



Journal of Applied Sciences

ISSN 1812-5654

science
alert

ANSI*net*
an open access publisher
<http://ansinet.com>

Stretched-exponential Localization Induced by Electric Field in Disordered System with Correlated

A. Yedjour and F. Hamdache

Laboratoire de Physique Des Plasmas Matériaux Conducteurs et Leurs Applications,
Department of Physics, Faculty of Science, Mohamed Boudiaf USTO,
University of Science and Technology of Oran Mohammed Boudiaf USTO,
P.O. Box 1505, El Menaour, Oran, Algeria

Abstract: The purpose of this study was to investigate the transport properties of a random dimer model in 1D. A Kronig-Penney model with δ -peak potentials is used to examine how the resonance energy is affected by the electric field. We discussed the influence of an electric field on the nature of the electronic states and compared the result to the case without field. We found that there are important differences, mainly for a large system size L . Localized wave functions have been obtained at particular energies which make transition between extended and stretched-exponential localized states. The most important conclusion so obtained is that the electric field applied to such systems suppresses progressively the effect of the correlation, the transmission coefficient decreases leading to the absence of transport in this kind of electrified chains.

Key words: Disordered system, random potential, electric field, localization, electronic states

INTRODUCTION

There is a growing interest for questions pertaining to wave spread in disordered lattices, which are related to the search of optical or acoustic localization and recently of cold atom localization (Economou and Alkire, 1988; (John and Stephen, 1983; Skipterov *et al.*, 2008). The well-studied case of electronic systems with independent site disorder does not fully cover all cases of such wavelike excitations in complex media. A well-known result of the Anderson model for the site energy is the absence of long-range transport in one dimensional system. All electronic states in one dimension are exponentially localized regardless of the amount of disorder (Anderson, 1958).

Much attention has been paid to special disorder correlations for which new phenomena are expected to appear. For instance, although Anderson localization occurs in one dimension, one finds partial delocalization even for an infinitesimal amount of disorder in the presence of correlations (Datta *et al.*, 1993; Sanchez *et al.*, 1994). A number of recent works dealing with tight-binding Hamiltonian strongly suggest that the occurrence of correlations in neighbour random parameters are not independent with a correlation length (Evengelou, 1990),

(Dunlap *et al.*, 1990; Wu and Phillips, 1991). Furthermore, the existence of a mobility edge between extended and localized states was found for 1D random system with weak long-range correlated disorder) (Molina, 2005; Esmailpour *et al.*, 2006). Long-range disorder induces the appearance of delocalization and long range transport.

The Random Dimer Model (RDM) can be shown to be an example of the correlated disordered system. In this 1D random model, the site energy takes one out of two possible values, one of which is distributed at random to pairs along the chain, so that the correlation length coincides with lattice spacing. On the basis of this interest the authors claimed that the RDM has \sqrt{N} states which are extended over the whole sample, with N the number of sites in the system. A discrete number of extended states was found numerically (Evengelou and Wang, 1993), (Evengelou and Economou, 1993) and was observed recently in the experiment with semiconductor random superlattices (Bellani *et al.*, 1999). In Kronig Penney model, the electronic field delocalizes the eigenstates where the wave functions decay with a power (Soukoulis *et al.*, 1983; Cota *et al.*, 1985), in this regime, the resistance was checked experimentally. For sufficiently large field strengths, the eigenstates become extended (Markos and Kramer, 1993; Markos and

Corresponding Author: Afifa Yedjour, Laboratoire de Physique Des Plasmas Matériaux Conducteurs et Leurs Applications,
Department of Physics, Faculty of Science, Mohamed Boudiaf USTO,
University of Science and Technology of Oran Mohammed Boudiaf USTO, P.O. Box 1505,
El Menaour, Oran, Algeria

Henneke, 1994). When the electric field vanishes, it is well-known that the spectrum is then pure and dense), (Abrahams *et al.*, 1979; Landauer, 1970). The wave functions are exponentially localized, with a localization length that decreases with increasing disorder (Mott, 1968). The transmission coefficient has been used successfully to analyze the nature of the electronic states. The effect of exponentially localized eigen-states can be observed in the exponential decreases of the transmission coefficient with the length of the system (Anderson *et al.*, 1980; Thouless, 1974). Moreover, the connection between resistance, or more precisely, conductance and transmission coefficient can be carried out via Landauer formula (Landauer, 1970). One find:

$$\ln(1 + \rho) = -\frac{\ln T}{L}$$

with ρ the resistance. Such a behaviour can be expected from the self-averaging of the Lyapunov exponent γ which is the inverse of the localisation length in 1D systems. This characteristic length l is defined as:

$$l_c^{-1} = -\frac{1}{2L} \ln T$$

This parameter is always positive and describes the spatial scaling properties of a disordered system (Soukoulis and Economou, 1981; Lifshitz *et al.*, 1988).

