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Importance of Assessing the Model Adequacy of Binary Logistic Regression

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Abstract: Logistic regression is a sophisticated statistical tool for data analysis in both control experimentation and observational studies. The goal of logistic regression is to correctly predict the category of outcome for individual cases using the most parsimonious model. To accomplish this goal, a model is created that includes all predictor variables that are useful in predicting the response variable. The logistic regression model is being used with increasing rate in various fields in data analysis. In spite of such increase, there has been no commensurate increase in the use of commonly available methods for assessing the model adequacy. Failure to address model adequacy may lead to misleading or incorrect inferences. Therefore, the goal of this study is to present an overview of a few easily employed methods for assessing the fit of logistic regression models. The summary measures of goodness-of-fit as Likelihood Ratio Test, Hosmer-Lemeshow goodness-of-fit test, Osius-Rojek large sample approximation test, Stukel test and area under Receiver Operating Characteristic curve indicate that the logistic regression model fits the data quite well. However, recommendations are made for the use of methods for assessing the model adequacy in different aspects before proceed to present the results from a fitted logistic regression model.

Key words: Binary response variable, covariate pattern, sensitivity, specificity, ROC curve

INTRODUCTION

Logistic regression is a statistical technique for predicting the probability of an event, given a set of predictor variables. The procedure is more sophisticated than the linear regression procedure. The binary logistic regression procedure empowers one to select the predictive model for dichotomous dependent variables. It describes the relationship between a dichotomous response variable and a set of explanatory variables. The explanatory variables may be continuous or discrete. The logistic model, as a non-linear regression model, is a special case of generalized linear model (McCullagh and Nelder, 1989) where the assumptions of normality and constant variance of residuals are not satisfied. Logistic regression models have demonstrated their accuracy in many classification frameworks (Bose and Pal, 2006; Cook *et al*., 2006; De Andres *et al*., 2006; Kiang, 2003).

Many social phenomena are discrete or qualitative rather than continuous or quantitative in nature. Binary response models are major importance in social sciences as well as demography. Such binary discrete phenomena usually take the form of a dichotomous indicator or dummy variable. Many such analysis involve an outcome or dependent variable that is dichotomous and in these studies the logistic regression model has become the statistical model of choice. The reasons for this are the ease of interpretation of the estimated coefficients as adjusted log-odds ratios, the ability to estimate the probability that a particular subject will develop the outcome and the wide availability of easily used and reliable software to perform the computations. It is now common to find, in an article using logistic regression, a table of estimated coefficients, estimated odds ratios and associated confidence limits for the odds ratio (Pampel, 2000; Hosmer and Lemeshow, 2000).

The validity of inferences drawn from modern statistical modeling techniques depends on the assumptions of the statistical model being satisfied. In order for our analysis to be valid, our model has to satisfy the assumptions of logistic regression such as the true conditional probabilities are a logistic function of the explanatory variables, the observations are independent, the explanatory variables are not linear combinations of each other, errors are binomially distributed. When these assumptions of logistic regression analysis are not met, we may have problems, such as biased coefficient estimates or very large standard errors for the logistic regression coefficients and these problems may lead to invalid statistical inferences. A critical step in assessing the appropriateness of a model is to examine its fit, or how well the model describes the observed data. Without such

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an analysis, the inferences drawn from the model may be misleading or even totally incorrect. Unfortunately, in some instances the results of fitting a logistic regression model have been published without enough satisfactory assessment of the goodness-of-fit test. Therefore, before we can use our model to make any statistical inference, we need to check the underlying assumptions involved in logistic regression and model fits sufficiently well with different aspects because logistic regression model is a powerful statistical tool and it must be used with caution (Hosmer *et al*., 1991).

The term covariate pattern is used to describe a single set of values for the covariates in a model. The number of covariate patterns may be some number less than the sample size. During the model development it is not necessary to be concerned about the number of covariate patterns, but the number of covariate patterns may be an issue when the fit of the model is assessed via logistic regression goodness-of-fit. It is substantial because goodness-of-fit is assessed over the constellation of fitted values determined by the covariate patterns in the model, not the total collection of covariates (Menard, 2002).

