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Effects of Uncertain Inflationary Conditions on an Inventory Model for Deteriorating Items with Shortages

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Abstract: This study proposes an inventory model with stochastic internal and external inflation rates for deteriorating items and allowable shortages. The many economic, political, social and cultural variables affect the inflation rates. For instance, economic factors such as changes in the world inflation rate, demand level, labor cost, cost of raw materials, exchange rates, unemployment rate, productivity level, tax, liquidity, etc are effective in this direction. Therefore, the assumption of constant inflation rates is not valid, especially, when the time horizon is long. This model considers stochastic inflationary conditions. Numerical examples are used to illustrate the theoretical results, which are further clarified through a sensitivity analysis on the model parameters. It has been shown that the optimal solution is highly sensitive to considerable uncertainty of the inflation rates.

Key words: Inventory, inflation, stochastic, deterioration, shortages

INTRODUCTION

The classical inventory models have not been considered the inflation and time value of money. However, consequence of high inflation, it is important to investigate how time-value of money influences various inventory policies. Since, 1975 a series of related papers appeared that considered the effects of inflation on the inventory system. Before the 1990s, the earlier efforts have been considered simple situations. The first attempt in this field has been reported by Buzacott (1975) that dealt with an Economic Order Quantity (EOQ) model with inflation subject to different types of pricing policies. Misra (1979) developed a discounted-cost model and included internal (company) and external (general economy) inflation rates for various costs associated with an inventory system. Sarker and Pan (1994) surveyed the effects of inflation and the time value of money on order quantity with finite replenishment rate. Some efforts were extended to consider variable demand, such as Uthayakumar and Geetha (2009), Maity (2010), Vrat and Padmanabhan (1990), Datta and Pal (1991), Hariga (1995), Hariga and Ben-Daya (1996) and Chung (2003).

In above cases, it has been implicitly assumed that the rate of inflation is known with certainty. Yet, inflation enters the inventory picture only because it may have an impact on the future inventory costs and the future rate of inflation is inherently uncertain and unstable. Horowitz (2000) discussed an EOQ model with a normal distribution for the inflation rate.

Certain types of commodities either deteriorate or become obsolete throughout course of time and hence are unstable. For example, the commonly used goods like fruits, vegetables, meat, foodstuffs, perfumes, alcohol, gasoline, radioactive substances, photographic films, electronic components, etc. where deterioration is usually observed during their normal storage period. Inventoried goods can be broadly classified into four meta-categories (Goyal and Giri, 2001):

- Obsolescence refers to items that lose their value through time because of rapid changes of technology or the introduction of a new product by a competitor
- Deterioration refers to the damage, spoilage, dryness, vaporization, etc. of the products
- Amelioration refers to items whose value or utility or quantity increase with time
- No obsolescence/deterioration/amelioration

There are several studies of deteriorating inventory models under inflationary conditions. Chung and Tsai (2001) presented an inventory model for deteriorating items with the demand of linear trend considering the time-value of money. Wee and Law (2001) derived a deteriorating inventory model under inflationary conditions when the demand rate is a linear decreasing function of the selling price. Chen and Lin (2002) discussed an inventory model for deteriorating items with a normally distributed shelf life, continuous time-varying demand and shortages under an inflationary and time discounting environment. Chang (2004) established a deteriorating EOQ model when the supplier offers a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. Yang

(2004) discussed the two-warehouse inventory problem for deteriorating items with a constant demand rate and shortages. Moon *et al.* (2005) considered ameliorating/deteriorating items with a time-varying demand pattern. Maiti *et al.* (2006) proposed an inventory model with stock-dependent demand rate and two storage facilities under inflation and time value of money where the planning horizon is stochastic in nature and follows the exponential distribution with a known mean. Lo *et al.* (2007) developed an integrated production-inventory model with assumptions of varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries. A Two storage inventory problem with dynamic demand and interval valued lead-time over a finite time horizon under inflation and time-value of money considered by Dey *et al.* (2008). Maity and Maiti (2008) developed a numerical approach to a multi-objective optimal inventory control problem for deteriorating multi-items under fuzzy inflation and discounting.

