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Review on Stress and Vibration Analysis of Composite Plates

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Abstract: A review on stress and vibration analysis of composite plates is presented in this study. The literature review is devoted on the free vibration and dynamic analysis, buckling analysis, failure analysis of composite laminated plates. Optimization is also considered in this review. The future research is summarized finally.

Key words: Laminated composite plates, free vibration, buckling analysis

INTRODUCTION

The use of composite materials in all types of engineering structures has led to an increased interest in the theory, analysis, design and manufacturing of structural components made of composite materials. The last few decades have seen a major effort to develop composite material systems and analyze and design structural components made from them.

The analysis of natural frequencies of composite plates/shells plays an increasingly important role in the design of structures in mechanical, civil and aerospace engineering applications. A thorough study of the dynamic behaviors of these structures is essential in assessing their full potential. Therefore, it is necessary to develop appropriate models capable of accurately predicting their dynamic characteristics.

Great progress has been made over past decades towards better understanding of the vibration characteristics of laminated composite plates/shells (Bert, 1984; Chen *et al*., 1989; Cheung and Kwok, 1975). Due to limited availability of analytic solutions for practical applications, numerical approximate methods have become the most effective tools. The Finite Element Method (FEM) is considered to be a very effective and versatile approach for these problems. There is a vast amount of literature on free vibration analysis of laminated plates/shells which is too large to list here.

Bert (1984) have conducted surveys and provided details on the development of the FEMs for modeling and modal analysis of laminated plates/shells. Further extensive references on shells can be found in the excellent review of Zhang and Yang (2009).

OVERVIEW

This review contain vibration analysis of composite plates, buckling and post buckling analysis and optimization in composite plates.

In this study, we have discussed the free vibration analysis of composite laminated plates.

The first-order shear deformation theory (FSDT) has been employed widely to establish finite element models for free vibration analysis of the composite laminated plates. The effects of lamination and extension-bending coupling, shear and twist-curvature couplings on the lowest frequencies and corresponding mode shapes for free vibration of laminated anisotropic composite plates was investigated using a finite element method with quadratic interpolation functions and five engineering Degrees of Freedom (DOF). The free and forced vibration response of laminated composite folded plate structures was predicted by a nine-node Lagrangian plate-bending finite element with five engineering DOF per node that incorporated rotary inertia.

In this study, we discuss Buckling and postbuckling analysis of laminated composite plates. The buckling of laminated composite plates is an important consideration in the design process; however the critical value of load given by linear buckling analysis may not accurately represent the load-carrying capability of a plate. Although composite laminated plates generally possess less load-carrying capacity after buckling compared to their metallic counterparts, the total load during the postbuckling of a composite laminated plate is still several times that of the critical buckling load. In order to get the practical limits of the load-carrying capability of the

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composite laminated plates, the postbuckling behavior has been studied to establish the sustained additional loads after buckling. Considerable efforts have been made for the numerical analysis of the buckling and postbuckling analysis over the years.

In this study, we discuss Failure analysis. Under normal operating conditions, local failures such as matrix cracks, fibre breakage, fibre matrix debonding and inter-layer delamination, may be developed in the laminated composite structures and the failure may cause permanent loss of integrity within the laminate and result in loss of stiffness and strength of the material. Prediction of the failure process, the initiation and growth of the damages and the maximum loads that the structures can withstand before failure occurs is essential for assessing the performance of composite laminated plates and for developing reliable and safe design. In particular, the first-ply failure analysis of laminated composite plates has been actively investigated in recent years and the mechanical behaviour and the first-ply failure load of laminated composite plates subjected to in-plane loading conditions, such as tension, compression, shear and out-of-plane loading such as transverse loads have been studied.

VIBRATION ANALYSIS

Free vibrational characteristics of layered circular plates are considered by Venkatesan and Kunukkasseril (1978). The equations incorporating shear deformation and rotatory inertia are developed for the asymmetric motion. For axisymmetric motion, exact closed form solutions are obtained. Timothy and Nayfeh (1996) developed the analysis and numerical calculations for the exact free vibration characteristics of simply supported, rectangular, thick, multilayered composite plates and assumed that each layer of the composite plate is of arbitrary thickness, is perfectly bonded to adjacent layers, possesses up to orthotropic material symmetry and that its material crystallographic axes are oriented either parallel or perpendicular to the plate's boundaries. Exact formal solutions are obtained for the individual layers which are, in turn, used to relate the field variables at the upper and lower layer surfaces. The solution is carried through by the successive application of appropriate interfacial continuity conditions between adjacent lamina.

