical parameters and the fatigue damage shall be as accurate as possible and the component durability tests shall be as short as possible.

In order to prove the suitability of this Wavelet Transform (WT)-based algorithm in extracting fatigue features for automotive applications, a random fatigue strain signal loading history, or also known as fatigue strain signal, was used as a case study. The WT approach is probably the most recent solution to overcome the nonstationary signals. This time-frequency technique is applied by cutting time domain signal into various frequency components through the compromise between time and frequency-based views of the signal. It presents information in both time and frequency domain in a more useful form (Valens, 1999; Percival and Walden, 2000; Addison, 2002).

The WT analysis is started with a basic function (called the mother wavelet) scaled and translated to represent the signal being analyzed (Berry, 1999). The transform shifts a window along the signal and calculates the spectrum for every position. The process is repeated many times with a slightly shorter (or longer) window for every new cycle. In the end, the result will be a collection of time-frequency representations of the signal with different resolutions. The WT provides information on when and at what frequency the change in signal behavior occurs (Valens, 1999). Obviously, the WT represents a windowing technique with variable-sized regions. This technique allows the use of long time intervals (more precise low frequency information) and shorter regions (high frequency information). It means the wavelet method solves the resolution problem because the window length is long for low frequency and short for high frequency. Therefore, the frequency resolution is good for low frequency (at high scales) and the time resolution is good at high frequency (at low scales). The

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major advantage is the ability to analyze a localized area of larger signal or also known as the local analysis (Misiti *et al*., 2008).

MATERIALS AND METHODS

This study was conducted from July 2009 to September 2009. In this study, the fatigue feature extraction using the Morlet wavelet was focused on the following main stages: analyzing the Morlet wavelet coefficients, extracting significant segments and generating a new shortened mission signal.

In the first stage, the Morlet wavelet coefficients were calculated and they were then presented as in the time-frequency domain localization. Then, the plot was transposed into time domain signal. The lower Morlet wavelet coefficient amplitude was eliminated based on the gate value for summarizing the signal length without compromised the original fatigue damaging potential. While the higher Morlet wavelet coefficient amplitude should be retained for further fatigue durability analysis. At the end of the process, the retained segments were joined together to be a single loading which retains the fatigue damaging content in the mission signal and plays a part in determining the degree of the fatigue damaging occurring. From the analysis of the mission signal, the optimum gate value was determined based on the capability of the gate value (reflect to the mission signal) to produce the shortest signal with minimum the signal statistical parameter and fatigue damaging deviation. The flowchart of the extraction process is schematically shown in Fig. 1.

In the case of the fatigue research, the signals consist of a measurement of cyclic loads i.e., force, strain and stress against time. A time series typically consists of a set of observations of a variable were taken at equally spaced intervals of time. Global signal statistical parameters are frequently used to classify random signals and monitor the pattern of analysed signals. For a signal with a numbers of data point n in a sampled sequence, the mean $\bar{x}$ is given by:

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j \tag{1}$$

For a fatigue signal, the calculation of the root-mean-square (rms) and the kurtosis are important in order to retain a certain amount of the signal amplitude range characteristics. The r.m.s. value is the signal 2nd statistical moment used to quantify the overall energy content of the oscillatory signal. The rms relationship is defined as:

