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Nile River Flow Forecasting Based Takagi-Sugeno Fuzzy Model

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Abstract: In this study, a fuzzy model for forecasting the Nile river flow is developed. The fuzzy model is represented by a set of rules based on the Takagi-Sugeno type. The Gustafsson-Kessel (GK) algorithm was applied to determine the antecedent membership functions and least-square estimation was used to determine the consequence parameters. The performance of the fuzzy model was tested using a set of measurements recorded at Dongola station in Egypt. The readings span over the period from 1975 to 1993. These measurements were split in to two groups one for training and another for testing. The performance of the developed proposed fuzzy model was checked in both training and testing cases. The developed fuzzy logic model showed a better modeling capability compared to traditional modeling.

Key words: Forecasting, fuzzy model, Nile river, Dongola station, Egypt, Sudan

INTRODUCTION

Time variations of river flow rate have always been predicted for actual use in advance of the daily power system's operation by using various methods, to be able to convert the cleaner energy stored in water reservations into electric energy as effectively as possible in hydro-power plants. It is also important to mention that models for river flow forecasting are fundamental tools in water resources studies, since they determine and provide the basis in establishing future reservoir water inflows. These predictions are of significance importance in the planning of water resources system, being responsible for the optimization of the system as a whole. This is why forecasting river flows is a fundamental topic in many engineering applications like constructing dams, analysis and forecasting, planning and designing of reservoirs, hydro-power generation, irrigation, water management, controlling floods and others (Sheta and Mahmoud, 2001; Sheta and De Jong, 2001).

A variety of models have been proposed for forecasting the annual flow of rivers. Few of them use the Linear Prediction (LP) model (Kothyari *et al.*, 1993; Zeng and Singh, 1996; Said, 1993) Auto-Regressive Moving Average (ARMA) model, Kalman Filter, Neural Networks (NNs) and Neuro-Fuzzy (Karunanithi *et al.*, 1994; Burlando *et al.*, 1993; Baareh *et al.*, 2006; El-Shoura *et al.*, 1998; EL-Shafie *et al.*, 2007). The importance of estimating and forecasting river flows to the

people living in the communities and villages around and beside the rivers made them study and record their levels since earliest history. In fact, there are records for the flow of the Nile river dating back to around 3000 BC (Said, 1993). It is also important to mention that the ancient Egyptians recorded the annual peak river levels for the years 3050 till 2500 BC. This has produced one of the longest time series of a natural phenomenon. It can therefore, being used as a benchmark time series for studying and comparing forecasting algorithms. Another important reason to study the Nile river flow is that the Nile river's average discharge is 3.1 million L (680,000 gallons) per second. This is why it is important to study, model and analyze the flow of the Nile river and be able to build a model for its forecasting, simulation and for projections.

In this study, we explore the use of fuzzy logic model to build a suitable model for forecasting the Nile river flow. Modeling nonlinear dynamics using fuzzy systems has increasingly been recognized as a distinct and important system identification paradigm (Klir and Folger, 1992; Dubois and Prade, 1992). It refers to a process whereby a dynamical system is modeled and not in the form of conventional differential and difference equations, but in the form of a set of fuzzy rules and corresponding membership functions. A detailed description of the forecasting problem will be provided. Also, the methodology for building our fuzzy model is introduced.

Characteristic of the flow problem: Flow in streams and rivers are complex random processes. They are random because they do not attain a stable form. Their flow value differs from a year to year and from month to month even from day today. Most river flows exhibit seasonal nature. Forecasting period is determined according to the river flow nature and its importance. They are complicated processes because they are influenced by many factors such as vegetation, cover, soil types, channel characteristics, ground aquifers, prediction distribution, rainfall, evaporation losses, consumption of river water and other factors. Many difficulties and problems arise when trying to solve the stream flow forecasting (Fan, 1994). The main reason is flow are multi-dimensional processes and they are defined by many factors like magnitude, velocity and timing duration.

