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The Measurement of Productivity Growth in the Academic Departments using Malmquist Productivity Index

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Abstract: The purpose of this study is to analyze efficiency and productivity growth of the academic departments in two periods of time 2003-2004 and 2008-2009 academic years. Efficiency measures are calculated by a non-parametric approach known as Data Envelopment Analysis (DEA). Productivity is measured by the Malmquist index. The study shows how DEA-based Malmquist productivity index can be employed to evaluate the technology and productivity changes resulted in the university. Total productivity growth has been found, but there is a variation among individual units, also increasing the frontier productivity and Technical Efficiency Change (TEC) indices which are parts of the decomposed total productivity have been shown.

Key words: Data envelopment analysis, performance assessment, malmquist productivity index, technical efficiency change, productivity

INTRODUCTION

Productivity growth is one of the major sources of economic development and a thorough understanding of the factors affecting productivity is very important. In recent years the measurement and analysis of productivity change has enjoyed a great deal of interest among researchers studying firm performance and behavior.

This study measures productivity growth in university, in addition, focus on the investigation of the causes of productivity change and on its decomposition. Such decompositions promote the understanding of the determinants of better performance and provide valuable information for managers and planners in both the private and the public sector.

In recent research, the malmquist index has become the standard approach to productivity measurement within the non-parametric literature. The concept of this index was first introduced by Malmquist (1953) and has further been studied and developed in the non-parametric framework by Caves *et al.* (1982), Fare *et al.* (1994a, b) and Cooper *et al.* (2007).

The Malmquist index approach to productivity measurement has many advantages. It is an index representing Total Factor Productivity (TFP) growth of a Decision Making Unit (DMU), in that it reflects (1) progress or regress in efficiency along with (2) progress or regress of the frontier technology between two periods

of time. It is based on multi input-output frontier representations of the production technology (Chames *et al.*, 1978). In the empirical context, the results are obtained using mathematical programming techniques (DEA) that rely on minimum assumptions regarding the shape of the production frontier. Finally, the index decomposes into multiple components to give insights into the root sources of productivity change. DEA-based Malmquist productivity index measures the technical and productivity changes over time.

DATA ENVELOPMENT ANALYSIS

Data Envelopment Analysis (DEA) has been recognized as an excellent method for analyzing performance and modeling organizations and operational processes, particularly when market prices are unavailable. Unlike the statistical regression method that tries to fit a regression plane through the center of the data, DEA floats a piecewise linear surface to rest on top of the data by linear programming techniques (Odeck, 2000). In other words, the statistical regression method estimates the parameters in the assumed functional form by a single optimization over all Decision Making Units (DMUs) whereas DEA uses optimizations for different DMUs without a priori assumptions on the underlying functional forms. Because of this unique feature, DEA has been applied to various areas of efficiency evaluation, for example, individual physician practice, program

evaluation, macroeconomics performance of countries or cities, pollution prevention, reorganization of forest districts and pupil transportation and others. The main advantages of DEA that make it suitable for measuring the efficiency of DMUs are: (1) it allows the simultaneous analysis of multiple outputs and multiple inputs, (2) it does not require an explicit a priori determination of a production function, (3) efficiency is measured relative to the highest observed performance rather than against some average and (4) it does not require information on prices.

DEA can be applied to panel data to measure the productivity changes between two periods of activities fulfilled by a specific set of DMUs. For example, Fare *et al.* (1994a, b) studied the productivity change in Swedish individual hospitals operating in a non-market environment. The specific approach used is called Malmquist productivity index in which DEA efficiency scores are used. Chen and Ali (2004) applied the DEA Malmquist productivity measure to the computer industries by the CCR model to assess the four distance functions of Malmquist productivity. They further analyzed the properties of two ratios of frontier shift, the backward and forward frontier shifts. The DEA models used in the Malmquist productivity index can either be input or output oriented. Consequently, the Malmquist productivity index can be input-oriented when the outputs are fixed at their current levels or output-oriented when the inputs are fixed at their current levels.