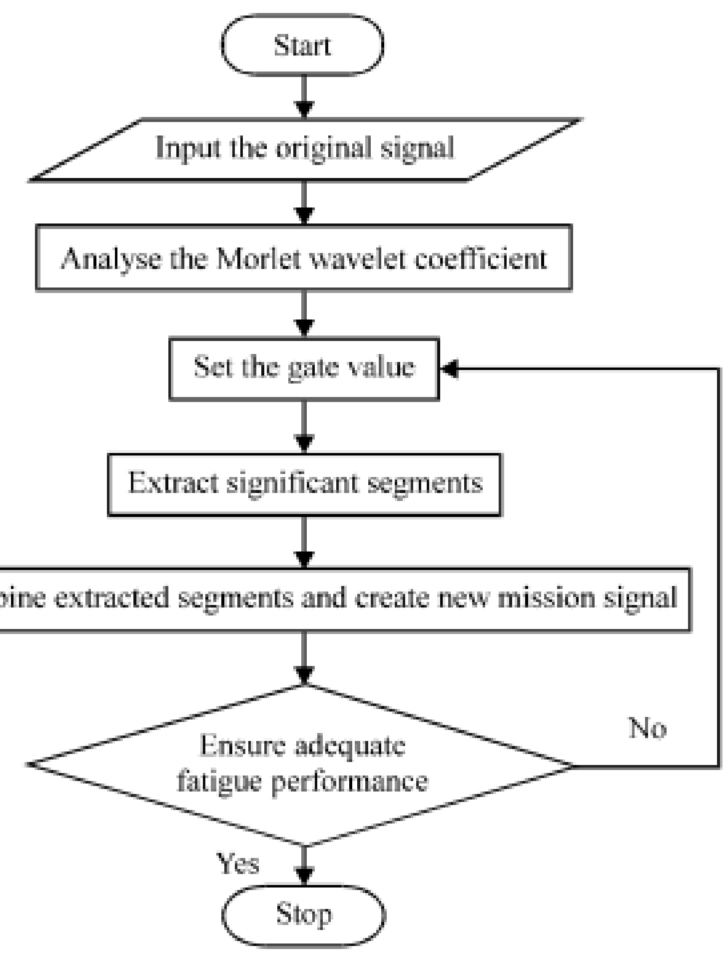

Fig. 1: Simplified flowchart of the Morlet wavelet extraction process

$$\text{rms} = \left\{\frac{1}{n}\sum_{j=1}^{n} x_j^{\,2}\right\}^{1/2} \tag{2}$$

The kurtosis is the signal 4th statistical moment. In engineering field, it is used as a measure of nongaussianity for detection of fault symptoms since, it is highly sensitive to spikiness or outlier signal among the instantaneous values. Mathematically, the kurtosis expression is defined as (Nuawi *et al*., 2009):

$$K = \frac{1}{n(\text{rms})^4}\sum_{j=1}^{n}\left(x_j - \bar{x}\right)^4 \tag{3}$$

where, x_j is the amplitude of signal

One common mean stress correction effect model is the Smith-Watson-Topper (SWT). The model appears to give good results for a wide range of materials and is a good choice for general use. For loading sequences that are predominantly tensile, the approach is more conservative and therefore recommended. The model is mathematically defined as the following expression (Smith *et al*., 1970):

$$\sigma_{max}\varepsilon_a = \frac{(\sigma'_f)^2}{E}(2N_f)^{2b} + \sigma'_f\varepsilon'_f(2N_f)^{b+c} \tag{4}$$

where, σ_{max} is the maximum stress for the particular cycle, ε_a is the true strain amplitude, σ'_f is the fatigue strength coefficient, E is the material modulus of elasticity, N_f is the numbers of cycle to failure for a particular stress range and mean, b is the fatigue strength exponent, ε'_f is the fatigue ductility coefficient and c is the fatigue ductility exponent.

For strain-based fatigue life prediction, current industrial practice uses the Palmgren-Miner linear cumulative damaging rule normally applied with the established strain-life fatigue damaging models. The fatigue damage caused by each cycle of repeated loading is calculated by reference to material life curves, such as S-N or ε-N curves. The total fatigue damage ΣD caused by cycles is expressed as (Palmgren, 1924; Miner, 1945):

$$\Sigma D = \Sigma \frac{N_i}{N_f} \quad (5)$$

where, N_i is the numbers of cycle within a particular stress range and mean.

The Morlet wavelet is one of functions that are generally used in the Continuous Wavelet Transform (CWT) analysis (Gao *et al*., 2001). Basically, the name of the wavelet family is written morl. The wavelet decomposition calculates a resemblance index between signal being analysed and the wavelet, called coefficient. It is a result of a regression of an original signal produced at different scales and different sections on the wavelet. It represents correlation between the wavelet and a section of the signal. If the index is large, the resemblance is strong, otherwise it is slight (Misiti *et al*., 2008).

The WT of any time-varying signal f(t) is defined as the sum of all of the signal time multiplied by a scaled and shifted version of the wavelet function ψ(t) (Kim *et al*., 2007). The CWT is expressed by the following integral:

$$CWT_{(a,b)} = \int_{-\infty}^{+\infty} f(t)\psi_{a,b}(t)dt \quad (6)$$

The parameter a represents the scale factor which is a reciprocal of frequency, the parameter b indicates the time shifting or translation factor and t is time.