Mirzazadeh (2010) assumed the inflation is time-dependent and demand rate is assumed to be inflation-proportional. Roy *et al.* (2009) prepared an inventory model for a deteriorating item with displayed stock dependent demand under fuzzy inflation and time discounting over a random planning horizon. Another research has been performed by Ameli *et al.* (2011) with considering an economic order quantity model for imperfect items under fuzzy inflationary conditions. Other efforts on inflationary inventory systems for deteriorating items have been made by Hsieh and Dye (2010), Sana (2010), Su *et al.* (1996), Chen (1998), Wee and Law (1999), Sarker *et al.* (2000), Yang *et al.* (2001, 2009), Liao and Chen (2003), Balkhi (2004a, b), Hou and Lin (2006), Shah (2006), Hou (2006), Jaggi *et al.* (2006), Yang (2006) and Chern *et al.* (2008).

In this study, a detailed analysis has been done for surveying the effect of uncertain inflationary conditions on the optimal ordering policy under stochastic inflationary conditions and arbitrary probability density functions (pdfs) for the internal and external inflation rates. Deteriorating items and shortages have been considered. A numerical example and a sensitivity analysis are used to illustrate the model.

This study concluded that the No. of replenishments and the expected value of cost are sensitive to the external inflation rate and the optimal solution is sensitive to the uncertainty of the inflation rates when the standard deviations of the inflation rates are sufficiently high. Particular cases of the problem, which follow the main problem and correspond to the situation of (1) a single inflation rate for all cost components, (2) no shortages, (3) no deterioration and (4) all the three previous cases together are discussed.

THE MATHEMATICAL MODEL AND ANALYSIS

The following assumptions are used throughout this study:

- The internal and external inflation rates are random variables with known distribution
- The demand rate is known and constant
- Shortages are allowed and fully backlogged except for the final cycle
- The replenishment is instantaneous and lead time is zero
- The system operates for prescribed time-horizon of length H
- A constant fraction of the on-hand inventory deteriorates per unit time

The cost components may be divided into internal and external classes. Van Hees and Monhemius (1972) have given the breakdown of the various costs of inventory system. In the real world, internal and external costs exhibit different behaviours, so that the internal cost changes by the current inflation rate of the company and the external cost varies with the inflation rate of the general economy (or of the supplier company). Therefore, two different pdfs for the inflation rates can be used in this model. The notations described as follows:

i_m	:	Internal (for m = 1) and external (for m = 2) inflation rates
$f(i_m)$	:	The pdf of i_m
r	:	The discount rate
D	:	The demand rate per unit time
A	:	The ordering cost per order at time zero
c_{lm}	:	The internal (for m = 1) and external (for m = 2) inventory carrying cost (for l = 1) and shortage cost (for l = 2) per unit per unit time at time zero
p	:	The external purchase cost at time zero
θ	:	The constant deterioration rate
$M_{im}(Y)$	:	The moment generating function of i_m for m = 1 and 2
H	:	The fixed time horizon

Other notations will be introduced later. It is assumed that the length of the planning horizon is H = nT, where, n is an integer for the number of replenishments to be made during period H and T is an interval of time between replenishment. The unit of time can be considered as a year, a month, a week, etc. and k ($0<k\leq 1$) is the proportion

of time in any given inventory cycle which orders can be filled from the existing stock. Thus, during the time interval [(j-1)T,jT], the inventory level leads to zero and shortages occur at time (j+k-1)T. Shortages are accumulated until jT before they are backordered and are not allowed in the last replenishment cycle. The optimal inventory policy yields the ordering and shortage points, which minimize the total expected inventory cost over the time horizon.