Some summary measures of goodness-of-fit as they are routinely provided as output with any fitted model and give an overall indication of the fit of the model. Summary statistics, by nature, may not provide information about the individual model components. A small value for one of these statistics does not rule out the possibility of some substantial and thus interesting deviation from fit for a few subjects. On the other hand, a large value for one of these statistics is a clear indication of a substantial problem with the model. The summary statistics based on the Pearson chi-square provide a single number that summarizes the agreement between observed and fitted values. The advantage as well as disadvantage of these statistics is that a single number is used to summarize considerable information. Therefore, before concluding that a model fits, it is crucial that other measures be examined to observe if fit is supported over the entire set of covariate patterns (Hosmer *et al*., 1988; Pregibon, 1981).

The goal of this study is to present an overview of a few easily employed methods for assessing the fit on the basis of summary measures of goodness-of-fit of the logistic regression models. Overall assessment of fit be examined using a combination of Likelihood Ratio Test, Hosmer-Lemeshow goodness of fit test, Osius and Rojek normal approximation to the distribution of the Pearson chi-square statistic, Stukels test and ROC curve analysis for adequacy of the fitted model. Thus, the perspective of this article is to illustrate concisely the fitting process of binary logistic regression model under standard assumptions and evaluate the predictive ability of the model under different aspects.

MATERIALS AND METHODS

The Bangladesh Demographic and Health Survey (BDHS-2004) is part of the worldwide Demographic and Health Surveys program, which is designed to collect data on fertility, family planning, maternal and child health. The BDHS are intended to serve as a source of population and health data for policymakers and the research community. The study was conducted during 2003-2004 and the data were published in 2005. The study will utilize data from BDHS-2004 employed nationally representative, two stage sample that was selected from the master sample maintained by the Bangladesh Bureau of Statistics (BBS) for the implementation of survey before the next census 2011. Macro International Inc. of Calverton, Maryland, USA provided technical assistance to the project as part of its International Demographical and Health Surveys program and financial assistance was provided by The United States Agency for International Development (USAID), Bangladesh. A total of 10,523 households were selected for the sample, of which 10,500 were successfully interviewed. In those households, 11,601 women were identified as eligible for the individual interview and interviews were completed for 11,440 of them (Mitra *et al*., 2005). But in this analysis there are only 2,212 eligible women those are able to bear and desire more children are considered. The women under sterilization, declared in fecund, divorced, widowed, having more than and less than two living children are not involved in the analysis. Those women who have two living children and able to bear and desire more children are only considered here during the period of global two children campaign.

In BDHS-2004, there are three types of questionnaires, namely the households, women and mens. The informations obtained from the field are recorded in their respective data files. In this study, the informations corresponding to the womens data file is used. The variable age of the respondent, fertility preference, place of residence, highest year of education, working status and expected number of children are considered in the analysis. The variable fertility preference involving responses corresponding to the question, would you like to have (a/another) child? The responses are coded 0 for no more and 1 for have another is considered as desire for children which is the binary response variable (Y) in the analysis. The age of the respondent (X_1), place of residence (X_2) is coded 0 for urban and 1 for rural, highest year of education (X_3), working status of respondent (X_4) is coded 0 for not working and 1 for working and expected number of children (X_5) is coded 0 for two or less and 1 for more than two are considered as covariates in the binary logistic regression model.

The goal of logistic regression modeling is to select those variables that result in a best model within the scientific context of the problem. In order to achieve this goal we must have a basic plan for selecting the variables for the model. At one time, stepwise logistic regression was an extremely popular method for model building. In recent years there has been a shift away from deterministic methods for model building to purposeful selection of variables. However, in the analysis the socially important variables associated with the response variable are selected under purposeful selection.