$\Psi_{a,b}$ (t) denotes the mother wavelet, i.e., (Purushotham *et al*., 2005):

$$\psi_{a,b}(t) = 1/\sqrt{|a|}\,\psi((t-b)/a) \;\; a,b \in R; a \neq 0 \quad (7)$$

$$CWT_{(a,b)} = \int_{-\infty}^{+\infty} f(t)\left(1/\sqrt{a}\right)\psi((t-b)/a)dt \quad (8)$$

In addition, the wavelet coefficient indicates how energy in the signal is distributed in the time-frequency plane (Darpe, 2007). The energy spectrum (the energy density over frequency) is plotted in order to observe the signal behavior and its content gives significant information about the random signal pattern.

RESULTS

The original signal was measured at a front lower suspension arm of a road vehicle driven over urban surface proving ground manoeuvres and rough road surface. The signal (in the unit of microstrain) was assumed to be sampled at 500 Hz for 30,000 data points. It gave the total record length of the signal of 60 sec, as shown in Fig. 2. The data represented load feature that might include turning and braking, rough road surface and speed bumps. This signal exhibited a lower frequency background that contained occasional shocks. This feature contributed to the higher fatigue damaging potential.

Equation 1-3 calculated the signal statistical parameters for the original signal. The signal was tensile data since it had the positive mean value i.e., 89.9 με. Furthermore, the rms and kurtosis values for the signal were 111.1 and 3.8 με, respectively. It indicated that the signal has nonstationary behavior. For the fatigue damaging calculation, the selected material for the simulation purpose was the SAE1045 carbon steel shaft. It was chosen as a common material used in automotive industries for fabricating a vehicle lower suspension arm structure (Khalil and Topper, 2003). The material properties and their definitions are given in Table 1 (nCode, 2005).

From the analysis, the numbers of cycle counted of the original signal were 4,051 cycles. Furthermore, the fatigue damaging potential for each cycle was calculated and was then accumulated in order to get the total fatigue damage for the loading using Eq. 5. These estimations obtained the total fatigue damage of 3.76×10^{-3} cycles to failure. Later part of this study, the fatigue life for the original signal was calculated, indicating how long a component can be lasted without failure under the given strain loading. Fatigue life calculation of a fatigue load history was ideally based on the numbers of the

Table 1: The mechanical properties of the SAE1045 carbon stell shaft

Properties	Values
Ultimate tensile strength (S_u) (MPa)	621
Modulus of elasticity (E) (GPa)	204
Fatigue strength coefficient (σ'_f) (MPa)	948
Fatigue strength exponent (b)	-0.092
Fatigue ductility exponent (c)	-0.445
Fatigue ductility coefficient (ε'_f)	0.26