In this study, we first discussed the delocalization induced by correlations in the Kronig Penney model. Here, we used an array of δ -function potentials with independent random strengths and study numerically the transmission properties for a finite length of the lattice. We derived exact results for the main characteristics of the model using a transfer matrix combined with a Poincaré map approach. Secondly, we examined the size dependence of the transmission coefficient of a linear RDM chain subject to electric field. When $F = 0$, the transmission coefficients at particular energy close to 1 and a deep minimum around the resonant energy in the resistance is found, indicating that the localization length of those states is large. For $F \neq 0$, we observed that the transmission decreases with increasing F where F is the electric strength. However, this minimum disappears and the values of resistance become extremely large. As soon as F is present, the electron gains the energy $V(x)$: $V(x) = -Fx$. This electrical potential suppresses the resonance energy induced by correlation. That induces a transition between extended and localized behaviour. Finally, we discussed our calculations of the Lyapunov exponent that indicate that all states around the resonance in the

presence of the electric field have a localization length smaller than the system size. We expected from this result a mobility edge that depends on the strength of the field in the RDM.

MODEL

Here we considered a 1D Kronig-Penney model with random δ -function potentials subject to an applied electric field. The problem is defined by the Schrödinger:

$$\left[-\frac{d^2}{dx^2} + \sum_{n=0}^L \lambda_n \delta(x - n) - Fx \right] \Psi(x) = E\Psi(x) \quad (1)$$

where, λ_n is a set of independent random variables that measures the strength of the δ -potentials. Here E is the energy of the electron measured in atomic units and Ψ is the wave function. We proceed with the problem of the disordered lattice containing a certain number of pair impurities placed randomly. We kept the positions of the δ -functions to be regularly spaced $\{x_n = n\}$ but we introduced a correlated disorder, for which λ_n takes only two values: λ and λ' , where λ' appears only in pairs of neighbouring sites (dimer impurities). The electronic potential $V(x)$ is given by $-Fx$ term in Eq. 1 with F denoting the electric field strength.

In this section, we presented a numerical study of the transmission coefficient of this model. Our approach is inspired by Soukoulis *et al.* (1983), Flores *et al.* (1989) who investigated the transmission coefficient and the nature of the electronic states in 1D disordered systems. They found that the transmission coefficient behaves as, with $f \approx \frac{1}{F}$. This reveals power-law localization.

Here, we calculated the transmission coefficient T in the above model (RDM) using the transfer matrix approach. We took an electron impinging from the left of a set of δ -function potentials with wave function $\Psi_0(x) = e^{iq_0x} + r_n e^{-iq_0x}$. The energy of the electron is $E = q_0^2$ with q_0 the momentum of the incident electron. The wave function in the right-hand side of the sample of length L is $\Psi(r) = T_N e^{iq_r x}$ Here $q_r = \sqrt{E + FL}$ with $L = N+2$; q_r denotes the momentum of the emerging wave. t_n and r_n are the transmission and the reflection amplitudes of the RDM with N scatterers respectively between two impurities, we will replace $V(x)$ by a constant value so that the solution between two impurities are plane wave functions (Soukoulis *et al.*, 1983; Cota *et al.*, 1985).

However, this is valid only when the electric potential between the ends of a sample is infinitesimally small.

The solution of Eq. 1 can be computed recursively for both transmission and reflection amplitude using well-known transfer-matrix technics (Kirilov and Trott, 1994). Then, the transmission amplitude can be written as:

$$A_n = \left(\alpha_n + \frac{\alpha \beta_n}{\beta_{n-1}} \right) A_{n-1} - \left(\frac{\beta_n}{\beta_{n-1}} \right) A_{n-1} \quad (2)$$

where $A_n \equiv t_n$ and:

$$\alpha_j = \left[1 - i \left(\frac{1}{2q_n} \right) \lambda_j \right]; \beta_j = -i \left(\frac{1}{2q_n} \right) \lambda_j e^{-iq_n} \quad (3)$$

Equation 2 supplied two boundary conditions, $A_0 = 1$ and $A_1 = 1$ to determine the amplitudes completely. q_n is the momentum of the electron at the site n. Finally, the transmission coefficient can be calculated for each chain from:

$$T = \frac{q_0}{q_t} \frac{1}{|t_N|^2} \quad (4)$$

RESULTS AND DISCUSSION

We first discussed our numerical results on the transmission coefficient for a RDM and investigated what changes occur when the electric field is applied along the linear chain.