Model identification problem: The problem of forecasting could be interpreted as a model identification problem. Selecting a suitable model structure for flow forecasting depends on many factors like the size of the basins of interest, availability of data records for the flow in that basin, availability of remote sensing instruments such as weather radar, satellites and etc. Present goal is to use set of measurements, known as training set, to build a model for the flow and then test the performance of the developed model using another set of measurements, called testing set. The selection of a model structure from which the process model can be obtained is an important task in any system identification problem (Ljung, 1987). Having established that the process exhibits nonlinear characteristics, a choice of nonlinear model set must be made.

Models which are linear in the parameters have received a considerable amount of attention in recent years than hierarchical multilevel models, block oriented models or models based upon functional series expansions (Haber and Unbehauen, 1990). This is because models which are linear in the parameters, in comparison to hierarchical multilevel models, offer simple structure identification algorithms, a better option to the measured output signal and easier incorporation of a priori knowledge into the model. Block oriented models such as Hammerstein and Wiener models can only be applied if the process has, or can be approximated by, a block-oriented structure, which restricts the class of system that can be represented (Wigren, 1993, 2007).

Fuzzy modeling and identification: In this study, we concentrate on approximation of a nonlinear system by a

set of local linear models. Each local model is valid for a certain range of operating conditions and an interpolative scheduling mechanism combines the outputs of the local models into a continuous global output. Such a model structure can be conveniently represented by means of fuzzy If-Then rules. Using membership functions, the antecedent of the rule defines a fuzzy region in the product space of the antecedent variables in which the rule is valid. The antecedent variables must convey information about the process operating conditions. The consequent of the rule is typically a local linear regression model. The overlap of the antecedent membership functions of different rules provides a smooth interpolation of the rules' consequents.

A rule-based fuzzy model requires identification of the antecedent and consequent structure, of the membership functions for different operating regions and estimation of the consequent regression parameters. While the latter task can be solved using linear estimation techniques, the construction of the membership functions is a nonlinear optimization problem. The presented approach does not require any prior knowledge about the operating regimes and also an appropriate number of rules can be determined automatically. If a sufficiently rich identification data set covering the operating ranges of interest is not available, the rules obtained from data can be combined with prior knowledge, if any, transformed into the membership functions for the relevant operating regions and the local models. The models provided by the user can also be nonlinear (semi) mechanistic models based on the first principles.

Fuzzy model structure: Takagi-Sugeno (TS) fuzzy models are suitable to model a large class of nonlinear systems (Kosko, 1994; Babuska *et al.*, 1998). Consider a nonlinear type system given its input and output, we can determine what the next output will be. In the discrete-time system we can write the relationship between the system input $u(k)$ and output $y(k)$ at time k as follows:

$$y(k) = f(u(k-1), y(k-1)) \quad (1)$$

$$y(k) = f(y(k-1), y(k-2), \dots, y(k-n+1), u(k-1), u(k-2), \dots, u(k-m+1)) \quad (2)$$

where, $u(k-1), \dots, u(k-m+1)$ and $y(k-1), \dots, y(k-n+1)$ represents the past model inputs and outputs, respectively, n and m are integers related to the model order. For example, a singleton fuzzy model of a dynamic system may consist of rules of the following form:

Ri: If $y(k-1)$ is A_1 and ... and $y(k-n+1)$ is A_n
and $u(k)$ is B_1 and ... and $u(k-m+1)$ is B_m
then $y(k)$ is c_i (3)

Here, we are more interested in the special case of the NARX which is the NAR (Nonlinear Auto Regressive) model which can be represented as follows:

$$y(k) = f(y(k-1), y(k-2), \dots, y(k-n+1)) \quad (4)$$

A singleton fuzzy model of a dynamic system may consist of rules of the following form:

Ri: If $y(k-1)$ is A_1 and ... and $y(k-n+1)$ is A_n then $y(k)$ is c_i (5)

Since, fuzzy models can approximate any smooth function to any degree of accuracy (Wang, 1992) models of the type NARX can approximate any observable and controllable models of a large class of discrete-time nonlinear systems (Leontaritis and Billings, 1985).

