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A Study on the Existence of an Optimal Distribution of Stiffness over the Height of Mid-to High-Rise Buildings to Minimize the Seismic Input Energy

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Abstract: In the Energy Based Design (EBD) of structures Housner's assumption has a primary importance. Based on this assumption the average input energy from earthquakes to a building, modeled as a SDOF system, is related mainly to total mass of the building, having the damping ratio of the building as small as 0.03 and its period of vibration more than 0.4 sec (For smaller periods input energy is function of period also). For higher damping ratios, the maximum energy attained decreases as the damping ratio increases. Thus, mainly the seismic input energy per unit mass of the system is only related to the ground motion characteristics. Housner and Akiyama assumed that the seismic input energy to multistory structures can be estimated by input energy to an equivalent single degree of freedom system. The validity of this assumption was demonstrated experimentally. This study attempts to show analytically the range of validity of these assumptions in linear systems and the possibility of existence of an optimal stiffness distribution over the height of high rise buildings to minimize the seismic input energy. For this purpose it will be shown that from the spectral standpoint, input energy spectra generally is a function of natural period of vibration and if so, input energy is related also to the stiffness of structure, as to the mass, damping ratio and ground motion characteristics.

Key words: Seismic input energy, modal analyses, input energy spectrum, velocity spectrum

INTRODUCTION

Housner was first who used concept of input energy as seismic design criteria (Housner, 1956). He presented his pioneering work in the 1st WCEE in 1956. Three main conclusions of his paper are of special concern in this study:

- The seismic energy input to a SDOF structure with specified damping, if it seen from spectral or average standpoint, is basically constant and independent from its period, especially for low damping ratios
- Seismic design of structures may by thought as satisfying the following inequality: Energy absorption capacity > seismic input energy. On the other hand, the amount of energy input to an elastic system is the upper bound of energy input to hysteretic systems with the same liner properties. So, seismic design of structures dose not means providing too strong elements with the capability of converting kinetic energy of structure to elastic strain energy and instead it is enough to supply sufficient capacity of energy absorption via plastic deformations in structural elements
- Seismic energy input to MDOF systems basically depends to their total mass and therefore it is equals to energy input to an equivalent SDOF with the same mass and main period of vibration

Based on Housner's work, Akiyama published his very important book Earthquake Resistant Limit State Design for Buildings in 1985. He expanded Housner's assumptions and showed their limitations and strong points (Akiyama, 1985). He developed input energy spectrum for different site soils. Those spectrums are basically constant with respect to the period of vibration except for periods smaller than predominant periods for the ground. As Housner, Akiyama tried to simplify seismic design of structures by supposing and demonstrating that input energy to structures is related mainly to earthquake excitation but scarcely to structural features. Most of researchers adapted this assumption

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and equation proposed by Kuwamura and Galambos (1989), Fajfar *et al.* (1989), Uang and Bertero (1990) Kuwamura *et al.* (1994), for establishing the earthquake input energy are based on the ground motion characteristics only.

Parallel to these works in estimating the energy demand, other researchers focused on the mechanism of dissipation of the input energy in structural elements by the hysteretic action. Park and Ang (1985) related their damage index to the energy dissipation via hysteretic loops. This means that reduction in the hysteretic dissipation of the input energy which is a fraction of the total input energy, reduces the structural damage. So it arises an important question:

Is it possible to minimize the seismic input energy to structures by a specific design pattern?

To answer the question, classical approach to the input energy demand problem must be reexamined the task which is the main purpose of this study.