The free vibration analysis of mass-loaded rectangular composite laminates plate with mixed boundaries was performed by Chang and Wu (1997) by using the orthogonal polynomial functions and Ritz method and developed the subdomain method to derive the governing eigenvalue equation. In the solution process, we used the subdomain weighted residual to satisfy the compatibility at the interconnect boundaries for two adjacent subdomain and carry out continuity matrices, then we adopted the Gram-Schmidt orthogonalization process to find the orthogonal functions set which satisfy the simply subdomain boundary condition. Finally, used the continuous matrices to develop the global energy functional and applied the Ritz method to obtain the governing eigenvalue equation. By solving the governing eigenvalue equation, the natural frequencies and mode shapes of the composite laminates are obtained. Axisymmetric Vibrations of Orthotropic Composite Circular Plates is analysed by Greenberg and Stavsky (1978). In this a sixth order system of equations of motion is formulated in terms of the radial and transverse displacements for axisymmetric vibrations of circular plates laminated of polar orthotropic plies. Previous results for heterogeneous isotropic circular plates are included as a special case in the present theory.

A finite element model is developed by Liu *et al*. (1999) for the shape control and active vibration suppression of laminated composite plates with integrated piezoelectric sensors and actuators. The model is based on the classical laminated plate theory and the principle of virtual displacements. Four-node rectangular Non-conforming plate bending elements are used to model the laminated composite plate. A computationally efficient and highly accurate numerical method is proposed by Cheung and Zhou (2001) to analyze the vibrations of symmetrically laminated rectangular composite plates with intermediate line supports. A set of admissible functions are developed from the static solutions of a beam with intermediate point supports under a series of sinusoidal loads, in which the beam may be considered to be a unit width strip taken from the plate in a direction parallel to the edges of the plate. In addition to satisfying both the geometric boundary conditions of the plate and the zero deflection conditions at the line supports, this set of static beam functions, being different from the existing admissible functions, can also properly describe the discontinuity of the shear forces at the line supports, so that more accurate results can be expected for the dynamic analysis of laminated rectangular plates with intermediate line supports. The governing eigen frequency equation of the plates is derived by using the Rayleigh-Ritz approach.

A meshless approach based on the reproducing kernel particle method is developed for the flexural, free vibration and buckling analysis of laminated composite plates. In this approach, the first-order shear deformation theory (FSDT) is employed and the displacement shape functions are constructed using the reproducing kernel approximation satisfying the consistency conditions. The essential boundary conditions are enforced by a singular kernel method. Numerical examples involving various boundary conditions are solved by Wang *et al.* (2002) to demonstrate the validity of the proposed method. Comparison of results with the exact and other known solutions in the literature suggests that the meshless approach yields an effective solution method for laminated composite plates.

The dynamic performance of a multi-layered composite plate with embedded shape memory alloy (SMA) wires has been investigated in terms of the changes in its relative fundamental natural frequency by Arkadiusz *et al.* (2003). A sensitivity analysis has been carried out on the influence of various geometrical parameters and material properties on the plate's dynamic performance, as well as the influence of the form of boundary condition. The use of the Active Property Tuning (APT) method and the Active Strain Energy Tuning (ASET) method has also been discussed within the study. The finite element method has been used for the analysis and a new element has been exploited for modelling multi-layered composite plates. It has been found that the dynamic performance of the multi-layered composite plate with embedded SMA wires strongly depends on the plate geometry and the form of boundary condition; however, the dynamics can be successfully controlled and influenced by an optimal selection of the geometrical parameters and material properties.

Aydogdu and Timarci (2003) has studied the vibration analysis of cross-ply laminated square plates subjected to different sets of boundary conditions. The analysis is based on a five-degree-of-freedom shear deformable plate theory. The requirement of the continuity conditions among the layers for the symmetric cross-ply laminated plates are fulfilled by the use of the shape functions incorporated into this theory which, also, unifies the two-dimensional shear deformable plate theories developed previously. Initially, the governing equations obtained by use of Hamilton's principle for the vibration of cross-ply laminated plates with simply supported boundary conditions at all of their edges are solved by an exact analytical method.

A finite element model, based on third-order shear deformation theory, is used in this study by Latheswary *et al.* (2004a) studied the linear and the non-linear free vibration analysis of laminated composite plates. This study has been motivated by the lack of open literature on large amplitude dynamic analysis of laminated plates based on higher-order theory. Moreover, the effect of various plate parameters on the linear and non-linear fundamental frequency of vibration has not been studied in detail so far. The non-dimensional frequency of vibration is found to increase with increase in plate width-to-thickness ratio, both in the linear and the nonlinear range. The effect of non-linearity is seen significant for plates of width-to-thickness ratio greater than 40. The in-plane edge conditions have significant influence on the non-linear frequency of vibration.