MALMQUIST PRODUCTIVITY INDEX

Suppose we have n DMUs, each DMU_j (j = 1, ..., n) produces a vector of outputs $y_j^t = (y_{1j}^t, \dots, y_{kj}^t)$ by using a vector of inputs $x_j^t = (x_{1j}^t, \dots, x_{mj}^t)$ at each time period t. The CCR DEA model can be expressed as (Charnes *et al.*, 1978):

$$\begin{aligned} \theta^t = \text{Min} \quad & \theta_0 \\ \text{St.} \quad & \sum_{j=1}^n \lambda_j x_{ij}^t \leq \theta x_{i0}^t \quad i = 1, 2, \dots, m \\ & \sum_{j=1}^n \lambda_j y_{rj}^t \geq y_{r0}^t \quad r = 1, 2, \dots, k \\ & \lambda_j \geq 0 \quad j = 1, 2, \dots, n \end{aligned} \tag{1}$$

where, $x_0^t = (x_{10}^t, \dots, x_{m0}^t)$ and $y_0^t = (y_{10}^t, \dots, y_{k0}^t)$ are the input and output vectors of DMU₀ among others. Model (1) is input-oriented, because it considers the possible radial reductions of all inputs when the outputs are fixed at their current levels.

If $\theta^t = 1$; then DMU₀ is efficient in time period t; otherwise, if $\theta^t < 1$; then DMU₀ is inefficient. It can be

seen that (1) if $\theta^t = 1$; then DMU₀ is unable to proportionally reduce its inputs and therefore DMU₀ is on the Empirical Production Frontier (EPF); (2) if $\theta^t < 1$; then DMU₀ can reduce its inputs and therefore DMU₀ is operating below the EPF. By replacing x_j^t and y_j^t with x_j^s and y_j^s , respectively, we have the technical efficiency of θ^s_0 for DMU₀ at the time period s, From t to s; DMU₀'s technical efficiency may change or (and) the EPF may shift. Based upon model (1), the Malmquist productivity index can be calculated via (Fare *et al.*, 1994a, b)

- Comparing x_0^t to EPF at time t; namely, calculating $\theta^t_0 = \theta^t_0(x_0^t, y_0^t)$
- Comparing x_0^s to EPF at time s; namely, calculating $\theta^s_0 = \theta^s_0(x_0^s, y_0^s)$
- Comparing x_0^t to EPF at time s; that is, calculating $\theta^s_0 = \theta^s_0(x_0^t, y_0^t)$ through the following linear program:

$$\begin{aligned} \theta^t_0 = \text{Min} \quad & \theta_0 \\ \text{St.} \quad & \sum_{j=1}^n \lambda_j x_{ij}^s \leq \theta x_{i0}^t \quad i = 1, 2, \dots, m \\ & \sum_{j=1}^n \lambda_j y_{rj}^s \geq y_{r0}^t \quad r = 1, 2, \dots, k \\ & \lambda_j \geq 0 \quad j = 1, 2, \dots, n \end{aligned} \tag{2}$$

- Comparing x_0^s to EPF at time t; namely, calculating $\theta^t_0 = \theta^t_0(x_0^s, y_0^s)$ through the following linear program:

$$\begin{aligned} \theta^s_0 = \text{Min} \quad & \theta_0 \\ \text{St.} \quad & \sum_{j=1}^n \lambda_j x_{ij}^t \leq \theta x_{i0}^s \quad i = 1, 2, \dots, m \\ & \sum_{j=1}^n \lambda_j y_{rj}^t \geq y_{r0}^s \quad r = 1, 2, \dots, k \\ & \lambda_j \geq 0 \quad j = 1, 2, \dots, n \end{aligned} \tag{3}$$

The technical efficiency change (catch-up) term relates to the degree to which a DMU improves or worsens its efficiency, while the frontier-shift (or innovation) term reflects the change in the efficient frontiers between the two time periods:

$$\text{Technical efficiency change} = \text{TEC} = \theta^s_0$$

TEC > 1 indicates progress in relative efficiency from period t to s, while TEC = 1 and TEC < 1, respectively indicate no change and regress in efficiency.

In addition to the technical efficiency change term, we must take account of the frontier-shift (innovation) effect in order to fully evaluate the productivity change since the technical efficiency change effect is determined by the efficiencies being measured by the distances from the respective frontiers:

$$\text{Frontier shift} = FS_o = \sqrt{\frac{\theta_o^s \theta_o^t}{\theta_o^s \theta_o^t}}$$

This indicator, also decompose to parts

$$\frac{\theta^t(x^s, y^s)}{\theta^s(x^s, y^s)}$$

and

$$\frac{\theta^t(x^t, y^t)}{\theta^s(x^t, y^t)}$$

which is called the backward and forward frontier shifts, respectively.

$FS_o > 1$ indicates progress in the frontier technology around DMU_o from period t to s ($t < s$), while $FS_o = 1$ and $FS_o < 1$, respectively indicate the status quo and regress in the frontier technology.