Various aspects of seismic input energy and its calculation, like absolute and relative energies, time interval for integration of related equations and so forth, have been discussed in the literature (Kuwamura and Galambos, 1989; Uang and Bertero, 1990; Khashaee, 2003). In this study some basic assumptions and definitions which are widely used in the literature are adopted as follow:

- Relative, rather than absolute input energy is studied
- Input energy is defined as the energy induced to the structure by strong ground motion from beginning (t = 0) of motion to the end of motion (t = t0). Note that definitions of beginning and ending moments of ground motion are not unique in the literature, but this is not important in this study. In this study beginning and end of motion are assumed to be coincide to the beginning and end of record. Also it is demonstrated that the maximum input energy may be attained not necessarily at the end of motion (Uang and Bertero, 1990). However, as mentioned earlier, the input energy of the system at the ending moment of ground motion is considered as seismic input energy. So the seismic input energy to a SDOF system with mass m, frequency ω and damping ratio ζ is defined mathematically as:

$$E_I(m, \omega, \zeta) = -\int_0^{t_0} m\ddot{y}_g \dot{y}\, dt \qquad (1)$$

where, $\ddot{y}_g$ and $\dot{y}$ are, respectively ground acceleration and system velocity. For a system with unit mass Eq. 1 can be written as:

$$E_I(1, \omega, \zeta) = -\int_0^{t_0} \ddot{y}_g \dot{y}\, dt \qquad (2)$$

It is helpful to use an equivalent velocity V_E, defined based on the input energy, as (Housner, 1956):

$$V_E = \sqrt{\frac{2E_I}{m}}$$

where, E_I is input energy to the SDOF.

The damping matrix is diagonal and $c_{ii} = c \approx$ const.

By using the abovementioned assumptions, in the following sections of the paper at first some input energy spectra are obtained and then possibility of existence of optimal stiffness distribution demonstrated mathematically.

INPUT ENERGY SPECTRA

Based on definition in of the input energy, given in the previous section, some input energy spectra are obtained using ten typical earthquake ground motion records shown in Table 1. All of these records have been extracted from peer strong motion database.

All records have been normalized to 1.0 g. Figure 1-3 show equivalent velocity V_E spectra versus period of vibration respectively for ζ = 0, 0.05 and 0.10. Design Input Energy Spectrum (DIES), proposed by Akiyama (1985), is also shown in the Fig. 1. It should be mentioned that the shown DIES values are for very stiff site soil and damping ratio of 10%.

Two important conclusions may be taken from these Fig. 1-3. First is the low decay rate of average spectrum with increase in the period of vibration, especially in the practical range of periods of high rise buildings, say 0.8 to 5 seconds. For example, it can be seen in Fig. 3 that the amount of input energy at T = 5 sec is half of that at T = 0.9, but typically, spectral pseudo acceleration at T = 5 sec is less than one fifth of respect value at T = 0.9 (considering UBC97 design spectrum). Second important observation is very low sensitivity of the input energy to

Table 1: List of records used to obtain input energy spectra

Name	Location	Date	Duration (sec)	PGA (g)
Chi-Chi, W	Taiwan	09/20/99	89.99	0.9675
Duzci, 90	Turkey	11/12/99	55.89	0.8224
El Centro, 180	USA	5/19/40	39.99	0.3129
Gazli, 90	Uzbekistan	5/17/76	16.20	0.7175
Kobe, 00	Japan	01/16/95	47.98	0.8213
Landers, 00	USA	6/28/92	41.91	0.7848
Loma Prieta, 00	USA	10/18/89	39.94	0.6437
Manjil, Long	Iran	06/20/90	53.50	0.5146
Northridge, 142	USA	1/17/94	39.99	0.6125
Tabas, Tr.	Iran	09/16/78	32.82	0.8518