Cugnoni *et al.* (2004) presents efficient C^0-compatible finite elements for modelling laminated composite shells under free vibrations. Derived from the first-order shear deformation theory (equivalent single-layer laminate model), the elements are well adapted for evaluating the global dynamic response (natural frequencies and mode shapes) of moderately thick multilayered shells. The components of their structural matrices are based on an exact integration per layer, which results in a higher solution accuracy than with standard explicit through-the-thickness schemes. The described finite element formulation, which can be easily implemented in commercial finite element codes, is next validated by means of several experimental modal test cases on thin to relatively thick plates or shells.

A discrete method is developed by Huang *et al.* (2005) for analyzing the free vibration problem of orthotropic rectangular plates with variable thickness. The Green function, which is obtained by transforming the differential equations into integral equations and using numerical integration, is used to establish the characteristic equation of the free vibration. The effects of the aspect ratios, boundary conditions and the variation of the thickness on the frequencies are considered.

For free vibrations of polar orthotropic plate, simple approximate closed form solutions for mode shapes and its natural frequencies were obtained by Kang *et al.* (2005) using the Rayleigh-Ritz method. Coordinate function satisfying the natural boundary conditions and the predetermined coefficients was adapted, which results in compact expressions and enables to readily calculate symmetric and nonsymmetric natural frequencies for arbitrary values of the elastic constants. The derived formulation can be used in designing of circular plates such as wood disk, which are naturally endowed with material orthotropy as well as fiber reinforced composite materials.

A theoretical approach for the free vibration analysis of delaminated unidirectional sandwich panels is developed by Schwarts-Givli *et al.* (2007). The theoretical model accounts for the flexibility of the core in the out of plane (vertical) direction and the resulting high-order displacement, acceleration and velocity fields within the core. The analytical approach is based on Hamilton's variational principle along with the high-order unidirectional sandwich panel theory and the modified Galerkin method. The two types of models investigated include delaminated regions with and without contact. The ability of the model to describe the high-order effects such as the pumping phenomenon and the localized effects in the vicinity of the delaminated regions is examined.

The state-vector approach is proposed by Chen *et al.* (2007) to analyze the free vibration of magneto-electro-elastic laminate plates. The extended displacements and stresses can be divided into the so-called in-plane and out-of-plane variables. Once the state equation for the out-of-plane variables is obtained, a complex boundary value problem is converted into an equivalent simple initial value problem. Through the state equation, the propagator matrix between the top and bottom interfaces of every layer can be easily derived. The global propagator matrix can also be assembled using the continuity conditions. It is obvious that the order of global propagator matrix is not related to the number of layers. Consequently, this approach possesses certain virtues including simple formulation, less expensive computation, etc. To test the formulation, the developed solution is then applied to a simply supported multilayered plate constructed of piezoelectric and/or piezomagnetic materials. The natural frequencies and corresponding mode shapes are computed and compared with existing results.

A mixed layerwise theory and Differential Quadrature (DQ) method (LW-DQ) for three-dimensional free vibration analysis of arbitrary laminated circular cylindrical shells is introduced by Malekzadeh *et al.* (2008). Using the layerwise theory in conjunction with the three-dimensional form of Hamilton's principle, the transversely discretized equations of motion and the related boundary conditions are obtained.

The free vibrations characteristics of simply supported anisotropic composite laminates are investigated by Ganapathi *et al.* (2009) using analytical approach. The formulation is based on the first-order shear deformation theory and the shear correction factors employed are based on energy consideration that depends on the layup as well as material properties. The governing equations are obtained using energy method.

A study of static deformations and free vibrations of shear flexible isotropic and laminated composite plates with a first-order shear deformation theory is presented by Ferreira *et al.* (2009). The analysis is based on collocation with a Deslaurier Dubuc interpolating basis to produce highly accurate results. Numerical results for isotropic and symmetric laminated composite plates are presented and discussed for various thickness-to-length ratios. The Discrete Singular Convolution (DSC) method for the free vibration analysis of laminated trapezoidal plates is studied by Murat *et al.* (2009). The plate formulation is based on first-order shear deformation theory (FSDT). The straight-sided trapezoidal domain is mapped into a square domain in the computational space using a four-node element by using the geometric transformation. The frequency parameters are obtained for symmetric angle-ply and cross-ply laminated trapezoidal plate. The accuracy of the present method is demonstrated by comparing with numerical and analytical solutions available in the literature.

Amabili and Farhadi (2009) studied (1) the classical Von Karman theory, (2) the first-order shear deformation theory and (3) the higher-order (third order) shear deformation theory are compared for studying the nonlinear forced vibrations of isotropic and laminate composite rectangular plates. In particular, the harmonic response in the frequency neighborhood of the fundamental mode of rectangular plates is investigated and the response curves computed by using the three different theories are compared.