The mathematical formulation: Let ECP, ECH, ECS and ECR denote the expected present values of the purchasing, carrying, shortage and replenishment costs, respectively. The detailed analysis of each cost function is given as follows:

Expected present value of the purchasing cost: The expected present value of the purchasing cost for the j-th period (j = 1, 2, …, n-1), as shown in Appendix A, is equal to:

$$\begin{aligned} ECP_{j-1} &= (pD/\theta)\, e^{(1-j)rT}(e^{\theta kT}-1)\, M_{i_2}((j-1)T) \\ &\quad + pDT(1-k)e^{-rjT}M_{i_2}(jT), \text{for } j=1,\ldots,(n\text{-}1) \end{aligned} \tag{1}$$

and for the last period is given by Appendix A:

$$ECP_{n-1} = (pD/\theta)\, e^{-r(n-1)T}\,(e^{\theta T}-1)\, M_{i_2}((n-1)T) \tag{2}$$

Therefore, the total purchase cost for all cycles can be written as follows:

$$ECP = ECP_{n-1} + \sum_{j=1}^{n-1} ECP_{j-1} \tag{3}$$

Expected present value of the inventory cost: The inventory carrying cost is divided into internal (for m = 1) and external (for m = 2) classes. From Appendix B the expected present value of the inventory carrying cost for the j-th cycle for the m-th class can be written as:

$$ECH_{jm} = c_{1m}DE\left[\frac{e^{-R_m(j-1)T}\left(1+e^{(\theta-R_m)kT}\left(kT(\theta-R_m)-1\right)\right)}{(\theta-R_m)^2}\right], \tag{4}$$
$$\text{for } j=1,\ldots,(n\text{-}1),\ R_m = r\text{-}i_m,\ m=1,2$$

In the last period the inventory level comes to zero at the end of period. Therefore:

$$ECH_{nm} = c_{1m}DE\left[\frac{e^{-R_m(n-1)T}\left(1+e^{(\theta-R_m)T}\left(T(\theta-R_m)-1\right)\right)}{(\theta-R_m)^2}\right], \tag{5}$$
$$R_m = r\text{-}i_m,\ \text{for } m=1,2$$

The total internal and external carrying costs for all cycles can be given as follows:

$$ECH = \sum_{m=1}^{2}\sum_{j=1}^{n-1} ECH_{jm} + \sum_{m=1}^{2} ECH_{nm} \tag{6}$$

Expected present value of the shortages cost: The shortages cost may be divided to internal and external classes similar to the holding cost. The expected present value of the shortages cost for the j-th cycle for the m-th class can be formulated as follows (Appendix C):

$$ECS_{jm} = c_{2m}DE\left[\frac{e^{-R_m jT} + ((1-k)R_mT-1)e^{-R_mT(k+j-1)}}{{R_m}^2}\right], \tag{7}$$
$$\text{for } j=1,\ldots,(n\text{-}1),\ R_m = r\text{-}i_m,\ m=1,2$$

The total shortages cost during the entire planning horizon H can be written as follows:

$$ECS = \sum_{m=1}^{2}\sum_{j=1}^{n-1} ECS_{jm} \tag{8}$$

Expected present value of the ordering cost: The expected present value of the ordering cost for replenishment at time (j-1)T for the j-th cycle is:

$$ECR_j = Ae^{-rjT}M_{i_1}(jT), \text{for } j=1,\ldots,(n\text{-}1) \tag{9}$$

The total replenishment cost can be given as follows:

$$ECR = \sum_{j=0}^{n-1} ECR_j \tag{10}$$

Hence, the total expected inventory cost of the system during the entire planning horizon H is given by:

$$ETVC(n,k) = ECP + ECH + ECS + ECR \tag{11}$$

THE SOLUTION PROCEDURE

The problem is determining n and k to lead the minimum of the total expected inventory system cost. For a given value of n, the necessary condition of optimality is as follows:

$$\begin{aligned} \frac{dETVC(n,k)}{dk} &= pDT\sum_{j=1}^{n-1}\left[e^{((1-j)r+\theta k)T}M_{i_2}((j-1)T) - e^{-rjT}M_{i_2}(jT)\right] \\ &\quad + \sum_{m=1}^{2}\left[c_{1m}DkT^2e^{-r(k-1)T+\theta kT}\sum_{j=1}^{n-1}\left[e^{-rjT}M_{i_m}((k+j-1)T)\right]\right] \\ &\quad + \sum_{m=1}^{2}\left[c_{2m}DT^2(k-1)\sum_{j=1}^{n-1}\left[e^{-rT(k+j-1)}M_{i_m}(T(k+j-1))\right]\right] = 0 \end{aligned} \tag{12}$$