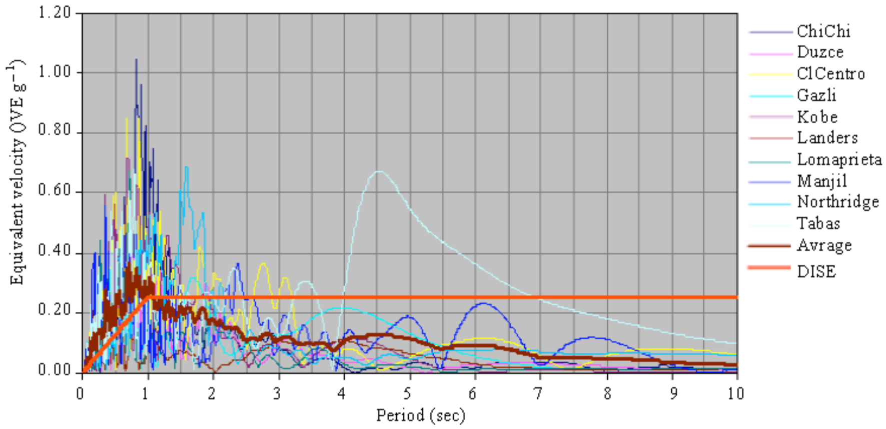

Fig. 1: Input energy spectra for ζ = 0%. Input energy spectra damping ratio = 0.00

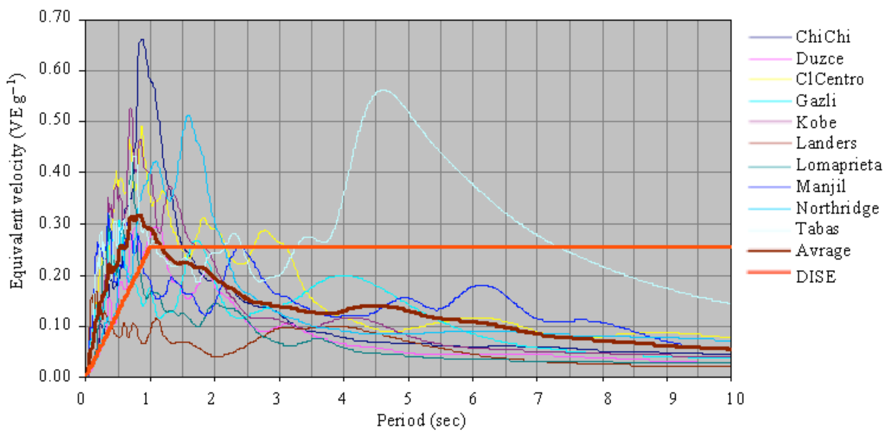

Fig. 2: Input energy spectra for ζ = 5%. Input energy spectra damping ratio = 0.05

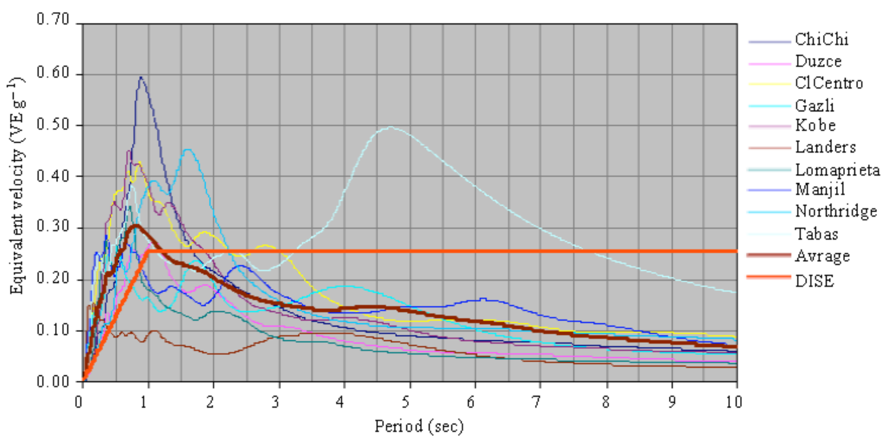

Fig. 3: Input Energy spectra for ζ = 10%.Input energy spectra damping ratio = 0.10

the damping of structures. Thus, the DIES can be assumed to be essentially constant over a wide range of periods, which is in agreement with Housner's pioneering statements. However, it is important to note that the matter is looked at from the design spectrum point of view. This means that the results are valid for average