A Ritz approach has been used by Liz and Ricardo (2007) for the study of the vibration of angle-ply symmetric laminated composite rectangular plates with edges elastically restrained against rotation and translation. Also, new numerical results are presented and some results are compared with existing values in the literature. The free vibration characteristics of laminated composite and sandwich plates with embedded and/or surface-bonded piezoelectric layers is studied by Topdar *et al.* (2007), where a hybrid plate theory is proposed for modelling the structural system. It involves a problem of coupled electromechanical field. The variation of mechanical/structural displacements across the thickness are modelled by an efficient plate theory, which ensures inter-laminar shear stress continuity as well as stress free condition at the plate top and bottom surfaces. Levy-type (semi-analytical) finite element analyses of free vibration and stability of laminated composite rectangular plates based on both classical and first-order shear deformation theories are presented by Luccioni and Dong (1998). In this finite element version, discretization occurs in one coordinate direction (say the y-axis), leaving the behavior in the x-direction and in time undetermined at the outset. In this formulation, arbitrary boundary conditions may be imposed on the two opposite ends of the plate in the y-direction. Hamilton's principle is used to derive the stiffness, mass and initial stress matrices that enter into the equations of motion. Periodic solution forms are taken in the x-direction, whereupon the analyses take the form of algebraic eigen problems from which the frequencies and critical buckling loads may be extracted.

As the electronic products are desired to have many functions with low weight and small size increasingly, the ultra-thin and multi-layer Printed Circuit Boards (PCBs) are required to be used extensively in electronic packaging assemblies. Usually, these multi-layer PCBs consist of multiple layers of woven glass fiber reinforced epoxy resin composite substrate sandwiched between copper foils. The mechanical properties of these multi-layer PCBs can be represented basically by their bending stiffness. However, complex woven composite material properties complicate the bending stiffness analysis. In this research, a finite element analysis model was suggested to describe the bending behavior of woven fiber composite multi-layer PCB. Both finite element simulation and experiment were employed in this study by Li *et al.* (2008).

Analysis of thickness locking in classical, refined and mixed multilayered plate theories studied by Erasmo Carrera and Brischetto, (2008). This study has presented a numerical investigation on thickness locking in classical, refined and advanced mixed theories for one-layered and multilayered isotropic and orthotropic plates. Closed form solutions have been given. The following already known conclusions have been confirmed.

- The use σ_{zz} = 0 condition appears a suitable technique to contrast TL in thin-plate analysis. It is very effective to contrast TL in the case of TPT and FSDT applications for the evaluation of global parameters such as displacement amplitudes and/or circular frequencies
- σ_{zz} = 0 preserves its advantages if applied to plate theories (HOT) with linear distribution of transverse displacement field uz
- TL appears if and only if a plate theory shows a constant distribution of transverse normal strain ε zz; that is to avoid TL the plate theories would require at least a parabolic distribution of transverse displacement component uz

After constructing some elasticity models, a set of close three-dimensional linear analytical solutions, taking account of all of the normal stresses, shear stresses and satisfying all the equations of equilibrium, the mid-plane clamped boundary conditions and interfacial continuity conditions through-thickness, are presented for axially symmetrical homogeneous isotropic circular plates, laminates and sandwich plates under uniform transverse load by using the variable-separating method and formulating a set of displacement functions by Luo *et al.* (2004). Reasonability of the present solutions is demonstrated comparing with FEM analysis, pb-2 Ritz theory analysis and experimental results of sandwich plates.

Linear static analysis and finite element modeling for laminated composite plates using third order shear deformation theory is studied by Aagaah *et al.* (2003). In this study, deformations of a laminated composite plate due to mechanical loads are presented. Third order shear deformation theory of plates, which is categorized in equivalent single layer theories, is used to derive linear dynamic equations of a rectangular multi-layered composite plate. Moreover, derivation of equations for FEM and numerical solutions for displacements and stress distributions of different points of the plate with a sinusoidal distributed mechanical load for Navier type boundary conditions are presented.