The Malmquist Index (MI) is computed as the product of (Technical efficiency change) and (Frontier shift), i.e.,

$$MI_o = (\text{Technical efficiency change}) \times (\text{Frontier shift}) = \left[\frac{\theta_o^s \theta_o^t}{\theta_o^t \theta_o^s} \right]$$

MI_o measures the productivity change between periods t and s . Productivity declines if $MI_o < 1$; remains unchanged if $MI_o = 1$ and improves if $MI_o > 1$.

DATA DESCRIPTION AND FINDINGS

To evaluate educational system cannot be used of market evaluation mechanisms such as benefit assessment to determine DMU performance or inputs and outputs economic value, because inputs and outputs generally stand in the education, research and service departments which the measurement or presentation of an assessment unit is very difficult. DEA method also emphasizes university targets for inputs and outputs choice and makes possible the choice of qualities input and output indicators to the system.

In this study, the application of DEA method in educational assessment, Islamic Azad University, Zahedan unit's educational departments in two periods of time 2003-2004 and 2008-2009 academic years is studied. The Islamic Azad University, Zahedan unit's educational departments are viewed as DMUs (Table 1).

Our original data consist of the annual statistics for the years 2004 and 2009 collected in each of the 21 departments. From these data the outputs and inputs are as shown in Table 2.

The relative efficiencies of each DMU in the period t and s are calculated under model (1). Table 3 shows the results solved models, which are implemented in an MS-Excel worksheet and by using the DEA Solver software and LINDO software. The productivity indices for DMUs are determined.

We first look at the technical efficiency changes. The columns 3 and 4 in above Table 3 report the DEA technical efficiency and the associated the technical efficiency changes from 2004 to 2009. The average technical efficiency changes of the universities are 1.01685, which is improved 1.685%.

According to Fig. 1, which shows technical efficiency changes of DMUs, seven departments are efficient in each time period, that is, no technical efficiency change is indicated by TEC. However, we would like to point out that caution should be paid when a DMU is a frontier DMU in time period t and time period s , i.e., although, $TEC = 1$ indicates no improvement in technical efficiency, these departments stands for the industry best practice in each year. On the other hand, we note that $TEC > 1$ only indicates an improvement in technical efficiency (e.g., Midwifery, Judicial law, Geography,...). This does not necessary mean that these departments have a better performance in improving its technical efficiency than others that are efficient in each two period.

We next look at the frontier shift. The column 8 in Table 3 reports the Malmquist frontier shift component, FS. It can be seen that on average, the frontier shift improved 3.13% from 2004 to 2009. As indicated by FS (Table 4), nine of the departments show a positive shift and the other twelve of the departments show a negative shift (Chen and Ali, 2004).

Table 1: Input and output categories

Variables	Description
Inputs	
Scientific board	Including: professors, assistant professor, lectures and educational experts, who are full-time and half-time
Student admission	No. of registered students in the term of the academic years
No. of presented unit by guest lecturers	No. of presented units in each department by guest lectures
Outputs	
No. of graduates	No. of graduates in the academic year
No. of students passing to higher level	Such as a two-years course to BA, BA to MA and to PHD
Research	Including: books compilation and translation, published articles or presented in authentic conferences and reports and research projects

Table 2: The gathered information to assess educational departments of Islamic Azad University, Zahedan Unit, Academic year 2004 and 2009

Education department	No. of registered students in academic		Scientific board's concession		Guest lecturers No. of units		No. of graduates		No. of students passing to higher level		Research work (project,article, books)	
	2004	2009	2004	2009	2004	2009	2004	2009	2004	2009	2004	2009
Nursing	350	433	4	5	75	88	45	56	0	1	40	45
Midwifery	202	242	1	1	50	55	27	35	1	2	9	10
Laboratory sciences	190	233	1	2	31	38	35	46	4	6	42	40
Veterinary	305	406	1	1	60	65	45	51	3	6	1	0
Management	802	989	7	10	120	140	65	75	2	2	55	60
Judicial law	1004	1276	8	9	130	145	130	148	2	2	62	60
Accountancy	1800	2190	6	6	140	165	170	189	1	2	21	20
Geography	502	634	5	6	73	82	55	68	5	4	38	40
Physical education	659	757	4	4	70	78	80	103	2	2	9	10
Mathematics	352	650	10	12	50	64	17	25	3	2	135	130
Geology	456	525	5	6	80	90	36	43	9	8	118	120
Computer	899	1020	2	2	91	100	97	116	2	1	2	0
Civil	1602	1718	8	11	120	140	96	111	3	4	48	50
Industrial	901	1025	5	6	105	120	71	80	4	3	92	90
Agriculture	600	655	10	11	75	88	22	30	8	10	45	50
Electronics	852	924	3	5	85	92	28	39	0	1	22	20
Persian literature	301	360	4	6	62	72	25	31	1	1	18	20
Islamic knowledge	701	779	7	8	90	102	130	146	0	1	8	10
Elementary education	491	589	4	5	82	92	97	115	2	1	1	0
Empirical science	351	441	4	4	67	80	27	35	0	1	18	20
Arabic literature	175	258	7	5	40	50	7	11	1	1	20	20