The iterative methods such as Newton method can be used for calculating k. By increasing n, the objective function decreases to lead to minimum. The second-order condition for a minimum is:

$$\frac{d^2ETVC(n,k)}{dk^2} = pD\theta T^2 \sum_{j=1}^{n-1}\left[e^{((1-j)r+\theta k)T} M_{i_2}((j-1)T)\right] + \sum_{m=1}^{2}\left[c_{1m}DT^2\sum_{j=1}^{n-1}E\left[(1+kT(\theta-R_m))e^{(-R_m(k+j-1)+\theta k)T}\right]\right] + \sum_{m=1}^{2}\left[c_{2m}DT^2\sum_{j=1}^{n-1}\left[E\left[e^{-R_m(k+j-1)T}(TR_m(1-k)+1)\right]\right]\right] > 0 \quad (13)$$

NUMERICAL EXAMPLE

According to the results, the following example is providing. Let r = \$0.2/\$/year, D = 1000 units/year, A = \$100/order, c_{11} = \$0.2/unit/year, c_{12} = \$0.4/unit/year, c_{21} = \$0.8/unit/year, c_{22} = \$0.6/unit/year, p = \$5/unit, H = 10 years, θ = 0.01. The internal and external inflation rates have the normal distribution function with means of μ_1 = 0.08 and μ_2 = 0.14, standard deviations of σ_1 = 0.04 and σ_2 = 0.06, respectively. The results are shown in Table 1. The minimum expected cost over the planning horizon is 44 537.26 for n* = 41 and k* = 0.664623. Optimal interval of time between replenishment, T*, equals to H/n* = 0.244 year.

SENSITIVITY ANALYSIS

To study the effects of system parameters changes D, H, θ, r, μ_1, μ_2, σ_1, σ_2, A, p, c_{11}, c_{12}, c_{21} and c_{22} on the optimal cost, the replenishment time and k* which is derived by the proposed method, a sensitivity analysis was performed. This fact is done by increasing the parameters by 20, 50, 100% and decreasing the parameters to 20, 50, 90%, taking each one at a time and keeping the remaining parameters at their original values. The following conclusion can be derived from the sensitivity analysis based on Table 2:

- As the mean of the internal inflation rate increases, the number of replenishments (n) decreases and k increases. By increasing the mean of external inflation rate, increase the number of replenishments (n) and k. The optimal expected present value of cost (ETVC) increases when μ_1 and μ_2 increase but is highly sensitive to μ_2. Induction of this result is the purchase cost increasing by external inflation rate is more than other cost components, i.e., purchase cost constitutes considerable portion of total cost. Hence, the total cost has a similar role as the purchase cost
- Table 2 shows that the optimal value of k and number of replenishments (n) are insensitive to changes in the standard deviations of inflation rates. But, the operating doctrine is highly sensitive, to high values of σ_1 and σ_2. For example, considering σ_1 = 0.04 and σ_2 = 0.4, Table 3 shows that the optimal number of replenishments equals 1 and we have no shortages (n* = k* = 1). The numerical example has solved by considering σ_1 = 0.04, 0.1, 0.2, 0.3, 0.4, 0.5 and σ_2 = 0.06, 0.1, 0.2, 0.3, 0.4 and 0.5. The results are shown in Table 3. The derived model is sufficiently sensitive if the uncertainty of inflation rates is higher than a certain magnitude. In that situation, larger order quantity should be placed
- High uncertainty inflation rate is occurred in the real world, for instants, international financial Statistical Yearbook, 2004, shows that the mean change of wholesale prices indices in Russia in 1993-2001 was 1.91 and it's the standard deviation was 3.05. For another case, in Norway in 1997-2001 the mean change of wholesale prices indices were 0.08 and the standard deviation was 0.2
- The number of replenishments (n) is highly sensitive to the change of the parameters D, A and H, is slightly sensitive to changes in c_{12} and insensitive to changes in r, θ, p, c_{11}, c_{21} and c_{22}
- The optimal value of k is highly sensitive to the change of the parameters c_{12}, c_{21} and c_{22}, is moderately sensitive to r and c_{11} and is insensitive to D, θ, H, p and A
- The total expected inventory cost of the system is highly sensitive to the changes in the parameters D, r, H and p and insensitive to θ, A, c_{11}, c_{12}, c_{21} and c_{22}