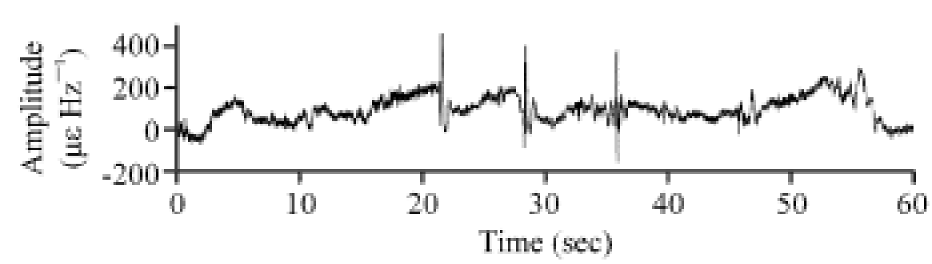

Fig. 2: The time history plot of the original test signal

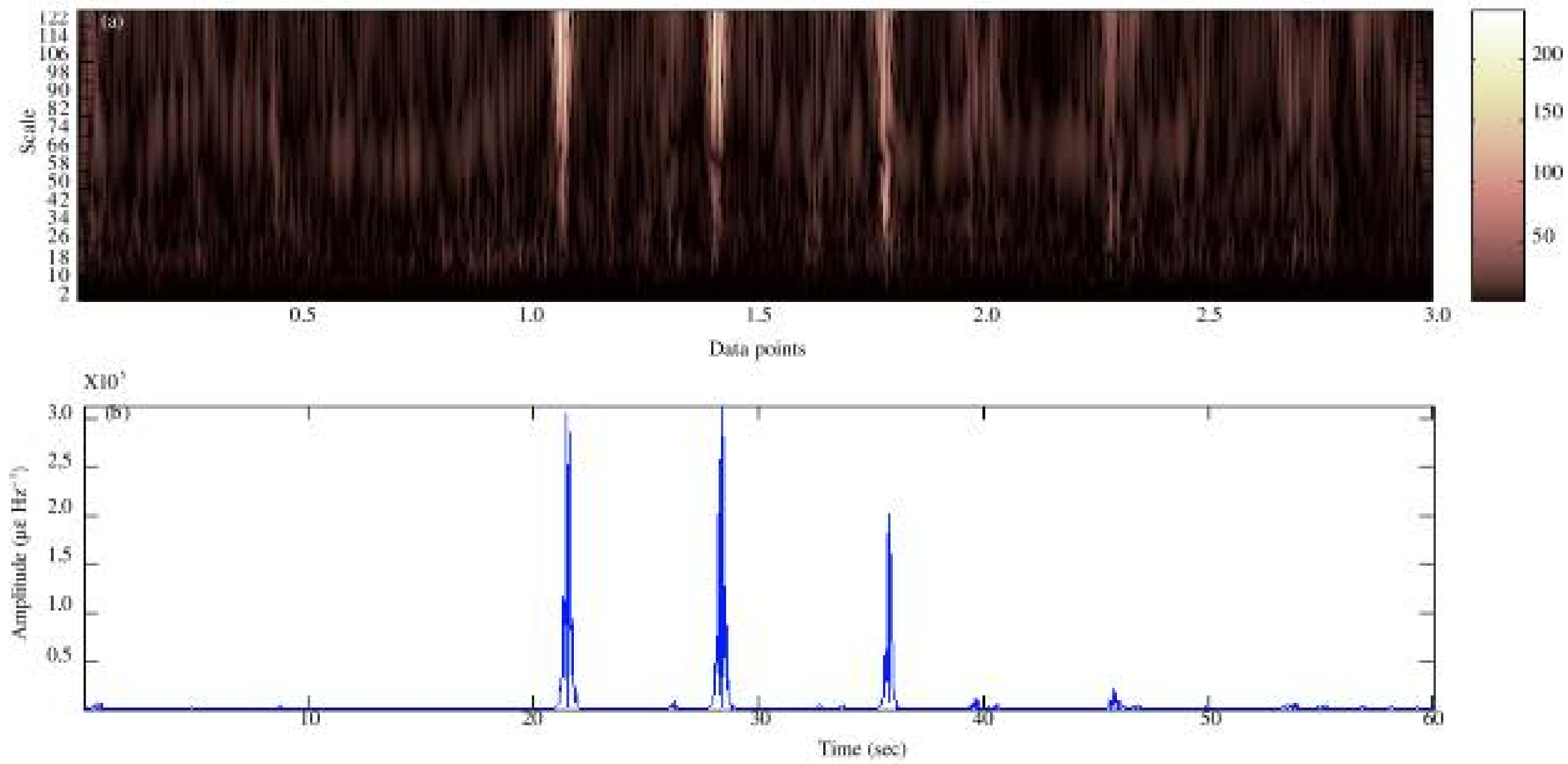

Fig. 3: The distribution of the Morlet wavelet coefficients (a) time-frequency representation and (b) time representation

meaningful cycle in a variable amplitude fatigue strain loading. For the SAE 1045 carbon steel shaft material, it gave the fatigue life of 265.7 reversals (blocks to failure).

The distribution of the Morlet wavelet coefficients was obtained using Eq. 8, as shown in Fig. 3a and b. In the presented scalogram, the x-axis denoted the time parameter and the y-axis represented the scale that has an inversely related to the frequency value. The color intensity at each x-y point was proportional to the absolute value of the wavelet coefficients as a function of the dilation and translation parameters. It provided the signal energy distribution display with respect to the particular time and frequency information.

Using the newly Morlet wavelet-based developed computational algorithm, the wavelet coefficient magnitude segments were transposed into time domain signal. The representation showed a two dimensional view of the energy distribution, as observed in time-frequency plane.

This fatigue signal summarizing computational algorithm uses peak to peak amplitude range as a parameter to determine gate value for the eliminating process, as shown in Fig. 4. The gate value obtained from the wavelet coefficient amplitude at cut off point or fatigue limit of the particular material is used to slice the original signal. The extracted segment identification is performed by searching two inversion points (one on either side of the peak value) which define the temporal extent of the extracted segment. After all the segments are identified, the time history fatigue signal is then sliced to remove the lower wavelet coefficient amplitude (less than the gate value) contained in the original time history range. For this reason, the majority of the original fatigue damage is retained in the edited signal.