Table 3: The productivity indices for the DMUs

No.	Education department	θ_t	θ_s	θ_t'	θ_s''	TEC	FS	MI_s
1	Nursing	0.687	0.655	0.6605	0.6863	0.953421	1.004702	0.957903
2	Midwifery	0.749	0.996	1.1833	0.8856	1.329773	1.002397	1.332961
3	Laboratory sciences	1.000	1.000	2.1000	1.2237	1.000000	1.310003	1.310003
4	Veterinary	1.000	1.000	1.0914	1.5000	1.000000	0.852995	0.852995
5	Management	0.451	0.437	0.4454	0.4381	0.968958	1.024321	0.992524
6	Judicial law	0.767	0.778	0.7694	0.7663	1.014342	0.994912	1.009180
7	Accountancy	1.000	0.927	0.9567	0.9807	0.92700	1.025841	0.950954
8	Geography	0.638	0.648	0.6010	0.6567	1.015674	0.949241	0.964120
9	Physical education	0.876	1.000	0.8562	1.4280	1.141553	0.724729	0.827316
10	Mathematics	1.000	1.000	1.3920	0.8255	1.000000	1.298557	1.298557
11	Geology	1.000	1.000	1.1800	0.9013	1.000000	1.144212	1.144212
12	Computer	1.000	1.000	0.9454	1.1959	1.000000	0.889120	0.889120
13	Civil	0.617	0.602	0.6138	0.6013	0.975689	1.022851	0.997984
14	Industrial	0.630	0.750	0.9200	0.5820	1.190476	1.152317	1.371806
15	Agriculture	0.827	0.720	0.6756	0.8807	0.870617	0.938679	0.817230
16	Electronics	0.285	0.334	0.3822	0.3304	1.171930	0.993515	1.164330
17	Persian literature	0.440	0.436	0.4207	0.4527	0.990909	0.968421	0.959617
18	Islamic knowledge	1.000	1.000	1.0110	1.0101	1.000000	1.000445	1.000445
19	Elementary education	1.000	1.000	1.0274	1.0091	1.000000	1.009027	1.009027
20	Empirical science	0.406	0.402	0.3896	0.4155	0.990148	0.973137	0.963549
21	Arabic literature	0.450	0.366	0.5000	0.3233	0.813333	1.378948	1.121544

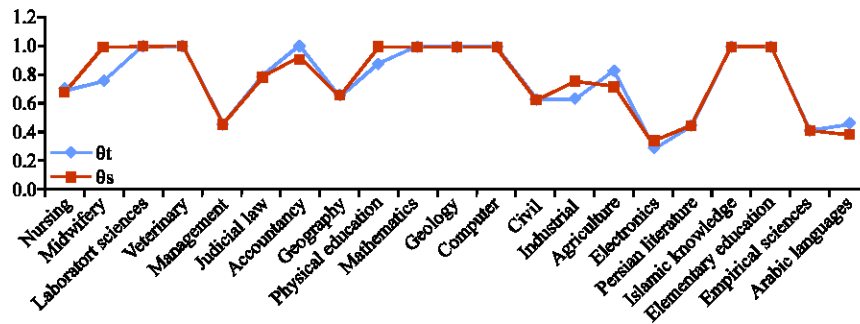


Fig. 1: The efficiency of DMUs in two periods of time

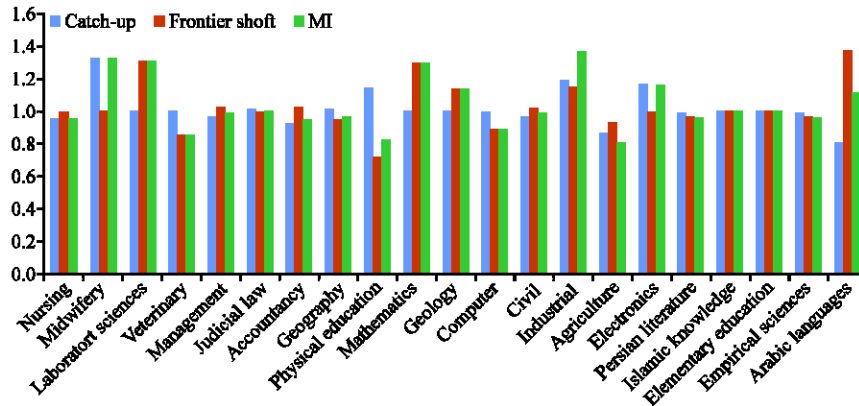


Fig. 2: Total productivity, technical efficiency change (catch-up) and frontier shift across units

Table 4 reports the component shifts in technology frontier and shows average the departments technology frontier has a pure negative shift.