BUCKLING AND POSTBUCKLING ANALYSIS

An semi-analytical solution method is presented for the buckling analysis of plates with a reinforced cutout under both uniform and non-uniform compression loading. The cutout is reinforced on both sides of the plate. The solution for buckling response is achieved in two subsequent steps: pre-buckling response from in-plane stress analysis and the critical buckling load and its mode from a buckling analysis that utilizes the prebuckling stresses. The pre-buckling stresses are obtained based on the principle of minimum potential energy while automatically satisfying the equilibrium equations and compatibility condition. Hence, the strain energy of the plate is directly achieved by boundary integrals only. However, there exist material and thickness discontinuities between the unreinforced and reinforced regions of the plate. Unlike the pre-buckling equilibrium equations, the buckling equations, which are obtained from the Treftz criterion, are highly complex and the potential functions automatically satisfying these equations do not exist. Hence, the Treftz criterion is applied with the assumed transverse displacement functions utilizing the full domain integration. In both pre-buckling and buckling analyses, the form of the assumed functions (real or complex) does not necessarily satisfy the kinematic boundary conditions. The kinematic boundary conditions are applied by introducing elastic springs along the boundaries and by enforcing the displacement field to satisfy kinematic boundary conditions through energy minimization.

A global/local analysis methodology for obtaining the detailed stress state of stepped, square plates with cutouts is presented by Kapania *et al*. (1997). The method is based on a global Ritz analysis and a local finite element analyses. The method is evaluated for an isotropic square plate with circular and elliptical cutouts and also for stepped plates with circular cutout. The method was also tested for the composite plate with a circular hole.

Parhi *et al*. (2001) studied the first ply failure analysis of laminated composite plates with arbitrarily located multiple delaminations subjected to transverse static load as well as impact. The theoretical formulation is based on a simple multiple delamination model. Conventional first order shear deformation is assumed using eight-noded isoparametric quadratic elements to develop the finite element analysis procedure. Composite plates are assumed to contain both single and multiple delaminations. For the case of impact, Newmark time integration algorithm is employed for solving the time dependent multiple equations of the plate. Tsai-Wu failure criterion is used to check for failure of the laminate for both the cases. To investigate the first ply failure, parametric studies are made for different cases by varying the size and number of delaminations as well as the stacking sequences and boundary conditions.

Static tests were carried out by Caprino *et al*. (2002) on moderately anisotropic, simply supported circular plates made of graphite fibre reinforced plastic laminates of various thicknesses, loading them at the centre by a hemispherical tup. A non-linear solution available for large deflections of isotropic plates was suitably modified to account for the Hertzian contact phenomena and adopted to model the plate behaviour in the elastic field. From the experimental results, an unacceptable error is made in predicting the force-deflection curve up to first failure, if the non-linear portion of the deflection is neglected. This error is the more evident when thin laminates are concerned. For thick laminates, the Hertzian contact plays a significant role in affecting the plate behaviour. Taking into account both the local deformation and non-linearity due to large deflections, a very accurate prediction of the load-deflection curve was obtained. Only in the case of the thinnest composites tested, a divergence of the theoretical curve from the experimental data was observed at sufficiently high loads. The analysis of the failure modes revealed that the discrepancy is seemingly attributable to internal damage not resulting in clearly discernible discontinuities in the load-deflection curve.

The behaviour of laminated composite plates under static loading is studied by Latheswary *et al*. (2004b) using a four-noded element with seven degrees of-freedom per node, based on higher-order shear deformation theory. The effects of plate width-to-thickness ratio, fibre orientation, number of layers, thickness ratio, aspect ratio and boundary conditions on the displacement and stress response of symmetric and anti-symmetric laminated composite plates subjected to uniformly distributed normal loads are presented. The non-dimensional central deflection is found to decrease with increase in plate width-to-thickness ratio. The central deflection approaches a minimum for 45°C fibre orientation. The number of layers does not have much influence on the central deflection beyond six layers. The thickness of individual layers plays an important role in the response of the plate.

Van Phu and Dat (2005) deals with the analysis of non-linear multilayered reinforced composite plates with simply supported along its four edges by Bubnov- Galerkin and Finite Element Methods. Numerical results are presented for illustrating theoretical analysis of reinforced and unreinforced laminated composite plates.

The high stress concentration at the edge of a cutout is of practical importance in designing of the engineering structures. There is not any closed form solutions for a plate with a general shape cutout. These types of cutout usually are determined either experimentally or numerically using finite element methods. The simple analytical stress analysis provides a numerical result for stress concentration factors for perforated plates. This study has presented an analytical solution for stresses in composite plates with special shaped cutouts. The stress concentration of isotropic and composite plates with variety of centrally located cutout was investigated by Rezaeepazhand and Jafari (2005). Analytical and numerical studies were conducted to investigate the effects of variation in cutout shape and geometries on the location and the value of the maximum stress in flat plate under uni-axial tension load. Leknitsjki's solution for circular and elliptical cutout is extended to special cutout shape using complex variable mapping. This complex variable function can be used in modeling and evaluation of stress distribution in perforated composite and isotropic plates. The stress concentration factor of perforated plates can be significantly change by using proper material properties and cutout parameters.