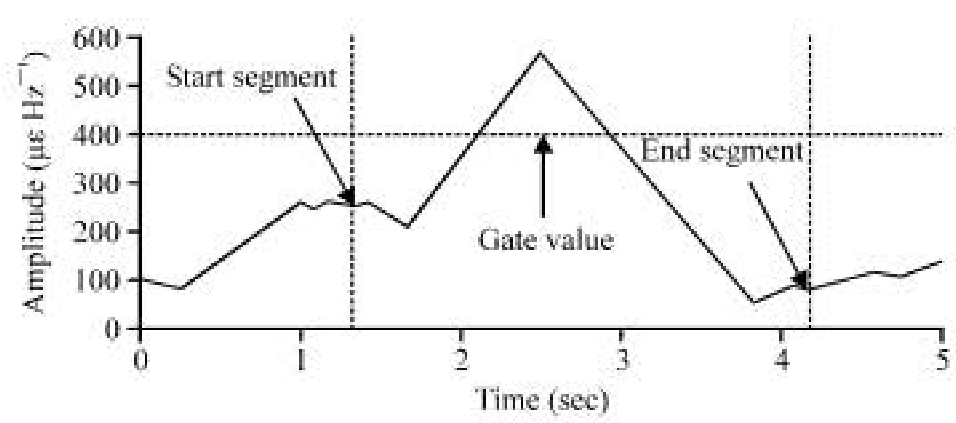

Fig. 4: The extracted segment identification

All the extracted segments (the complete section between the start and the end of the segments) selected based on time location of the wavelet coefficient amplitude are then combined together so produce a new mission time history. The mission signal replicates the signal statistical parameter and total fatigue damaging characteristics of the original time history. The optimum gate value is accordingly determined and it is based on the effectiveness of retaining the characteristics of the original signal in the mission signal. Ideally, the signal has shorter time length but equivalent in the characteristic values.

Various gate values were applied based on try and error method. At the first extraction, 5,000 µε Hz^{-1} was chosen as the gate value giving a signal length of 12.1 sec. From the signal statistical analysis, it was obtained the mean and r.m.s. values were 85.9 and 117.2 µε, respectively, with the differences of 4.5 and 5.5% compared to the original signal. The values were in the required range i.e., ±10% difference. Unfortunately, the kurtosis value of the mission signal changed of 17.1% (4.5 µε). Although, the total fatigue damage given by the mission signal was 3.43×10^{-3} cycles to failure (only 8.9%

reduction), this gate value could not be used as the parameter since, it changed the signal behavior.

Mission signal with higher gate value give an obvious deviation in retaining the originality of the signal statistical parameters and the fatigue damage. Nevertheless, by decreasing the gate value, the deviation was descended and almost reached the original signal behavior. Since at 5,000 $\mu\varepsilon$ Hz^{-1} gate value could not give an eligible mission signal, by necessity, the gate value was decreased. The 4,500 $\mu\varepsilon$ Hz^{-1} was chosen as the second gate value giving 13.8 sec mission signal. This signal gave 77% reduction in length compared to the original signal.