We choose for convenience the length $L = 1000$ and a dimer concentration equal to 20%. We fixed $\lambda = 1$ for the values of potential strength of the host lattice and $\lambda' = 1.5$ for the dimer impurities.

Present results are similar to the ones obtained in by Sanchez *et al.* (1994), Dunlap *et al.* (1990), where a unique energy was found in the allowed band (recall their model is a single band) and where a perfect transmission $T = 1$ was seen in the RDM. In such case, the system of electronic transport becomes ballistic. Thus, nondecreasing transmission coefficient for particular energy shows the existence of extended states around this one.

In Fig. 1, we showed the transmission coefficient versus energy for intervals near the first resonance. The spectrum of the Kronig Penney model follows the equation $|2q \cos q + \lambda \sin q| \leq 1$ (this is the condition to be able to move in the perfect lattice) when λ is fixed. Here, we have averaged the transmission coefficient for 1000 realizations with an accuracy of 1%. We found that around the first resonance $E_r = 3.75$ the transmission coefficient reaches values very close to 1. All realizations show the same peak around E_r . It is clear from Fig. 1 that

the states close to the resonant energy have good transmission properties, similar to those of the resonant energy.

When $F \neq 0$, there are some important differences with respect to the case $F = 0$. For the same concentration of dimer impurities, we observe that the transmission coefficient decreases for a field as small as $5 \cdot 10^{-4}$. In this case, the small F will only slightly shift the resonant energy and the transmission coefficient will completely vanish.

We showed in Fig. 2, the resistance of a RDM in both the presence and the absence of an electric field with the same concentration of impurities. The lower curve, corresponding to a dimer model with $F = 0$, exhibits a minimum resistance about ten orders of magnitude below the resistance for 10^{-4} (the middle curve). For the F considered in Fig. 2, the curve saturates to essentially a constant value with energy. However, the resistance becomes extremely big compared to the same RDM

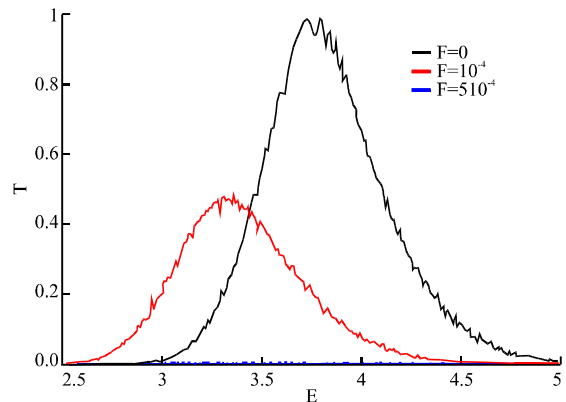


Fig. 1: Plot of resistance versus energy in RDM, with $L = 10^4$ for different values of the electric field

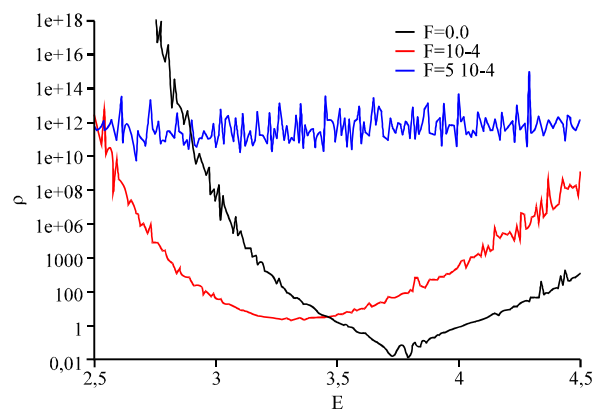


Fig. 2: Plot of resistance versus energy in RDM, with $L = 10^4$ for different values of the electric field

without electric field. Such as the localization scenario is quite different in the presence of an electric field.

The dependence of the resistance with system size is useful to study the spatial structure of the electronic states. Exponentially localized states lead to a nonohmic behaviour of the resistance, which increases exponentially with the system size.