Technology change at the DMU level shows the two ratios associated with the frontier change index are larger than 1 for nine of departments, indicating that these departments stay with a consistent operations strategy. Other departments show a move between two facets, indicating that these departments have a change in operations strategy. The technology of thirteen departments moves from a negative shift facet towards a positive shift facet, indicating a favorable strategy change.

We finally look at the Malmquist productivity index. The column 9 of Table 3 reports Malmquist productivity index, MI. It can be seen that from academic year 2003-2004 to 2008-2009, the total productivity indices for the average unit show an increase with scores of 1.0445, with standard deviation 0.167. Industrial department has maximum productivity growth, that is 1.3718 and Agriculture department has maximum regress in productivity, that is 0.8172. The values of inputs and outputs in two periods 2004 and 2009 represent increasing 18 and 12%, respectively. The distribution of total productivity, technical efficiency change and frontier shift across units is shown in Fig. 2.

Figure 2 shows that ten departments have productivity gain since, its MI is greater of 1 ($MI > 1$). In departments of midwifery, industrial and electronics, the malmquist productivity gain is from not only an efficiency improvement, but also a technology movement from negative shift section to positive shift section of the frontier, indicating that these departments have a favorable strategy shift and also gains relative efficiency with respect to a positively shift frontier.

Table 4: The backward and forward frontier shifts

No. of department	$\theta^1(x^s, y^s)$	$\theta^1(x^s, y^s)$
	$\theta^s(\theta^s, y^s)$	$\theta^s(\theta^s, y^s)$
1	1.05	1.04
2	0.89	0.63
3	1.22	0.48
4	1.50	0.92
5	1.00	1.01
6	0.98	0.99
7	1.06	1.05
8	1.01	1.06
9	1.43	1.02
10	0.83	0.72
11	0.90	0.85
12	1.20	1.06
13	0.99	1.01
14	0.78	0.68
15	1.22	1.22
16	0.99	0.75
17	1.04	1.05
18	1.01	0.99
19	1.01	0.97
20	1.03	1.04
21	0.88	0.90

CONCLUSION

This study explored the evolution of productivity of the university departments operating in the Islamic Azad University Zahedan Unit's education departments for the period 2004-2009.

Since, the Islamic Azad University Zahedan Unit's education departments are part of the public sector where economic behavior is uncertain and there is no price information on the services produced, the Malmquist index based on DEA approach is well suited for productivity measurement in this sector.

With regard to productivity indices, the picture that emerges is one of productivity growth due to an improvement of production possibilities. The decomposed

index shows that much of the observed progress in total productivity is explained by the frontier shift term.

REFERENCES

- Caves, D.W., L.R. Christensen and W.E. Diewert, 1982. The economic theory of index numbers and the measurement of input, output and productivity. *Econometrica*, 50: 1393-1414.
- Charnes, A., W.W. Cooper and E. Rhodes, 1978. Measuring the efficiency of decision making units. *Eur. J. Operat. Res.*, 2: 429-444.
- Chen, Y. and A.I. Ali, 2004. DEA Malmquist productivity measure: New insights with an application to computer industry. *Eur. J. Operat. Res.*, 159: 239-249.
- Cooper, W.W., L.M. Seiford and K. Tone, 2007. *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*. Kluwer Academic Publishers, Boston, pp: 490.
- Fare, R., S. Grosskopf and C.A.K. Lovell, 1994a. *Production Frontiers*. Cambridge University Press, Cambridge.
- Fare, R., S. Grosskopf, B. Lindgren and P. Roos, 1994b. *Productivity Developments in Swedish Hospitals: A Malmquist Output Index Approach Data Envelopment Analysis: Theory, Methodology and Applications*. Kluwer Academic Publishers, Boston.
- Malmquist, S., 1953. Index numbers and indifference surfaces. *Trabajos de Estadística y de Invest. Operativa*, 4: 209-242.
- Odeck, J., 2000. Theory and Methodology Assessing the relative efficiency and productivity growth of vehicle inspection services: An application of DEA and Malmquist indices. *Eur. J. Operat. Res.*, 126: 501-514.