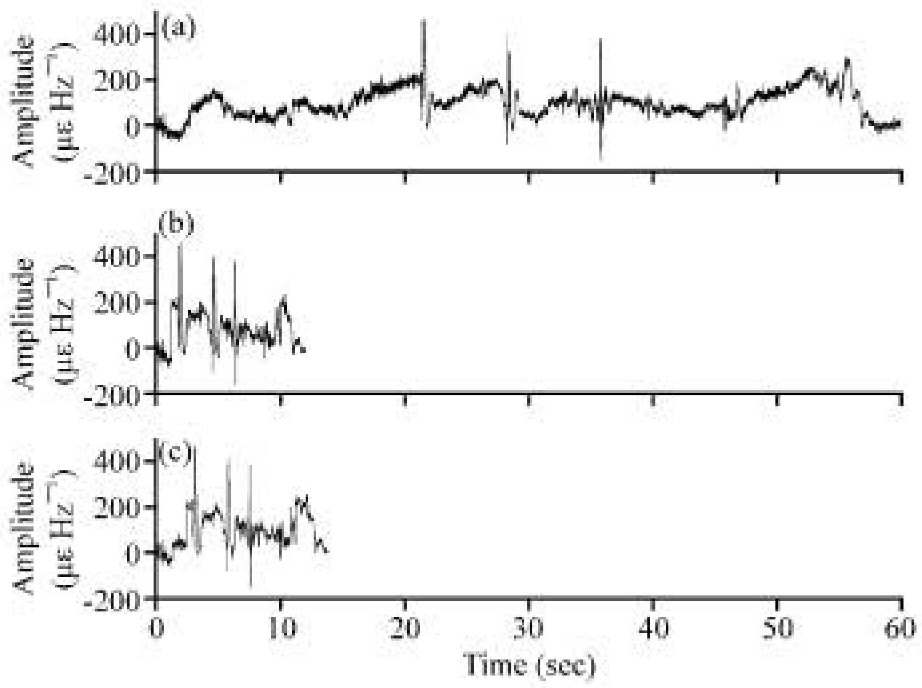

Fig. 5: Signal length comparison: (a) the 60 sec original signal, (b) the 12.1 sec mission signal and (c) the 13.8 sec mission signal

For the signal statistical parameter analysis, the mean, rms and kurtosis values of the mission signal were 86.2, 118 and 4 $\mu\varepsilon$, respectively. The differences of 4.1, 6.1 and 5.7% indicated that this gate value did not change the signal behavior. The numbers of cycle counted for the mission signal were 836 cycles, which were 79.4% less than the original signal. Furthermore, because of the decrement, the total fatigue damage was also decreased. The total fatigue damage was 3.43×10^{-3} cycles to failure, which was 8.7% reduction compared to the original signal. For the fatigue life assessment, it was obtained from the inverse calculation of the total fatigue damage. The numbers of life gathered for the mission signal were 291.1 reversals.

Figure 5a-c show the time history of the original signal compared to the mission signals of fatigue loading. The cycle counting histograms obtained from the original and shortest mission signal are presented in Fig. 6a and b. In this process, all the range less than the gate value were removed. When looking at each histogram, it can be seen that although the signal has been edited in the Morlet wavelet coefficient extraction process, the range and mean values in the mission signal remain unchanged compared to the original signal.