Model structure selection: The structure of the model i.e. the values of n, m are determined by the user on the basis of prior knowledge and/or comparing several candidate structure in terms of the prediction error or other selected criteria (Sheta and El-Sherif, 1999; Kumar *et al.*, 2004). Once the model structure is selected the next step is to estimate the parameters of the fuzzy model. These parameters include the antecedent membership functions and the consequence polynomials. An additional number of parameters need to be selected so that it is the number of rules (clusters) σ which needs to be specified by the user. The methodology to build a fuzzy model for forecasting the Nile river flow can be described in the following steps:

- Using the flow sequence of measurements $y(k-1)$, $y(k-2)$, $y(k-3)$, $y(k-4)$ and the user defined parameters we form the nonlinear regression problem to find $y(k)$
- Compute the antecedent membership function from the cluster parameters
- Given the antecedent membership functions, estimate the consequence parameters by the least-square method

This technique was introduced (Babuska *et al.*, 1996) and was successfully applied to solve variety of modeling and control of Multi Input Single Output (MISO) system process (Babuska and Verbruggen, 1996; Sosa *et al.*, 1997).

Regression matrix: Using the set of measurements N for the Nile river flow we build the regression matrix Φ and the output vectory.

$$\Phi = \begin{pmatrix} y(k-1) & \dots & y(k-4) \\ y(k) & \dots & y(k-3) \\ \dots & \dots & \dots \\ y(k+N-1) & \dots & y(k+N-4) \end{pmatrix}$$

$$y = \begin{pmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+N) \end{pmatrix}$$

Fuzzy clustering: Given the regression matrix and Φ the specified number of clusters σ Gustanfson-Kessel (GK) algorithm (Gustafso and Kessel, 1979) is applied. This algorithm computes the following:

- The fuzzy partition matrix $U = [\mu_{ik}] \sigma \times N$ with $\mu_{ik} \in [0, 1]$. i stand for the rule number
- The prototype matrix, $V = [v_1, \dots, v_\sigma]$
- The set of cluster covariance matrices $F = [F_1, \dots, F_\sigma]$, F_i are positive definite matrices in $R^{(p+1) \times (p+1)}$. p is the dimension of the antecedent space which equal:

$$\sum_{j=1}^{ny} n + \sum_{j=1}^{mu} m$$

where, n and m are the number delayed inputs and outputs, respectively and m_u, n_y are the number of inputs and output, respectively. Here, m_u equal 4, n_y equal 1

Given the triple, (U, V, F) the antecedent membership functions and the consequence parameters A_i, B_i and c_i can be computed.

Gustanfson-Kessel (GK) algorithm: In this section, we will describe the Gustanfson-Kessel (GK) algorithm for, the general case, of Multi-Input Multi-Output (MIMO) systems. Consider z , the input output data matrix, where $z = [u, y]$ and the number of clusters σ and some $\epsilon > 0$ are given. Assume j is the iteration number. Initialize the fuzzy partition matrix U^0 at random. Then, start the following procedure:

Repeat for $j=1, 2, \dots$

- **Step 1:** Compute cluster means:

$$v_i = \frac{\sum_{k=1}^N (\mu_{ik}^{j-1})^m z_k}{\sum_{k=1}^N (\mu_{ik}^{j-1})^m}$$

- **Step 2:** Compute covariance matrices:

$$F_i = \frac{\sum_{k=1}^N (\mu_{ik}^{j-1})^m (z_k - v_i)(z_k - v_i)^T}{\sum_{k=1}^N (\mu_{ik}^{j-1})^m}$$