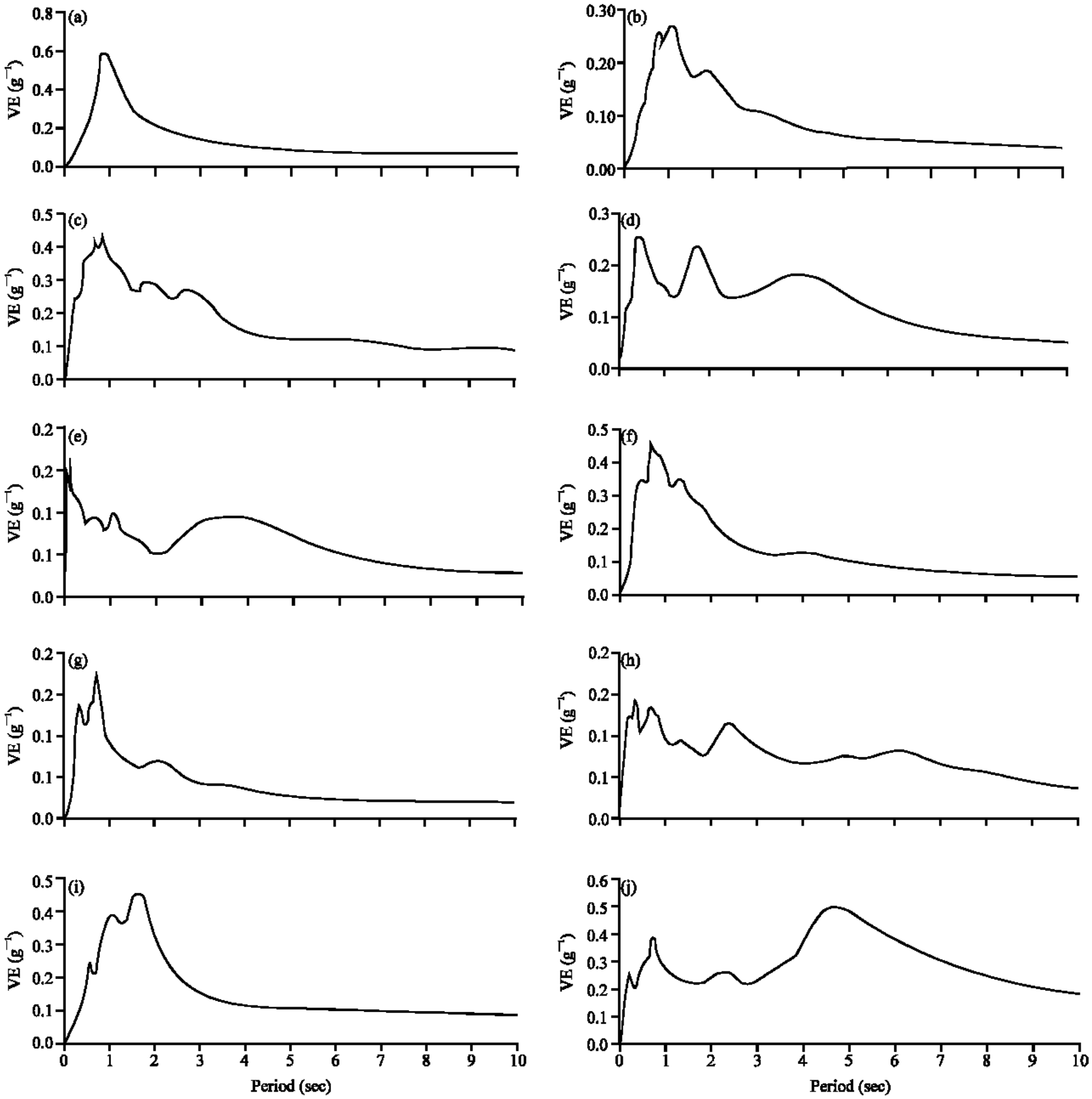

Fig. 4: Input energy spectra of the considered earthquakes for ζ=10. Input energy spectra damping ratio = 0.10. (a) ChiChi, (b) Duzce, (c) ClCentro, (d) Gazli, (e) Kobe, (f) Landers, (g) Lomaprieta, (h) Manjil, (i) Northridge and (j) Tabass

values obtained from past earthquakes, not for an individual record. Also, it should be noted that to obtain the input energy spectra in this study, records were selected regardless of their site specification features, which does not affect the outcome of the study, but some discrepancies may be seen between DIES and average of spectra (Climent *et al*., 2002).