The buckling behaviour of composite plates (unidirectional and cross-ply, symmetrical and non-symmetrical), with multiple delaminations of equal length, subjected to uniaxial compressive load, has been studied with FEM performing linear and non-linear analyses. More general cases than those present in the literature have been analysed. The influence of stacking sequence and of number, position and length of the delaminations, on the value of the critical load and on threshold values between global, mixed and local behaviours, has been investigated by Cappello and Tumino (2006). Linear and non-linear analyses give similar results when the unstable sublaminates are located in the external parts of the laminate. When these sublaminates are located internally results can be different: the discordance could be caused by possible interpenetration of plies during buckling, that occurs when using a linear analysis, which can give more conservative results, contrary to what happens when using a non-linear analysis and in reality.

In general, for a small delamination length ($a/L < 0.2$) the influence is small and the characteristics at buckling are similar to those of the laminates without cracks (global buckling). The influence increases as the delamination length increases and causes the local buckling of a sublaminate that precedes any other mode of instability because a sublaminate is slender enough when compared with laminate as a whole; in this case significant out-of-plane deflection occurs, generally, in the thinner sublaminates. Regarding to cross-ply laminates, numerical analyses demonstrate that with laminae at 0° on the surface the buckling load of the undamaged composite increases and a high strength to buckling can be obtained also in presence of delaminations at the interfaces with laminae at 90°.

The distributions of stresses and deflection in rectangular isotropic, orthotropic and laminated composite plates with central circular hole under transverse static loading have been studied by Mittal and Jain (2008a) using finite element method. The aim of author is to analyze the effect of D/A ratio (where, D is hole diameter and A is plate width) upon Stress Concentration Factor (SCF) and deflection in isotropic, orthotropic and laminated composite plates under different transverse static loading condition. Analysis has been done for symmetric and anti-symmetric composite laminates. The results are obtained for three different boundary conditions. The variations of SCF and deflection with respect to D/A ratio are presented in graphical form and discussed. The finite element formulation is carried out in the analysis section of the ANSYS package.

The Effect of fibre orientation on stress concentration factor in a rectangular composite laminate with central circular hole under transverse static loading has been studied by and Mittal and Jain (2008b) using Finite Element Method. The percent variations in deflection with fibre orientation are also compared with deflection in laminate without hole. A finite element study is made for whole analysis of laminate with a central hole under transverse static loading.

A finite strip method for non-linear static analysis based on the tangential stiffness matrix has been developed, Zahari and El-Zafrany (2008) using the new concept of polynomial finite strip elements, with Mindlin (first-order shear deformable element) plate-bending theory for composite plates. A progressive failure algorithm for composite laminates has been successfully developed for the new finite strip methods using a stress-based failure criterion, Tsai-Wu. A finite strip analysis programming package which is capable of performing non-linear progressive damage analysis for composite stiffened plates and shells has also been developed with Mindlin plate-bending element. Good agreement with the finite element results has been observed through various test cases, confirming the accuracy and reliability of the new developed method.

A higher-order layerwise theoretical framework was developed by Theofanis and Saravanos (2009) for predicting the through-thickness response of thick composite and sandwich composite plates. Linear, parabolic and cubic distributions of the in-plane displacements were assumed through the thickness of each discrete layer, thus, the composite laminate could be modeled using a small number of discrete layers compared to linear layerwise theories. Interlaminar shear stress compatibility conditions were explicitly imposed on the discrete layer stiffness matrices by using a propagating algorithmic procedure, enabling prediction of interlaminar shear stress at the interfaces between adjacent discrete layers. Validations of the current theory with 3-D exact solution and a linear layerwise theory for thick composite and sandwich composite plates were conducted. The higher-order layerwise theory proved to be robust, since it accurately predicted parabolic through-thickness distributions of displacements, strains and stresses at the interfaces and at free edges and efficient, since it used a small number of discrete layers to model the composite laminate through-thickness. The deviations observed indicated the effect of constant transverse displacement through-thickness assumed in the present formulation. Overall, the validations conducted illustrated the enhanced capabilities of the present higher order layerwise laminate theory, as well as, its range of applicability.

Based on the two-dimensional theory of elasticity, an accurate solution is presented by Malekzadeh (2009) for the static analysis of thick laminated deep circular arches with general boundary conditions. The formulations are general in the sense that the effects of the variation of arch curvature across the cross-section, the transverse shear and normal stresses are included. Fast rates of convergence of the method are demonstrated and its high accuracy with low computational efforts is exhibited by comparing the results with existing solutions in the literature. The effects of different parameters such as opening angle, thickness to-length and ply angle on the normalized stress and displacement components of the thick laminated arches with different set of boundary conditions are investigated. The solutions can be used as benchmarks for other numerical methods and also to clear the accuracy of the classical theories such as the first order shear deformation theory.