- **Step 3:** Compute distances: 1

$$d^2(z, v_i) = (z_k - v_i^j)^T \left(\det(F_i) \frac{1}{p+1} F_i^{-1} \right) (z_k - v_i^j)$$

- **Step 4:** Update partition matrix:
If $d^2(z, v_i) > 0$ for $1 \leq i \leq \sigma$ and $1 \leq k \leq N$,

$$\mu_{ik}^j = \frac{1}{\sum_{j=1}^{\sigma} (d(z, v_i^j) / d(z, v_i^j))^{2/(m-1)}}$$

Otherwise:

$\mu_{ik}^j = 0$ if $d^2(z, v_i^j) > 0$ and $\mu_{ik}^j \in [0, 1]$ with:

$$\sum_{i=1}^{\sigma} \mu_{ik}^j = 1$$

until $||U - U^{j-1}|| < \epsilon$

Consequent parameters: In our case, the fuzzy model inputs are $u = [y_1, y_2, y_3, y_4]$, y_1 stand for $y(k-1)$ and the model output is y . Here, there are several possibility to estimate the consequence parameters A_i , B_i and c_i (Babuska and Verbruggen, 1996). Here, we adopt the weighted least-square estimation. Let θ^T be the vector which has the coefficient of the consequence polynomial A_i , B_i and c_i . Let Φ be the matrix $[\phi, 1]$ and the matrix W be a diagonal matrix with dimension $R^{1 \times 1}$ having a membership degree μ_k as its k th diagonal element. Assuming that the column of the matrix X are linearly independent and $\mu_k > 0$ for $1 = k = 1$, then:

$$\theta = (\Phi^T W \Phi)^{-1} \Phi^T W y \quad (6)$$

where, θ is the least-square solution of the equation $y = X\theta + \delta$ where the k th data pair(u, y) is weighted by μ_k .

The Nile river flow data: To build a fuzzy model for the Nile river flow, we used measurements of the average daily flow volume for each ten day period at the

Dongola station located in Egypt. The readings span over the period from 1975 to 1993. These measurements were split in to two groups one for training and another for testing. A Matlab toolbox for modeling of fuzzy systems (Babuska, 1998a) was used to implement the following results. The routines of the toolbox contain the Gustanfson-Kessel (GK) clustering algorithm, whose implementation is given (Gustafso and Kessel, 1979).

FUZZY FORECASTING MODEL

The Fuzzy Model Identification (FMID) Matlab toolbox developed by Babuska (1998b) was used to produce our results. The developed fuzzy model has the advantage over traditional models such as the auto-regression models and other time series model is that we have now number of models (three models in our case) to represent the variations in the flow. Each model is capable to provide accurate estimation to the Nile river flow within its domain of measurements. This makes the developed fuzzy model able to represent the river flow better and can provide an accurate forecasting measure.

The input-output training data was used to build the regression matrix ϕ and the output vector, respectively. The number of clusters σ need to be set in advance. σ is a scalar number, since, we have a single output system. The rest of the toolbox parameters are optional. The termination tolerance for the clustering algorithm can be set priori:

The set of rules which describe the relationship between u and y is given as:

- If $y(k-1)$ is A11 and $y(k-2)$ is A12 and $y(k-3)$ is A13 and $y(k-4)$ is A14 then $y(k) = 1.22 \cdot 10^0 y(k-1) - 5.40 \cdot 10^{-1} y(k-2) + 2.90 \cdot 10^{-2} y(k-3) + 4.94 \cdot 10^{-2} y(k-4) + 14.6$
- If $y(k-1)$ is A21 and $y(k-2)$ is A22 and $y(k-3)$ is A23 and $y(k-4)$ is A24 then $y(k) = 8.97 \cdot 10^{-1} y(k-1) - 2.11 \cdot 10^{-1} y(k-2) + 2.12 \cdot 10^{-1} y(k-3) + 9.92 \cdot 10^{-3} y(k-4) - 23.5$
- If $y(k-1)$ is A31 and $y(k-2)$ is A32 and $y(k-3)$ is A33 and $y(k-4)$ is A34 then $y(k) = 6.38 \cdot 10^{-1} y(k-1) + 8.94 \cdot 10^{-1} y(k-2) - 8.34 \cdot 10^{-1} y(k-3) - 5.82 \cdot 10^{-1} y(k-4) + 264$

The consequent parameters and the cluster centers are given in Table 1 and 2.