Consider a collection p explanatory variables be denoted by the vector X' = (X_1, X_2 ...Xp). Let the conditional probability that the outcome is present be denoted by $P(Y = 1|X) = \pi$. Then the logit or log-odds of having Y=1 is modeled as a linear function of the explanatory variables as:

$$\ln\left(\frac{\pi_i}{1-\pi_i}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p \; ; \; 0 \le \pi_i \le 1 \qquad (1)$$

where, the function:

$$\pi_i = \frac{\exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p)}{1 + \exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p)}$$

is known as logistic function. The most common method used to estimate the unknown parameters in linear regression is the Ordinary Least Squares (OLS). Under usual assumptions, least square estimations have some desirable properties. But when the OLS method is applied to estimate a model with dichotomous outcome the estimators no longer have these properties. In such a situation, the most commonly used method of estimating the parameters of a logistic regression model is the method of Maximum Likelihood (ML). In logistic regression, the likelihood equations are non-linear explicit function of unknown parameters. Therefore, we use a very effective and well known Newton-Raphson iterative method to solve the equations which is known as iteratively reweighted least square algorithm.

In general, the sample likelihood function is defined as the joint probability function of the random variables. Specifically, suppose (y_1, y_2...y_n) be the n independent random observations corresponding to the random variables (Y_1, Y_2... Y_n). Since the Y_i is a Bernoulli random variable, the probability function of Y_i is $f_i(Y_i) = \pi_i^{Y_i}$ $(1-\pi_i)$ $1-y_i$; $Y_i = 0$ or 1; i = 1, 2, ..., n, since Y's are assumed to be independent, the joint probability function or likelihood function is given by:

$$g(Y_1, Y_2, \cdots Y_n) = \prod_{i=1}^{n} \pi_i^{Y_i} (1-\pi_i)^{1-Y_i}$$

the log-likelihood function as:

$$L(\beta_0, \beta 1 \ldots \beta_p) = l_i \text{ (say)},$$

$$= \sum_{i=1}^{n} Y_i(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p) - \sum_{i=1}^{n} \ln\{1 + \exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p)\} \qquad (2)$$

The most effective and well known Newton-Raphson iterative method can solve the equations. The solution of the likelihood equations requires special software that is available in most, but not all, statistical packages. In this study, SPSS 11.5 for Windows is used to obtain the solution of the unknown parameters.

The computer output (Table 1) shows the coefficients βs, their standard errors, the Wald chi-square statistic, associated p-values and odds ratio exp (β). In order to determine the worth of the individual regressor in logistic regression, the Wald statistic denoted as (Hosmer and Lemeshow, 2000; Hauck and Donner, 1977):

$$W = \frac{\hat{\beta}_i}{S.E(\hat{\beta}_i)}$$

However, several researchers have designed:

$$W = \frac{\hat{\beta}_i^2}{\left[S.E(\hat{\beta}_i)\right]^2}$$

which was different but equivalent form Rao (1973), Wald, (1943) and Jennings (1986). Under the null hypothesis H_0: $\beta_i = 0$ (i = 1, 2, ..., 5), the statistic W defined as later case is approximately distributed as chi-square with single degree of freedom. The Wald chi square statistics (Table 1) agree reasonably well with the assumption that all the individual predictors have significant contribution to predict the response variable.

Table 1: Analysis of maximum likelihood estimates

Variables	Coefficient β	Standard error	Wald chi-square statistics	df	p-value	Odds ratio exp (β)
X_1	-0.052	0.009	36.386	1	0.000	0.949
X_2	0.454	0.111	16.724	1	0.000	1.575
X_3	-0.083	0.014	35.659	1	0.000	0.920
X_4	-0.307	0.124	6.127	1	0.000	0.736
X_5	2.143	0.114	352.391	1	0.013	8.527
Intercept	0.381	0.261	2.129	1	0.000	1.464

The binary logistic regression model has been fitted under the assumption that we are at least preliminarily satisfied with our efforts at the model building stage. By this we mean that, to the best of our knowledge, the model contains those variables that should be in the model and the variables have been entered in the correct functional form. Hence, we may like to know how effectively the model has described the outcome variable. Once, the particular multiple logistic regression model has been fitted, we begin the process of model assessment. The likelihood ratio test is performed to test the overall significance of all coefficients in the model on the basis of test statistic:

$$G = \left[(-2\ln L_0) - (-2\ln L_1)\right] \quad (3)$$

where, L_0 is the likelihood of the null model and $L_1$is the likelihood of the saturated model. The statistic G plays the same role in logistic regression as the numerator of the partial F-test does in linear regression.