In Fig. 3, around the resonance, the resistance has a constant value which indicates that the band of state exist with very good transport for a dimer model without field. In this case, the effect of the correlation in the random dimer potential is dominant, the electron gains more kinetic energy behaving essentially as a free particle in a potential well.

Not only the resonant energy has a low resistance for any length of the chain (lower curve), but also when F is far from zero $F = 10$, the plot shows a relatively small resistance, exhibits a good behaviour (middle curve). For $F = 5 \cdot 10^{-4}$ and for large L the resistance rises quickly to large values.

To investigate the nature of electronic states around the resonance, we have analyzed the average scaling of $\ln T$ with the system size.

In Fig. 4, we showed the results for $\langle \ln T \rangle$ versus L for a fixed value of the energy and for different values of field. First, as was done for the resistance, we compared our results to the size dependence of transmission

coefficient when the electric field is present. We saw that for $F = 0$ the curve is flat and $\langle \ln T \rangle$ reaches a constant value. We concluded that the states are extended. These extended states are not of the Block-type encountered in periodic solids (Hilke and Flores, 1997; Xiuqing and Xintian, 1997).

When $F = 0$ on the other hand, we observed three things. For small $L < 2000$ we obtained similar behaviour as for $F = 0$. However, for increasing F , the value of $\langle \ln T \rangle$ changes considerably for relatively small changes of F which suggests exponential decreasing for transmission coefficient, with an exponent that depends on F . For $L > 700$, the electronic states are stretched exponential-localized. This means that this phase has a zero measure in the thermodynamic limit. For $F = 5 \cdot 10^{-4}$ this phase, will diverge for large L . Here, the system will be return to equilibrium.

We investigated the Lyapunov coefficient which represents the inverse of the localization length l_c . As is shown in Fig. 5, when $F = 0$, energies close to the resonant energy E_r have $\gamma < 10^{-4}$. This is in agreement with the notion that delocalization of the electronic states occurs $l_c > 10^4$.

When we increased F , we observed an increase of the Lyapunov exponent that stays much smaller than one. This effect coincides with the standard definition of γ (Liffshitz *et al.*, 1988). The localization can be explained by

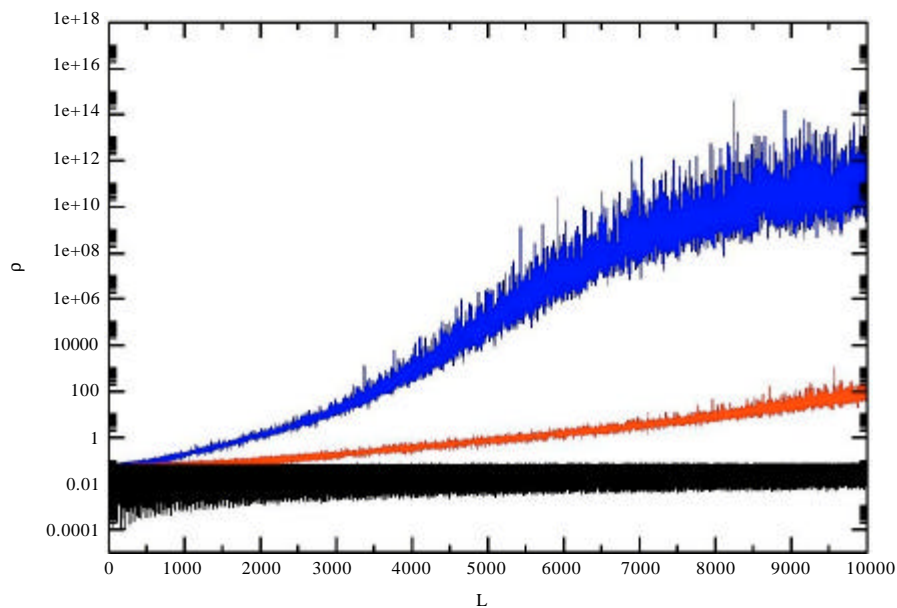


Fig. 3: Plot of the resistance ρ versus length L at the resonant energy E_r . For $F = 0$ (dark) a band of state exist with very good transport. For $F = 0$, a good transport exist for $L > 700$ (red). If $F = 5 \cdot 10^{-4}$ (blue) the resistance converges to a large value