In Fig. 1 we show the membership function in our case. We have used three clusters to build our model. Figure 2 and 3 show the actual and predicted Nile river flow in both training and testing cases. As a figure of merit, we considered the Variance-

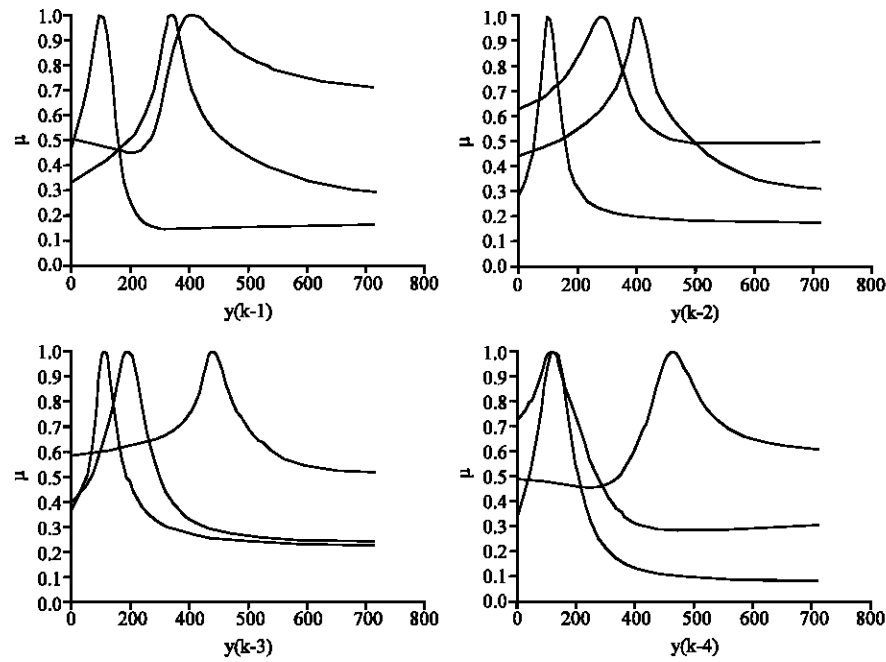


Fig. 1: Membership functions

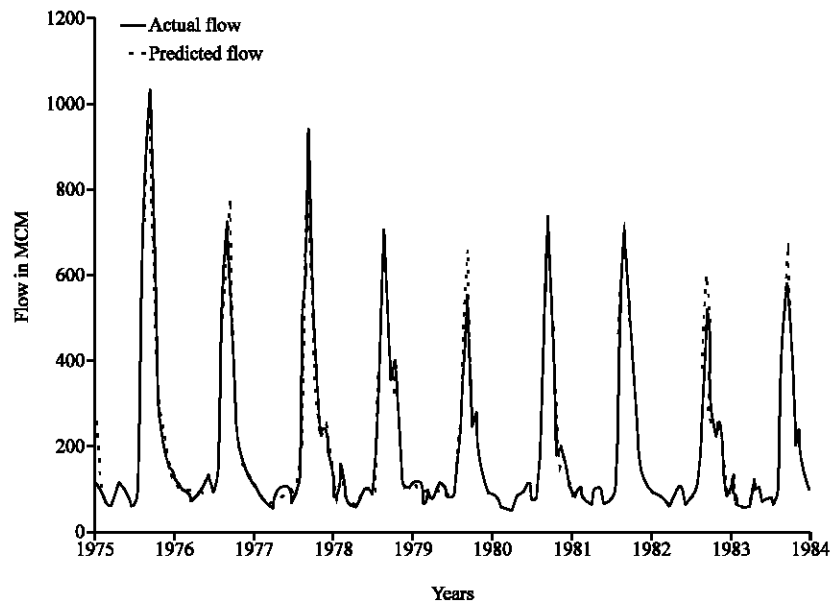


Fig. 2: Actual and predicted Nile river flow in the training case

Table 1: Consequent parameters

Rules	$y(k-1)$	$y(k-2)$	$y(k-3)$	$y(k-4)$	Offset
1	1.22×10^0	-5.40×10^0	2.90×10^{-2}	4.94×10^{-2}	1.46×10^1
2	8.97×10^{-1}	-2.11×10^{-1}	2.12×10^{-1}	9.92×10^{-3}	-2.35×10^1
3	6.38×10^{-1}	8.91×10^{-1}	-8.34×10^{-1}	-5.82×10^{-1}	2.64×10^2

Table 2: Cluster centers

Rules	$y(k-1)$	$y(k-2)$	$y(k-3)$	$y(k-4)$
1	1.01×10^2	1.07×10^2	1.13×10^2	1.19×10^2
2	3.40×10^2	4.11×10^2	4.83×10^2	5.32×10^2
3	4.08×10^2	2.88×10^2	1.93×10^2	1.30×10^2

Accounted-For (VAF) as a major of performance in the modeling process. The VAF is calculated as: The value