OPTIMIZATION IN COMPOSITE PLATES

The optimal control problem of minimizing the dynamic response of anisotropic symmetric or antisymmetric composite laminated rectangular plates with various boundary conditions is presented by Fares *et al.* (2002) using various plate theories. The objective of the present control problem is to minimize the dynamic response of the plate with minimum possible expenditure of force. The dynamic response of the structure comprises a weight sum of the control objective (the total vibrational energy) and a penalty functional of the control force. In addition to the active control, the layer thickness and the orientation angle of the material fibers are taken as optimization design variables. The explicit solutions for the optimal force and controlled deflections are obtained in forms of double series using the Liapunov-Bellman theory. The effectiveness of the proposed control and the behavior of the controlled structure are investigated. Various numerical results including the effect of boundary conditions, number of layers, anisotropy ratio, aspect ratio and side-to-thickness ratio on the control process for symmetric and antisymmetric laminates are presented.

Mackerle (2004) presented a bibliographical review of the finite element analyses and simulations of manufacturing processes of composite materials and their mechanical properties from the theoretical as well as practical points of view. Topics include: filament winding process; braiding, weaving and knitting; fiber preforms and resin injection; pultrusion; compression molding; injection molding; extrusion and other specific manufacturing processes and processes in general.

A strength-based multiple cutout optimizations in composite plates using fixed grid finite element method are studied by Liu *et al.* (2006). As a typical structural feature, interior cutouts are often indispensable in meeting such special technical requirements as laying fuel lines and electrical cables in aircraft wing spars, opening access holes for the service of interior parts in machine, ventilating the air of tubes and passing the liquid at the bottom of container. The presence of these holes, however, may significantly change the stress intensity and structural performance, which could to a certain extent affect the operational life of the composite structures. For this reason, one of the foremost design objectives can be to minimize the resulting stress-induced failure due to the introduction of cutouts. Based on the Tsai-Hill failure criterion of the first ply, this study presents a newly developed Fixed (FG) Grid Evolutionary Structural Optimization (ESO) method to explore shape optimization of multiple cutouts in composite structures. Different design cases with varying number of cutouts, ply orientations and lay-up configurations are taken into account in this study. The examples demonstrate that the optimal boundaries produced by FG ESO are much smoother than those by traditional ESO. The results show the remarkable effects of different opening numbers and various lay-up configurations on resulting optimal shapes. The study also provides an in-depth observation in the interactive influence of the adjacent cutouts on the optimal shapes.

The finite element formulation presented by Naghipour *et al.* (2008), which was based on shear deformation theory (FSDT) and predicts reasonably good results for the laminated plate of mobile bridge deck with different stacking sequence and fiber orientation. The results have also been compared with the result of finite difference method, based classic plat theory (CLPT). This comparison was done to determine the maximum deflection. The principle normal stress and shear stress of the central point was also determined using the finite element formulation. The studies reveal the influence of various parameters and show the following facts:

- The central deflection is a minimum for 0 fiber orientation
- The central deflection decreases with increase in stiffness ratio
- The normal stress and in-plane shear stress with increase in stiffness ratio
- The central deflection decreases with increase in number of layers, but the rate of decrease is negligible beyond 20 layers
- The normal stress is found to decrease and in-plane shear stress is found to increase with increase in the number of layers for plate of arrangements in all cases
- The variation of deflection, normal stress and inplane shear stress with stiffness ratio follow the same pattern for both simply supported and clamped conditions. But the variation of transverse shear stress with stiffness ratio for clamped plates is different from that for simply supported plates

Effect of fibre orientation: Four-layer symmetric and anti-symmetric laminates with the angle of fibre orientation varying from 0 to 45° with b/h = 10 and 100 are analysed. A change in fibre orientation angle from

0 to 45° leads to an increase in the fundamental frequency of vibration in the case of both thick (b/h = 10) and thin (b/h = 100) plates as in Fig.1. It can also be seen that the fundamental frequency of vibration for symmetric arrangement is less than that for antisymmetric arrangement, the difference being more for higher values of α.

Effect of number of layers: The effect of number of layers on non-linear fundamental frequency is studied by considering both thin and thick crossply and angle-ply laminates with anti-symmetric lay-up. The variation of frequency ratio with number of layers is shown in Fig. 2.