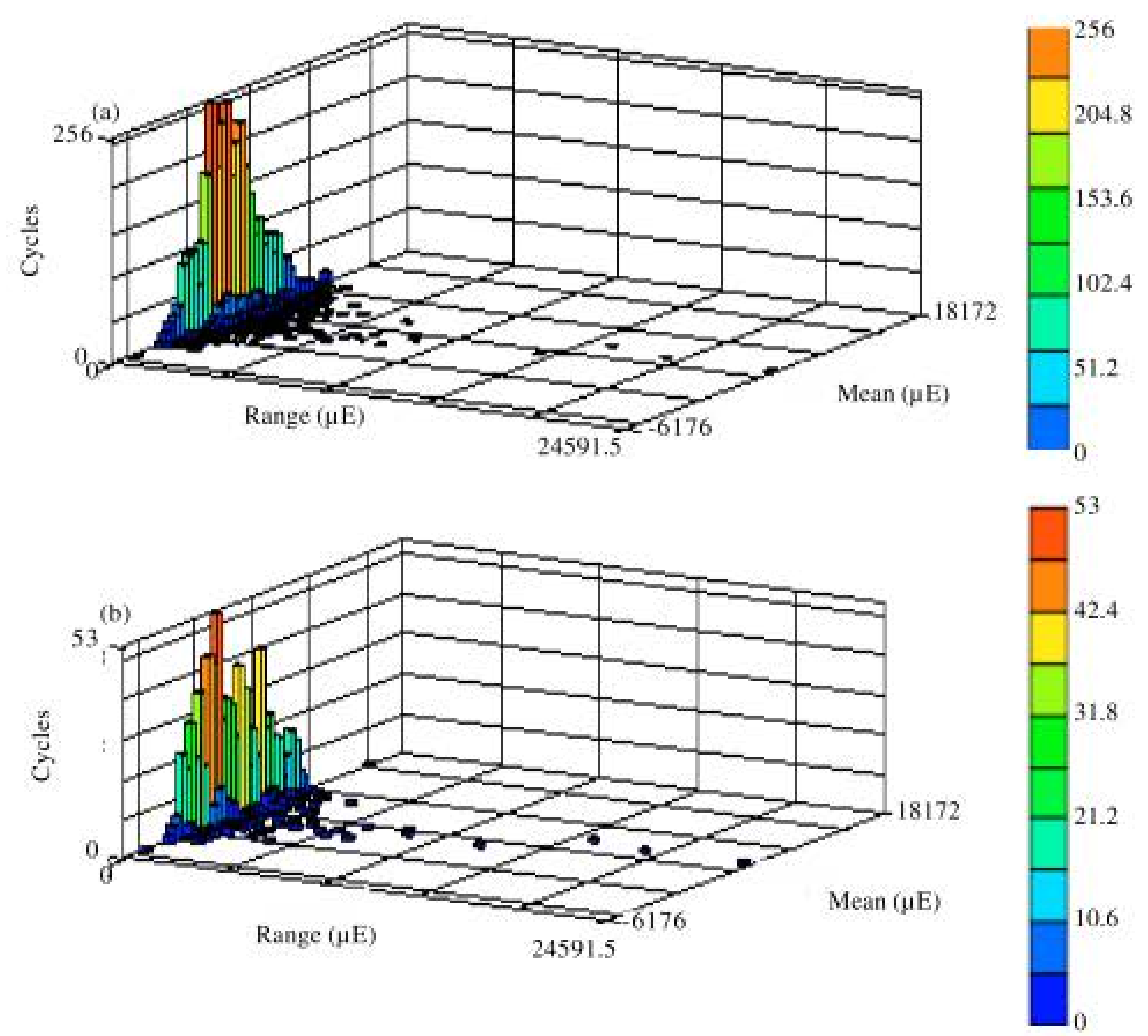

Fig. 6: The cycle counting histograms: (a) the 60 sec original signal and (b) the 13.8 sec mission signal