Individual input energy spectra and pseudo velocity for each record are shown respectively in Fig. 4 and 5 for damping ratio of 10%. As seen in these two figures, there is a strong correlation between the input energy and pseudo velocity spectrum. Therefore, pseudo velocity is an important index of seismic input energy as stated by Housner (1956).

Comparing Fig. 4a-j and 5a-j one can see that the general pattern of the input energy spectrum for each record is very similar to its corresponding pseudo velocity spectrum.

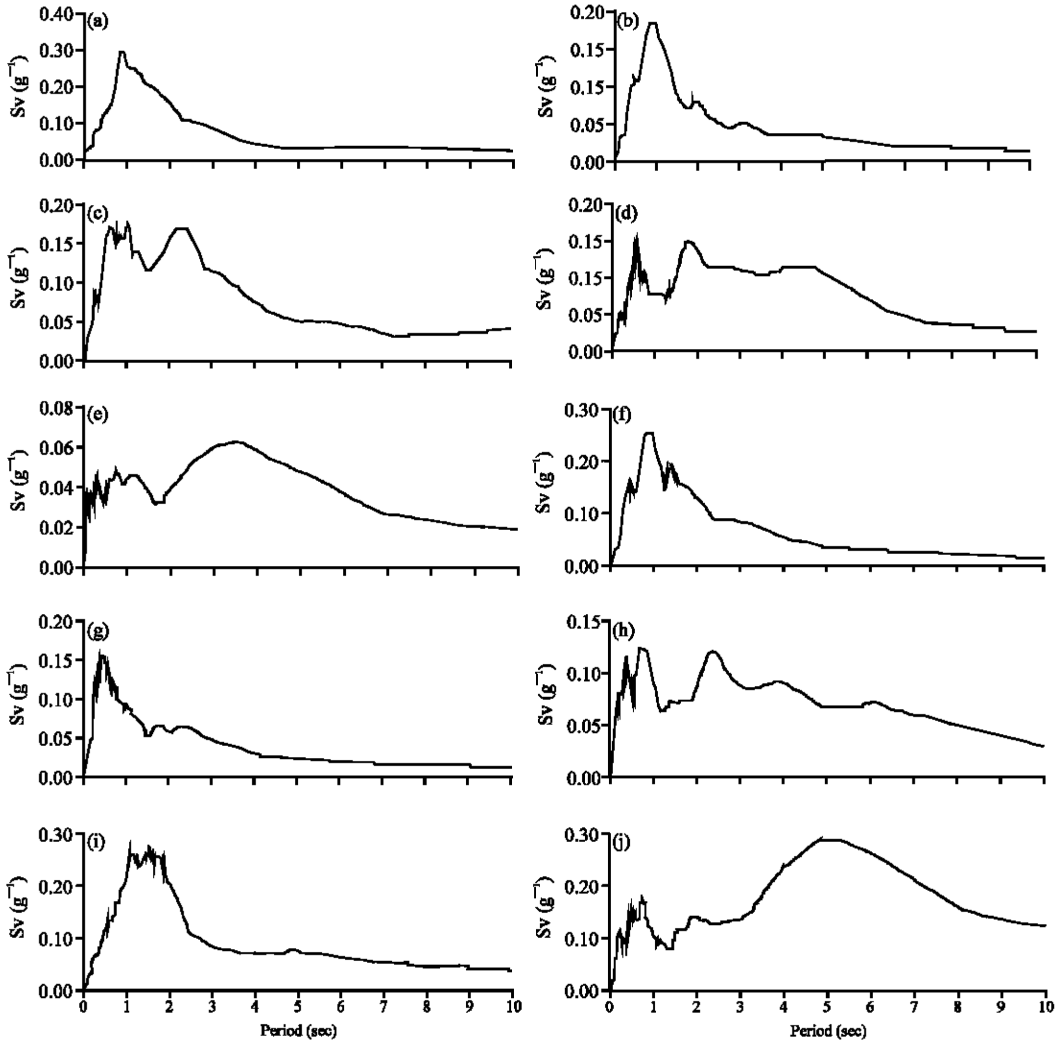

Fig. 5: Pseudo velocity spectra for ζ = 10%. Pseudo velocity spectra damping ratio = 0.10. (a) ChiChi, (b) Duzce, (c) ClCentro, (d) Gazli, (e) Kobe, (f) Landers, (g) Lomaprieta, (h) Manjil, (i) Northridge, (j) Tabas