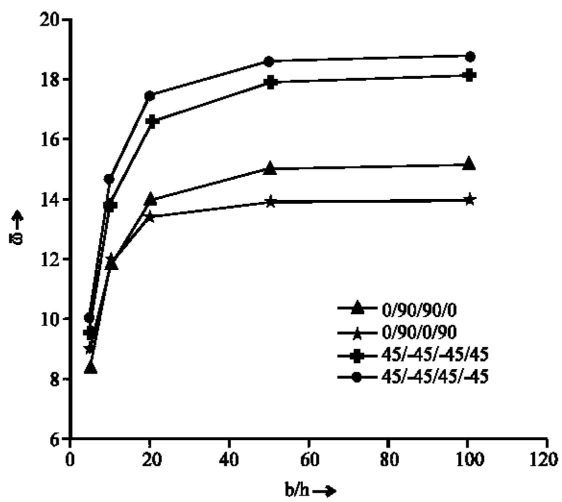

Fig. 1: Variation of non-dimensional fundamental frequency with b/h ratio

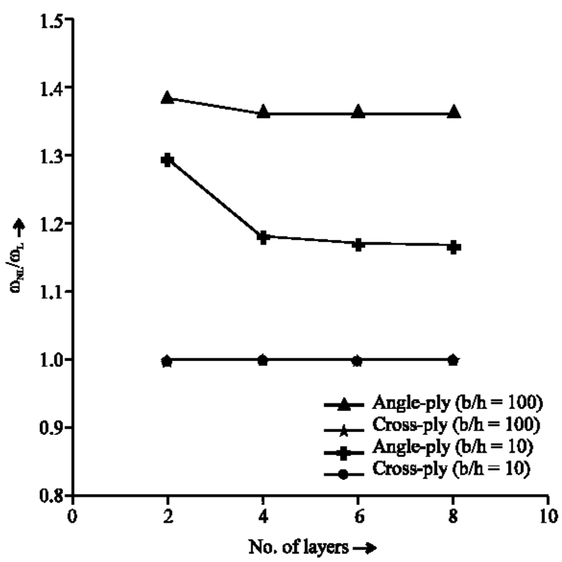

Fig. 2: Variation of frequency ratio with number of layers

Even though there is no change in the frequency ratio with increase in number of layers in the case of thick plates, there is a small change from two to four layers in the case of thin plates and afterwards the value remains practically constant. This is in agreement with the earlier observation that two-layer plates behave differently from multi-layered plates.

CONCLUSION

Linear and non-linear free vibration analysis of laminated composite plates using a finite element model, based on third order shear deformation theory and including Von-Karman nonlinear strains, is presented. The non-dimensional fundamental frequency of vibration is found to increase with increase in width-to-thickness ratio, material anisotropy and angle of fibre orientation. The edge conditions of the plate play an important role in the frequency of vibration of the system. The fundamental frequency of vibration decreases gradually with aspect ratio. The effect of number of layers is found to be insignificant beyond four layers. The frequency ratio increases with increase in width-to-thickness ratio. The Non-linear fundamental frequency of vibration decreases with increase in fibre orientation angle for thin plates, the maximum decrease being for α = 45°. The number of layers does not have any significant influence on non-linear frequency of vibration beyond four-layers. The effect of non-linearity is found to be significant for plates with width-to-thickness ratio greater than 40 aspect ratio. The effect of number of layers is found to be insignificant beyond four layers. The frequency ratio increases with increase in width-to-thickness ratio. The Non-linear fundamental frequency of vibration decreases with increase in fibre orientation angle for thin plates, the maximum decrease being for α = 45°. The number of layers does not have any significant influence on non-linear frequency of vibration beyond four-layers. The effect of non-linearity is found to be significant for plates with width-to-thickness ratio greater than 40.

FUTURE RESEARCH

The recent advances of the finite element analysis of composite laminated plates based on various lamination theories, with the focus on the free vibration and dynamics, buckling and postbuckling analysis, geometric nonlinearity and large deformation analysis and failure and damage analysis of composite laminated plates, are reviewed in this study. The development of buckling and postbuckling analysis under material nonlinearity and thermal effects are emphasised and in the failure analysis, the concentration is especially on the advances of the first-ply failure analysis. Based on the author's investigation, it has been found that the research on the following aspects of the composite laminated plates is relatively limited and may attract more interests in the future research.

- Material nonlinearity effects on structural behaviour of composite laminates
- Failure and damage analysis under viscoelastic effects such as thermal and creep effects
- Failure and damage analysis under cyclic loading
- Micromechanical approach for damage analysis
- Analysis of the damage evolution in composite laminates
- Multiscale modelling of crack initiation, propagation and overall structural failure
- The free vibration problem of antisymmetric laminated plates having translational as well as rotational edge constraints which appears to have not been studied as yet

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