Table 1: Optimal solution for numerical example

n	K	ETVC (n, k)	n	k	ETVC (n, k)
2	0.657362	98 743.29	40	0.664614	44 539.42
3	0.659947	76 905.97	41*	0.664623	44 537.26
5	0.661980	61 198.56	45	0.664656	44 556.16
10	0.663489	50 521.04	50	0.664689	44 629.30
15	0.663990	47 319.78	55	0.664716	44 743.37
20	0.664240	47 257.46	60	0.664739	44 888.07
25	0.664390	45 170.48	70	0.664774	45 243.00
30	0.664489	44 789.17	80	0.664801	45 657.88
35	0.664561	44 603.47	100	0.664838	46 595.31

Table 2: Effects of changes in model parameters on n, k and optimal expected system cost

		-90%	-50%	-20%	0%	20%	50%	100%
D	n	13	29	37	41	45	50	58
	k	0.663836	0.664472	0.664584	0.664623	0.664656	0.664689	0.664730
	ETVC	5 552.40	23 301.14	36 098.98	44 537.26	52 926.26	65 442.69	86 176.54
r	n	39	40	41	41	41	42	42
	k	0.745769	0.708673	0.681926	0.664623	0.647734	0.623124	0.583672
	ETVC	11 5611.20	73 225.55	53 797.50	44 537.26	37 360.53	29 398.54	20 908.19
μ_1	n	45	43	42	41	40	39	37
	k	0.649265	0.655729	0.660958	0.664623	0.668429	0.674403	0.684991
	ETVC	43 739.44	44 059.29	44 334.21	44 537.26	44 758.00	45 126.13	45 859.41
μ_2	n	37	38	40	41	43	45	50
	k	0.650107	0.654418	0.659778	0.664623	0.670675	0.682305	0.709432
	ETVC	27 898.19	33 779.60	39 668.73	44 537.26	50 358.13	61 364.32	88 405.41
σ_1	n	42	42	41	41	41	41	40
	k	0.664631	0.664631	0.664623	0.664623	0.664623	0.664623	0.664614
	ETVC	44 491.86	44 502.88	44 520.65	44 537.26	44 557.89	44 596.90	44 684.41
σ_2	n	41	41	41	41	41	41	40
	k	0.658875	0.660179	0.662418	0.664623	0.667511	0.673437	0.689420
	ETVC	42 548.68	43 015.07	43 790.72	44 537.26	45 488.20	47 357.34	51 990.19
θ	n	41	41	41	41	41	41	41
	k	0.667606	0.666277	0.665284	0.664623	0.663963	0.662976	0.661337
	ETVC	44 515.82	44 525.37	44 532.50	44 537.26	44 541.99	44 549.08	44 560.86
H	n	4	20	33	41	50	63	85
	k	0.673426	0.669066	0.666261	0.664623	0.663200	0.661471	0.659803
	ETVC	56 30.38	24 959.53	37 151.40	44 537.26	51 500.36	61 418.60	77 422.05
p	n	40	41	41	41	41	42	42
	k	0.684788	0.675807	0.669090	0.664623	0.660165	0.653504	0.642430
	ETVC	8 783.22	24 675.04	36 592.69	44 537.26	52 481.39	64 396.48	84 252.33
A	n	130	58	46	41	38	34	29
	k	0.664873	0.664730	0.664662	0.664623	0.664594	0.664548	0.664472
	ETVC	41 168.96	43 088.27	44 013.86	44 537.26	45 011.55	45 655.45	46 602.29
c_{11}	n	38	39	40	41	42	43	44
	k	0.721019	0.694817	0.676380	0.664623	0.653266	0.636935	0.611449
	ETVC	44 106.50	44 311.37	44 450.76	44 537.26	44 619.61	44 735.86	44 912.56
c_{12}	n	29	36	39	41	43	45	48
	k	0.832512	0.748448	0.695783	0.664623	0.636142	0.597722	0.543083
	ETVC	43 091.96	43 876.31	44 302.00	44 537.26	44 743.19	45 011.44	45 368.26
c_{21}	n	37	39	40	41	42	43	44
	k	0.511705	0.594053	0.639568	0.664623	0.686413	0.714249	0.751060
	ETVC	43 962.37	44 280.14	44 447.77	44 537.26	44 614.02	44 710.66	44 835.60
c_{22}	n	37	39	41	41	42	43	44
	k	0.519767	0.597051	0.640506	0.664623	0.685723	0.712830	0.748933
	ETVC	43 992.07	44 290.75	44 450.96	44 537.26	44 611.71	44 706.01	44 828.75