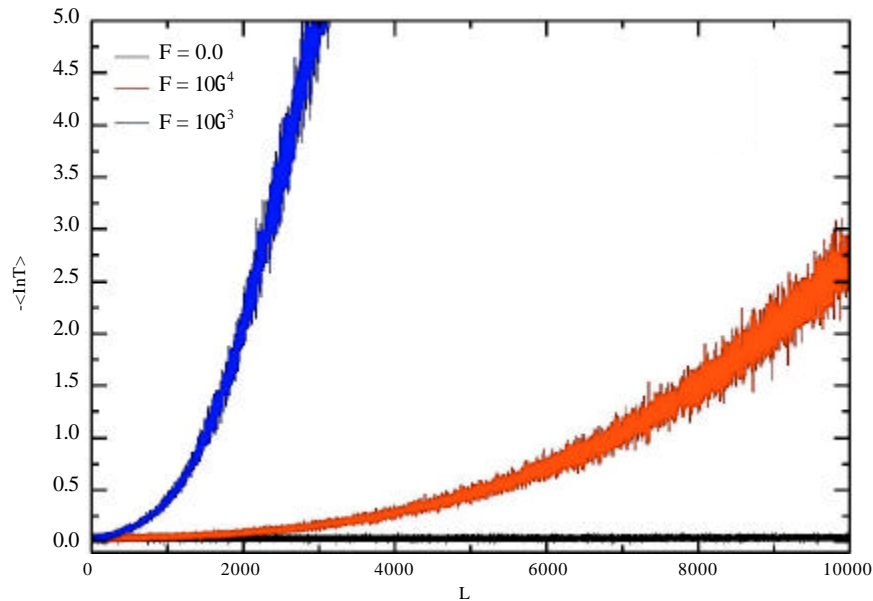


Fig. 4: Plot of $\langle \ln T \rangle$ versus length L at the resonant energy E_r for different values of F $F = 0$ (dark), $F = 10^{-4}$ (red) and $F = 5.10^{-4}$ (blue)

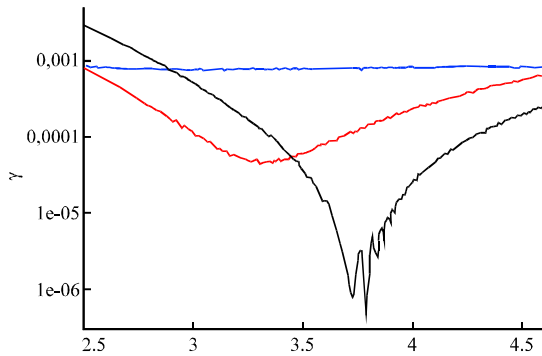


Fig. 5: Plot of Lyapunov exponent versus energy with $L = 10^4$ and for different values of strength of field $F = 0$ (dark), $F = 10^{-4}$ (red) and $F = 5.10^{-4}$ (blue)

the fact that when the electric field strength is increased, the effective potential component as $FN > E_r$ indicating that the states decay as exponential-law. We concluded that the delocalization-localization can be observed in dimer systems such as the electric field suppresses progressively the effect of the correlation.

CONCLUSION

We have studied the effect of electric field on a linear chain with correlated disorder. To analyze the properties of electronic transport, we have used the Kronig-Penny model. Based on the results, we have noted that the

electric field impedes the movement of the electrons in the presence of correlation. For relatively small field, we notice that the transmission is stretched exponential-law decaying with the length. This decaying depends on the strength of the electric field.

The electric field has an effect on the resonance energy that carries with it a variation of transmission coefficient which influences the nature of the electronic states. The Lyapunov exponent was also used to analyze the localization length, we have found out that when the electronic field increases, the Lyapunov exponent is saturated by a constant value that is lower than the system's size, which indicates a localization of the electronic states.

REFERENCES

- Abrahams, E., P.W. Anderson, D.C. Licciardello and T.V. Ramakrishnan, 1979. Scaling theory of localization: Absence of quantum diffusion in two dimensions. *Phys. Rev. Lett.*, 42: 673-676.
- Anderson, P.W., 1958. Absence of diffusion in certain random lattices. *Phys. Rev.*, 109: 1492-1505.
- Anderson, P.W., D.J. Thouless, E. Abrahams and D.S. Fisher, 1980. New method for a scaling theory of localization. *Phys. Rev. B*, 8: 3519-3526.
- Bellani, V., E. Diez, R. Hey, L. Toni and L. Tarricone *et al.*, 1999. Experimental evidence of delocalized states in random dimer superlattices. *Phys. Rev. Lett.*, 82: 2159-2162.