Under the global null hypothesis, H_0: $\beta_1 = \beta_2 = ... = \beta_5 = 0$ the statistic G follows a chi-square distribution with 5 degrees of freedom and measure how well the independent variables affect the response variable. In the study, summary measure provides G = 589.436 with p<0.001 (Table 2) indicate that as a whole the independent variables have significant contribution to predict the response variable.

In order to find the overall goodness-of-fit, Hosmer and Lemeshow (1980) and Lemeshow and Hosmer (1982) proposed grouping based on the values of the estimated probabilities. Hosmer-Lemeshow goodness-of-fit test divides subjects into deciles based on predicted probabilities and computes a chi-square from observed and expected frequencies (Table 3). Using this grouping strategy, the Hosmer-Lemeshow goodness-of-fit statistic, $\hat{C}$ is obtained by calculating the Pearson chi-square statistic from the g×2 table of observed and estimated expected frequencies. A formula defining the calculation of $\hat{C}$ is as follows:

$$\hat{C} = \sum_{k=1}^{g} \frac{(o_k - n'_k\bar{\pi}_k)^2}{n'_k\bar{\pi}_k(1-\bar{\pi}_k)} \quad (4)$$

where, g denotes the number of groups, n'_k is the number of observations in the kth group, o_k is the sum of the Y values for the kth group and $\bar{\pi}_k$ is the average of the ordered $\bar{\pi}_k$ for the kth group. Hosmer and Lemeshow (1980) demonstrated that under the null hypothesis that the fitted logistic regression model is the correct model, the distribution of the statistic $\hat{C}$ is well approximated by the chi-square distribution with g-2 degrees of freedom. This test is more reliable and robust than the traditional chi-square test (Agresti, 2002). The value of the Hosmer-Lemeshow goodness-of-fit statistic computed from the frequencies (Table 3) is $\hat{C}$ = 5.28 and the corresponding p-value computed from the chi-square distribution with 8 degrees of freedom is 0.73. The large p-value signifies that there is no significant difference between the observed and the predicted values of the outcome. This indicates that the model seems to fit quite reasonable. A comparison of the observed and expected frequencies in each of the 20 cells (Table 3) also shows close agreement within each decile. Hosmer *et al*. (1997) examined the distributional properties of their test via simulations.

They recommended that overall assessment of fit be examined using a combination of Hosmer-Lemeshow goodness-of-fit test, Osius and Rojek normal approximation test and Stukel test as adjunct of their tests. The large sample normal approximation to the

Table 2: Analysis of likelihood ratio χ^2 test as an overall goodness of fit

Variables	-2 log-likelihood	Change in deviance/ Model chi-square	df	p-value
Intercept	2917.932	-	-	-
X_1	2816.726	101.206	1	0.000
X_2	2780.482	137.450	2	0.000
X_3	2740.323	177.609	3	0.000
X_4	2732.980	184.952	4	0.000
X_5	2328.496	589.436	5	0.000