Table 3: Effects of changes in the standard deviations of inflation rates on n, k and optimal expected system cost

		σ_1					
σ_2		0.04	0.1	0.2	0.3	0.4	0.5
0.06	n	41	39	30	15	5	3
	k	0.640506	0.664604	0.664489	0.663989	0.661980	0.659946
	ETVC	44 450.96	44 804.75	46 257.91	51 864.27	68 571.72	88 683.42
0.1	n	41	39	30	15	5	3
	k	0.677768	0.677728	0.677476	0.676400	0.672259	0.668315
	ETVC	48 664.20	48 928.81	50 363.75	55 884.01	72 146.76	91 693.11
0.2	n	31	29	22	11	4	3
	k	0.833115	0.831904	0.826068	0.804142	0.749069	0.729375
	ETVC	83 619.22	83 814.55	84 864.81	88 690.88	98 429.21	11 1760.84
0.3	n	2	2	2	2	2	2
	k	0.810799	0.810799	0.810799	0.810799	0.810799	0.810799
	ETVC	147 455.25	147 461.45	147 489.75	147 568.31	147 804.78	148 648.35
0.4	n	1	1	1	1	1	1
	k	1.00	1.00	1.00	1.00	1.00	1.00
	ETVC	181 725.76	181 725.76	181 725.76	181 725.76	181 725.76	181 725.76
0.5	n	1	1	1	1	1	1
	k	1.00	1.00	1.00	1.00	1.00	1.00
	ETVC	181 725.76	181 725.76	181 725.76	181 725.76	181 725.76	181 725.76

SOME PARTICULAR CASES

Here, an attempt has been made to study some important special cases of the model.

Case (I): If the internal and external inflation rates have the same pdf, the expected present value of the total cost ETVC (n,k) can be obtained by deleting: $\sum_{m=1}^{2}$ in Eq. 11 and substituting:

$$c_{lm} = c_l, R_m = R, i_m = i \text{ , for } l = 1,2 \text{ and } m = 1, 2 \quad (14)$$

The previous numerical example assumes that the inflation rate has the normal distribution function with the mean of $\mu = 0.11$ and the standard deviation of $\sigma = 0.05$. The optimal solution in this case is as follows: n* = 38, k* = 0.666099, ETVC(n,k) = 39 296.36 and T* = 0.263 year. The number of replenishments and inventory system cost decrease and k increases.

Case (II): If shortages are not allowed, k = 1 can be substituted in expression (11) and the expected present worth of the total variable cost ETVC(n) can be obtained. The minimum solution of ETVC(n) for the discrete variable of n must satisfy the following equation:

$$\Delta ETVC(n) \le 0 \le \Delta ETVC(n+1) \quad (15)$$

where, ETVC(n) = ETVC(n)-ETVC(n-1). In the numerical example, using the above inequality, the following solution is obtained: n* = 50, ETVC(n) = 45 613.73 and T* = 0.2 year. It shows that n and ETVC increase in the without shortages case.