EQUATION OF MOTION AND INPUT ENERGY TO MDOF SYSTEMS

A schematic illustration of the simplified model of multi-story buildings, considered in this study, is shown in Fig. 6.

The equation of motion of the system shown in the Fig. 6 can be written as:

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=-\ddot{y}_g[M]\{r\}$$

or:

$$[M][\Phi]\{\ddot{z}\}+[C][\Phi]\{\dot{z}\}+[K][\Phi]\{z\}=-\ddot{y}_g[M]\{r\} \quad (3)$$

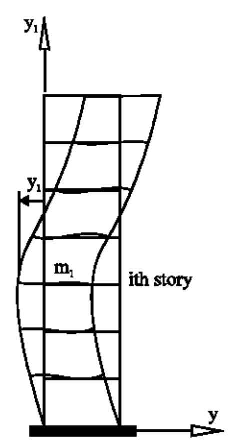

Fig. 6: Structural model

Where:

$$\{y\}=[\Phi]\{z\}$$

$$[\Phi]^T[M][\Phi]=[I] \quad ([\Phi] \text{ is orthonormalized and } [I] \text{ is unit matrix})$$

$$[\Phi]^T[C][\Phi]=[c] \quad (\text{A diagonal matrix with elements } c_{ii}=2\xi_i\omega_i \approx \text{const})$$

$$[\Phi]^T[K][\Phi]=[\omega^2]$$

$$\{r\}=\{1\}$$

It is obvious that the input energy to this system can be written as:

$$E=-\int_0^{y_0}\ddot{y}_g\{dy\}^T[M]\{r\}=-\int_0^{t_0}\ddot{y}_g\{\dot{y}\}^T[M]\{r\}dt=-\int_0^{t_0}\ddot{y}_g\left([\Phi]\{\dot{z}\}\right)^T[M]\{r\}dt$$

Thus:

$$E=-\int_0^{t_0}\ddot{y}_g\{\dot{z}\}^T[\Phi]^T[M]\{r\}dt=\sum_{i=1}^{n}\left(-\int_0^{t_0}\ddot{y}_g\dot{z}_i\{\varphi_i\}^T[M]\{r\}dt\right) \quad (4)$$

Now by decupling Eq. 2 results in:

$$[\Phi]^T[M][\Phi]\{\ddot{z}\}+[\Phi]^T[C][\Phi]\{\dot{z}\}+[\Phi]^T[K][\Phi]\{z\}=-\ddot{y}_g[\Phi]^T[M]\{r\}$$

$$[I]\{\ddot{z}\}+[c]\{\dot{z}\}+[\omega^2]\{z\}=-\ddot{y}_g[\Phi]^T[M]\{r\}$$

$$\ddot{z}_i+2\omega_i\xi_i+\omega_i z_i=-\ddot{y}_g\{\varphi_i\}^T[M]\{r\} \quad (5)$$

Equation 5 can be interpreted as equation of motion of a SDOF system with unit mass subjected to ground acceleration $\ddot{y}_g$, magnified by $\{\varphi_i\}^T[M]\{r\}$. It is evident that magnifying the excitation by $\{\varphi_i\}^T[M]\{r\}$ leads to magnification of the input energy by $\{\varphi_i\}^T[M]\{r\}$, thus considering Eq. 2 the input energy to the system shown by Eq. 5 may be written as:

$$-\int_0^{t_0}\ddot{y}_g\dot{z}_i\{\varphi_i\}^T[M]\{r\}dt=E_1(1,\omega_i,\zeta_i)\left(\{\varphi_i\}^T[M]\{r\}\right)^2 \quad (6)$$