of the VAF for both training and testing cases were 93.7383 and 90.3624, respectively.

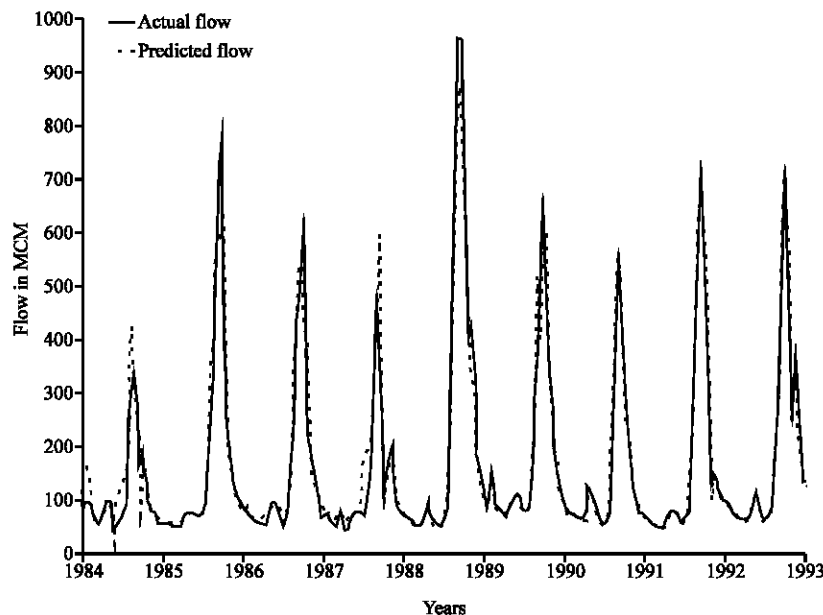


Fig. 3: Actual and predicted Nile river flow in the testing case

CONCLUSIONS

In this study we discussed the process of developing a fuzzy model for forecasting the Nile river flow in Egypt. The application and performance of the fuzzy model was tested using a set of measurements recorded at Dongola station in Egypt. The analysis of the results concluded that the fuzzy model was successfully able to build a relationship between the model input (i.e., historical measurements) and output (i.e., current measurements). The results for both training and testing cases had a high VAF value, which mean good modeling capabilities. The fuzzy rules were developed based on the Takagi-Sugeno type model. The Gustafson-Kessel (GK) algorithm was applied to determine the antecedent membership functions and the least-square estimation was used to determine the consequence parameters.

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