Case (III): If available inventory has no deterioration ($\theta = 0$) over time the cost function, after modeling, may be rewritten as follows:

$$\begin{aligned} ETVC(n,k,\theta=0) &= A\sum_{j=0}^{n-1}\left[e^{-rjT}M_{i_1}(jT)\right] \\ &+ pDT\left[e^{-rT(n-1)}M_{i_2}((n-1)T) + \sum_{j=1}^{n-1}\left[ke^{-rT(j-1)}M_{i_2}((j-1)T) - (k-1)e^{-rTj}M_{i_2}(jT)\right]\right] \\ &+ \sum_{m=1}^{2}\left[c_{1m}D\left[\sum_{j=1}^{n-1}E\left[\frac{e^{-R_m(j-1)T}\left(1-e^{-R_m kT}\left(kTR_m+1\right)\right)}{R_m{}^2}\right] + E\left[\frac{e^{-R_m(n-1)T}\left(1-e^{-R_m T}\left(TR_m+1\right)\right)}{R_m{}^2}\right]\right]\right] \\ &+ \sum_{m=1}^{2}\sum_{j=1}^{n-1}\left[c_{2m}DE\left[\frac{e^{-R_m jT} + ((1-k)R_m T - 1)e^{-R_m T(k+j-1)}}{R_m{}^2}\right]\right] \end{aligned} \quad (16)$$

The cost function can be minimized by the methods indicated in this study. For a given value of n, the necessary condition of optimality is:

$$\begin{aligned} \frac{dETVC(n,k,\theta=0)}{dk} &= pDT\sum_{j=1}^{n-1}\left[e^{-rT(j-1)}M_{i_2}((j-1)T) - e^{-rjT}M_{i_2}(jT)\right] \\ &+ \sum_{m=1}^{2}\left[c_{1m}DkT^2e^{-rT(k-1)}\sum_{j=1}^{n-1}\left[e^{-rjT}M_{i_m}((k+j-1)T)\right]\right] \\ &+ \sum_{m=1}^{2}\left[c_{2m}DT^2(k-1)\sum_{j=1}^{n-1}\left[e^{-rT(k+j-1)}M_{i_m}(T(k+j-1))\right]\right] = 0 \end{aligned} \quad (17)$$

and the sufficient condition is the second derivative is positive. In this case, the numerical result is obtained as follows: n* = 41, k* = 0.667940, ETVC(n,k) = 44 513.44 and T* = 0.244 year. Thus ETVC is decreased, k is increased and n is not changed in comparison to the main model.

Case (IV): Now assume that the internal and external inflation rates have the same pdf, no shortages allowed and $\theta = 0$. This may be solved by using Eq. 16 and 17, substituting k = 1 and considering (14). Therefore, the optimal solution is as follows: n* = 46, ETVC(n,k) = 40 391.48 and T* = 0.217.

CONCLUSIONS

Usually, in the inventory systems under inflationary conditions, it has been assumed that the inflation rates are constant over the planning horizon. But, many economic, political, social and cultural variables may also affect the future changes in the inflation rate. Therefore, assuming constant inflation rates is not valid, especially when the time horizon is more than two years. The current study considers stochastic inflation rates where any distribution function is applicable.

This model incorporates some realistic features that are likely to be associated with the inventory of certain types of goods. First, deterioration over time is a natural feature for goods. Second, occurrence of inventory shortages is a natural real situation phenomenon. In the solution process of the problem, the subject of moment generating function has been used. The numerical examples have been given and sensitivity analysis has been conducted to illustrate the theoretical results. The results of sensitivity analysis indicate that which of the parameters are more sensitive to the optimal solution. Also, the results indicate the importance of taking into account stochastic inflation, especially when there is considerable uncertainty associated with the inflation rates. Finally, four special cases have been discussed: identical inflation rates, no shortages situation, no deterioration and considering all the three cases simultaneously. These cases are compared with the main model through the numerical example.