By comparing Eq. 4 and 6 the following Equation can be written:

$$E=\sum_{i=1}^{n}E_1(1,\omega_i,\zeta_i)\left(\{\varphi_i\}^T[M]\{r\}\right)^2 \quad (7)$$

But, as indicated previously, in a wide range of relatively long to very long periods $E_1(1,\omega_i,\zeta_i)$ has no notable variation and may be taken as constant and thus Eq. 7 can be written as:

$$E=E_1(1,\omega_1,\xi_1)\sum_{i=1}^{n}\left(\{\varphi_i\}^T[M]\{r\}\right)^2 \quad (8)$$

Based on Eq. 8 it can be claimed that, considering E as a constant value, the summation term in the last this Equation must be a constant. In fact, it can be said that as the mode shapes of any system are bases of a vector space V, each vector, including {1}, in this space can be written as a linear combination of bases, which is:

$$\forall\{v\}\in V\ \exists a_i\in R \,|\, \sum a_i\{\varphi_i\}=[\Phi]\{a\}=\{v\}$$

Thus, if {v} = {1} then [Φ]{a} = {1} and one can write:

$$a_i=\{\phi_i\}^T[M]\{1\} \text{ and } \{a\}=[\Phi]^T[M]\{1\} \quad (9)$$

$$\{a\}^T\{a\}=\left([\Phi]^T[M]\{1\}\right)^T\left([\Phi]^T[M]\{1\}\right)=\{1\}^T[M]^T[\Phi][\Phi]^T[M]\{1\} \quad (10)$$

But, $[\Phi][\Phi]^T[M]$ in the right hand side of Eq. 10 must be a unit matrix because $[\Phi]^T[M][\Phi]=[I]$ and by pre-multiplication of both sides by [Φ] one can obtain the desired result. Thus, Eq. 10 may be rewritten as:

$$\{a\}^T\{a\}=\sum_{i=1}^{n}a_i^2=\{1\}^T[M]\{1\}=\sum_{i=1}^{n}m_{ii}=m_{total}$$
$$=\text{total mass of MDOF structure}$$

and finally, Eq. 8 can be written as:

$$E=m_{total}E_1(1,\omega_1,\xi_1)=E_1(m_{total},\omega_1,\xi_1)$$

This means that the seismic input energy to a MDOF system is the same as input energy to a SDOF system with same mass, main frequency and damping, provided that the following conditions are met:

- Input energy is calculated at a specified instant of a record for all modes, say at the end of record
- Input energy spectra are constant all over the wide range of periods

As it can be seen from Fig. 1-3, constancy of input energy spectra is a simplifying assumption which apparently is not fully in agreement with reality, particularly in the rage of short to medium periods. In fact, the input energy as expressed by Eq. 7 depends on the shape of spectra and therefore, on the structural features. On the other hand, calculation of the input energy at a specified instant of a record, say at its end, implies that unlike base shear and other desired quantities, the input energy is maximized simultaneously in all modes and therefore there is no need to use modal combination methods such as SRSS and CQC. This fact simplifies the problem and in conjugation with the inconstant input energy spectrum, demonstrates possibility of existence of an optimal distribution of stiffness in high rise buildings to minimize the seismic input energy to the structure.

CONCLUSIONS

Followings are the main conclusions of this study (1) Constancy of input energy spectrum is a simplified assumption which is not in agreement with reality, particularly in the rage of short to moderate periods, which correspond to majority of short- to mid-rise buildings, (2) Equality of the amount of energy, input to a MDOF system, with that of a SDOF system is limited to the validity of constancy of input energy spectrum and (3) It is conceptually possible to find an optimal distribution of stiffness over the height of a multi-story building to minimize the seismic input energy to the structure. However, it should be pointed out that the hysteretic energy dissipated by the structure is an important damage index and that; the hysteretic energy to be dissipated by the system can be expressed as a fraction of the total input energy.

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