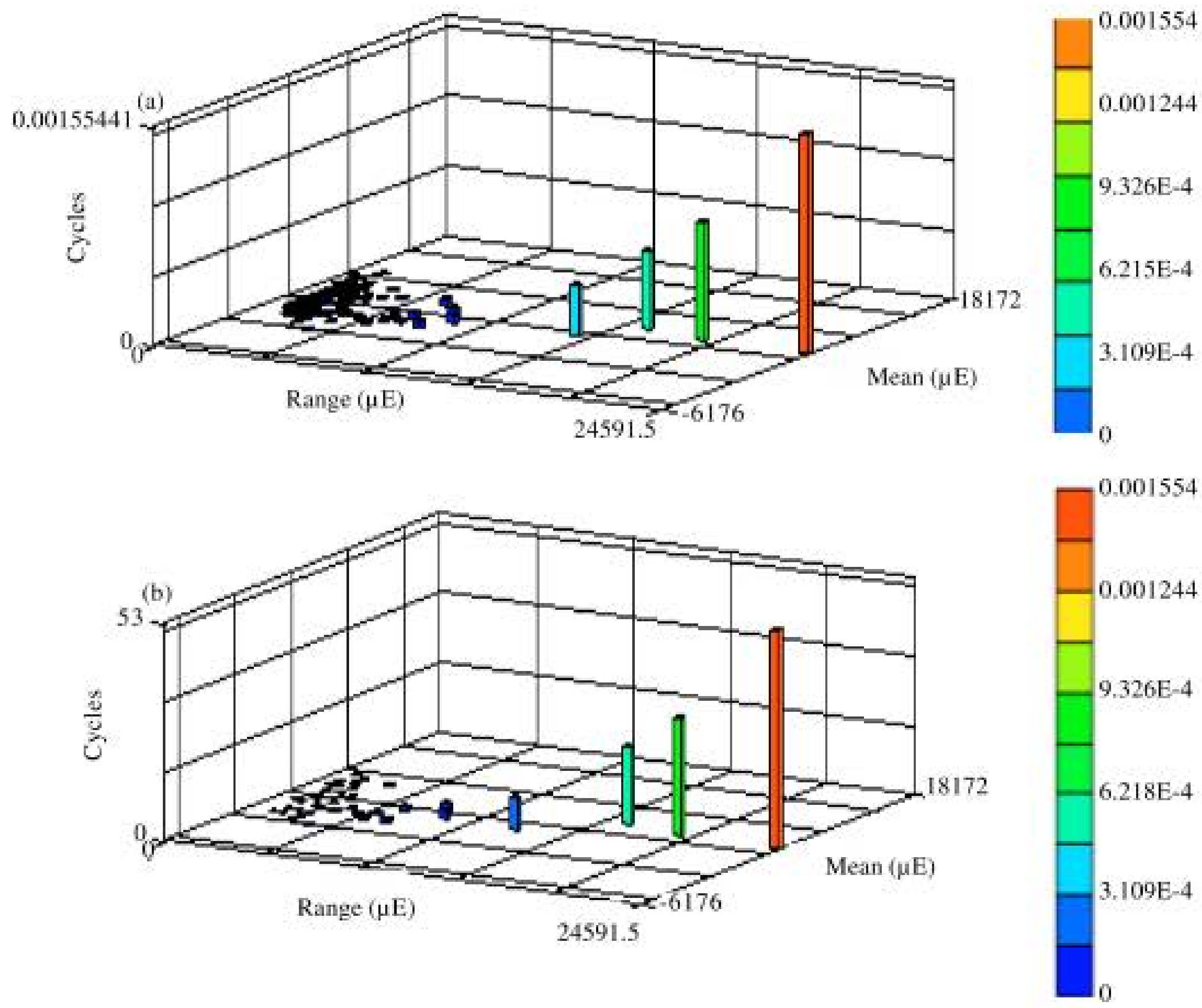

Fig. 7: The fatigue damaging histograms: (a) the 60 sec original signal and (b) the 13.8 sec mission signal

The histograms of the fatigue damaging potential distribution for each cycle are illustrated in Fig. 7a and b. The maximum fatigue damage has been contributed by cycles that have the highest range or the tallest column in the histogram plot. Figure 7 shows that at lower cycle range, the fatigue damage was zero or very minimal value and the departure of those cycles was not effect to the total fatigue damage.

According to the analysis findings, this technique has been found to be able in removing the undamaging segments from the original signal. This procedure coincidentally removed the lower Morlet wavelet coefficient amplitude which was not important in the fatigue damaging assessment. They caused a minimal or did not contribute to the fatigue damage since these cycles had the lower energy. Whereas, the higher Morlet wavelet coefficient amplitude was indicatives of the higher energy that contributes to the more fatigue damaging events. In other words, the higher Morlet wavelet coefficient presented damaging segment, otherwise, it was undamaging segment. It indicated that the relationship between the Morlet wavelet coefficient and the fatigue damage was strong and parallel.

DISCUSSION

This study discussed on the Morlet wavelet coefficient amplitude-based fatigue strain signal extraction. The computational algorithm was developed to identify and extract fatigue features, for which these features contributed to higher fatigue damaging potential. Consequently, the process removed the lower Morlet wavelet amplitude contained in the original signal. In order to verify the effectiveness of the mission signals produced from the difference gate values, the signal statistical parameters and the fatigue damage were determined. The fatigue damage was calculated by utilizing the SWT relationship.

As the result, based on the simulation analysis, the gate value of 4,500 $\mu\varepsilon$ Hz^{-1} was found to be an optimum gate value which contains at least 91.3% of the original fatigue damage in the 13.8 sec mission signal. With respect to the time retention, only 23.1% of the original signal time length was retained using this method. In addition, this process removed 79.4% of the original cycle numbers. The numbers of cycle counted were reduced from 4,051 to 836 cycles in the mission signal compared to the original signal. Although, the signal has edited using the Morlet wavelet extraction algorithm, it has the equivalent signal behavior and pattern as the original signal.

Accordingly, a lower scale indicated higher frequency and had small amplitude that means these cycles had lower energy, indicating minimal or no fatigue damaging potential. A large scale was indicative of lower frequency and higher amplitude that indicates these cycles had higher energy causing the fatigue damage. Obviously, the lower frequency indicated higher magnitude distribution and the lower magnitude distribution was presented at higher frequency event. It indicated that the fatigue damage and the Morlet wavelet had a strong correlation. In conclusion, by using this newly developed fatigue data editing algorithm, the large Morlet wavelet amplitude causing the majority of the fatigue damage were retained and thus only shortened loading which consists of the large Morlet wavelet amplitude produced. The higher wavelet coefficient presented the higher fatigue damage, otherwise, it was the lower fatigue damage. Finally, this fatigue signal summarizing computational algorithm was suggested to be used for the fatigue durability simulation.

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