APPENDIX A

During any given period, the order quantity is consisting of both demand and deterioration for the relevant period excluding shortage part of the period and the amount required to satisfy the demand during the shortage period in the preceding time interval. For the j-th cycle (j = 1, 2, ..., n-1) the expected present value of the purchase cost can be formulated as follows:

$$\begin{aligned} ECP_{j-1} &= E\left[pe^{-R_2(j-1)T}\int_{(j-1)T}^{(k+j-1)T} De^{\theta(t-(j-1)T)}dt + pe^{-R_2 jT}\int_{(k+j-1)T}^{jT} Ddt\right] \\ &= E\left[pD\left[e^{-R_2(j-1)T}(e^{\theta kT}-1)/\theta + e^{-R_2 jT}(1-k)T\right]\right] \\ &\text{for: } j = 1\ldots n\text{-}1,\ R_2 = r - i_2 \end{aligned} \quad \text{(A1)}$$

The above equation can be rewritten as Eq. 1. In the last period shortages are not allowable, therefore the expected present value of the purchase cost is:

$$\begin{aligned} ECP_{n-1} &= E\left[pe^{-R_2(n-1)T}\int_{(n-1)T}^{nT} De^{\theta(t-(n-1)T)}dt\right] \\ &= E\left[(pD/\theta)e^{-R_2(n-1)T}(e^{\theta T}-1)\right] \end{aligned} \quad \text{(A2)}$$

where $R_2 = r - i_2$. It can similarly be rewritten as Eq. 2.

APPENDIX B

The expected present value of the inventory carrying cost for the j-th cycle (j = 1, 2, ..., n-1) for the m-th class (m = 1,2) is:

$$\begin{aligned} ECH_{jm} &= E\left[c_{1m}\int_{(j-1)T}^{(k+j-1)T}(t-(j-1)T)De^{-R_m t}e^{\theta(t-(j-1)T)}dt\right] \\ &= c_{1m}DE\left[\left[(t-\frac{1}{\theta-R_m})\frac{e^{-R_m t+\theta(t-(j-1)T)}}{\theta-R_m}\right]_{(j-1)T}^{(k+j-1)T} - \left[\frac{(j-1)T}{\theta-R_m}e^{-R_m t+\theta(t-(j-1)T)}\right]_{(j-1)T}^{(k+j-1)T}\right] \\ &= c_{1m}DE\left[\frac{e^{-R_m(k+j-1)T+\theta kT}}{(\theta-R_m)^2}(k(\theta-R_m)-1) + \frac{e^{-R_m(j-1)T}}{(\theta-R_m)^2}\right] \\ &= c_{1m}DE\left[\frac{e^{-R_m(j-1)T}(1+e^{(\theta-R_m)kT}(kT(\theta-R_m)-1))}{(\theta-R_m)^2}\right] \end{aligned} \quad \text{(B1)}$$

where, $R_m = r - i_m$ and Eq. B 1 can be rewritten as Eq. 4. In the last period for the m-th class (m = 1,2) from similar machinations we have:

$$\begin{aligned} ECH_{nm} &= E\left[c_{1m}\int_{(n-1)T}^{nT}(t-(n-1)T)De^{-R_m t}e^{\theta(t-(n-1)T)}dt\right] \\ &= c_{1m}DE\left[\frac{e^{-R_m(n-1)T}(1+e^{(\theta-R_m)T}((\theta-R_m)T-1))}{(\theta-R_m)^2}\right] \\ &R_m = r - i_m,\ m = 1,2 \end{aligned} \quad \text{(B2)}$$

APPENDIX C

The expected present value of the shortages cost for the j-th cycle (j = 1, 2, ..., n-1) for the m-th class (m = 1,2) can be computed as:

$$\begin{aligned} ECS_{jm} &= E\left[c_{2m}\int_{(k+j-1)T}^{jT}(jT-t)De^{-R_m t}dt\right] \\ &= c_{2m}DE\left[\left[\frac{e^{-R_m t}}{-R_m}(jT-t-\frac{1}{R_m})\right]_{(k+j-1)T}^{jT}\right] \\ &= c_{2m}DE\left[\frac{e^{-R_m jT}}{R_m{}^2} - \frac{e^{-R_m(k+j-1)T}}{-R_m}((1-k)T-\frac{1}{R_m})\right] \\ &= c_{2m}DE\left[\frac{e^{-R_m jT}(1+((1-k)R_m T-1)e^{-R_m T(k-1)})}{R_m{}^2}\right] \end{aligned} \quad \text{(C1)}$$

where, $R_m = r - i_m$. It can be rewritten as Eq. 7.

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