- Cota, E., J.V. Jose and M.Y. Azbel, 1985. Delocalization transition in random electrified chains with arbitrary potentials. *Phys. Rev. B*, 32: 6157-6165.
- Datta, P.K., D. Giri and K. Kundu, 1993. Nonscattered states in a random-dimer model. *Phys. Rev. B*, 47: 10727-10737.
- Dunlap, D.H., H.L. Wu and P. Phillips, 1990. Absence of localization in a random-dimer model. *Phys. Rev. Lett.*, 65: 88-91.
- Economou, E.N. and R. Alkire, 1988. Effect of potential field on ion deflection and shape evolution of trenches during plasma-assisted etching. *J. Electrochem. Soc.*, 135: 941-949.
- Esmailpour, A., M. Esmaeilzadeh, E. Faizabadi, P. Carpena and M. Reza Rahimi, 2006. Metal-insulator transition in random Kronig-Penney superlattices with long-range correlated disorder. *Phys. Rev. B*, 74: 024206-024206.
- Evengelou, S.N., 1990. Scaling exponents at the mobility edge. *Phys. A*, 167: 199-214.
- Evengelou, S.N. and A. Z. Wang, 1993. Localization in paired correlated random binary alloys. *Phys. Rev. B*, 47: 13126-13136.
- Evengelou, S.N. and E.N. Economou, 1993. Reflectionless modes in chains with large-size homogeneous impurity. *Phys. A Math, Gen.*, 26: 2803-2803.
- Flores, C., J. V. Jose and G. Monsiavais, 1989. Statistical properties of disordered 1-D model in a field. *J. Phys. Soc.*, 27: 369-369.
- Hilke, M. and J.C. Flores, 1997. Delocalization in continuous disordered systems. *Phys. Rev. B*, 55: 10625-10630.
- John, S. and M.J. Stephen, 1983. Wave propagation and localization in a long-range correlated random potential. *Phys. Rev. B*, 28: 6358-6368.
- Kirilov, M. and M. Trott, 1994. K-space treatment of reflection and transmission at a potential step. *Am. J. Phys.*, 62: 553-558.
- Landauer, R., 1970. Electrical resistance of disordered one-dimensional lattices. *Philosophical Magazine*, 21: 863-867.
- Lifshitz, I.M., S.A. Gredeskul and L.A. Pasture, 1988. *Introduction to Theory of Disordered Systems*. Wiley, New York.
- Markos, P. and B. Kramer, 1993. Statistical properties of the Anderson transition numerical results. *Philosophical Magazine Part B*, 68: 357-379.
- Markos, P. and M. Henneke, 1994. Metal-insulator transition in the four-dimensional Anderson model. *J. Phys. Condens. Matter*, 6: L765-L765.
- Molina, M.I., 2005. Nonlinear surface impurity in a semi-infinite lattice. *Phys. Rev. B*, 71: 035404-035404.
- Mott, N.F., 1968. Conduction in glasses containing transition metal ions. *J. Non-Crystal. Solids*, 1: 1-17.
- Sanchez, A., E. Macia and F. Dominguez-Adame, 1994. Suppression of localization in Kronig-Penney models with correlated disorder. *Phys. Rev. B*, 49: 147-157.
- Skipetrov, S.E., A. Minguzzi and B. Shapiro, 2008. Anderson, localization of a Bose-Einstein condensate in a 3D random potential. *Phys. Rev. Lett.*, 100: 165301-165301.
- Soukoulis, C.M. and E.N. Economou, 1981. Static conductance and scaling theory of localization in one dimension. *Phys. Rev. Lett.*, 46: 618-621.
- Soukoulis, C.M., J.V. Jose, E.N. Economou and P. Sheng, 1983. Localization in one-dimensional disordered systems in the presence of an electric field. *Phys. Rev. Lett.*, 50: 764-767.
- Thouless, D.J., 1974. Electrons in disordered systems and the theory of localization. *Phys. Rep.*, 13: 93-142.
- Wu, H.L. and P. Phillips, 1991. Polyaniline is a random-dimer model: A new transport mechanism for conducting polymers. *Phys. Rev. Lett.*, 66: 1366-1369.
- Xiuqing H. and W. Xintian, 1997. Periodic wave functions and number of extended states in random dimer systems. *Phys. Rev. B*, 55: 11018-11021.