Table 3: Contingency table for Hosmer and Lemeshow goodness-of-fit test

		Desire for no more children		Desire for more children					
Deciles	$\bar{\pi}_k$	Observed	Expected	Observed (o_k)	Expected	Total (n'_k)	$\hat{C}\sim\chi^2$	df	p-value
1	0.0833	206	202.60	15	18.40	221	5.28	8	0.73
2	0.1453	186	188.87	35	32.13	221			
3	0.1884	178	178.56	42	41.40	220			
4	0.2281	161	169.81	59	50.19	220			
5	0.2669	167	162.75	55	59.25	222			
6	0.3113	153	156.05	74	70.95	227			
7	0.3721	149	141.28	76	83.73	225			
8	0.5478	104	99.94	117	121.06	221			
9	0.7497	53	55.32	168	165.68	221			
10	0.8326	34	35.82	180	178.18	214			

distribution of the Pearson chi-square statistic derived by Osius and Rojek (1992) may be easily computed in any package that has the option to save the fitted values from the logistic regression model and do a weighted linear regression. Suppose J denote the number of distinct values of X values observed in the covariate space which is known as covariate pattern and J<n. The essential steps in the procedure when we have J covariate patterns are as follows:

Step 1: Save the fitted values from the model, denoted as:

$$\hat{\pi}_j,\ j = 1, 2, \cdots J$$

Step 2: Create the variable

$$v_j = m_j\hat{\pi}_j\left(1-\hat{\pi}_j\right),\ j = 1, 2, \cdots J$$

Step 3: Create the variable:

$$c_j = \frac{\left(1-2\hat{\pi}_j\right)}{v_j},\ j = 1, 2, \cdots J$$

Step 4: Compute the Pearson chi-square statistic defined as:

$$\chi^2 = \sum_{j=1}^{J}\frac{\left(Y_j - m_j\hat{\pi}_j\right)^2}{v_j}$$

Step 5: Perform a weighted linear regression of c, defined in step 3, on the model covariates X_1, X_2, X_3, X_4 and X_5, using weights v, defined in step 2. It is important to note that the sample size for this linear regression is J, the number of covariate patterns. Let RSS denote the residual sum-of-squares from this linear regression

Step 6: Compute the correction factor for the variance, denoted for convenience as:

$$F = 2\left[J - \sum_{j=1}^{J}\frac{1}{m_j}\right]$$

Step 7: Compute the standardized statistic:

$$z = \frac{\left[\chi^2 - (J-p-1)\right]}{\sqrt{F + RSS}}$$

Step 8: Compute a two-tailed p-value using the standard normal distribution

Application of the eight-step procedure given by Osius and Rojek using the model in Table 1 yields $\chi^2 = 1070.67$, RSS = 884.39, F = 584.60, J = 1052, p = 5 and:

$$z = \frac{1070.67 - (1052-5-1)}{\sqrt{584.60 + 884.39}} = 0.64$$

The two-tailed p-value is p = 0.52. Again on the basis of large p-value, we cannot reject the null hypothesis that the model fits quite well.

Stukel (1988) proposed a two degree-of-freedom test to ascertain whether a generalized logistic model is better than a standard model fit to the data. Her test determines whether two parameters in a generalized logistic model are equal to zero. Briefly, the two additional parameters allow the tails of the logistic regression model that is the small and large probabilities to be either heavier or lighter than the standard logistic regression model. This test is not actually a goodness-of-fit test since, it does not compare observed and fitted values. However, it does provide a test of the basic logistic regression model assumption and in that sense it may be considered as a useful adjunct to the Hosmer-Lemeshow and Osius-Rojek goodness of fit tests. The test has not been implemented in any package but it can be easily obtained from four steps procedure as:

Step 1: Save the fitted values from the model, denoted as:

$$\hat{\pi}_j,\ j = 1, 2 \ldots J$$

Step 2: Compute the estimated logit:

$$g_j = \ln\left(\frac{\hat{\pi}_j}{1-\hat{\pi}_j}\right) = X_j'\hat{\beta},\ j = 1, 2, \ldots J$$

Step 3: Compute two new covariates $z_{1j} = 0.5\times\hat{g}_j^2\times I\left(\hat{\pi}_j \geq 0.5\right)$ and $z_{2j} = -0.5\times\hat{g}_j^2\times I\left(\hat{\pi}_j < 0.5\right)$, j = 1, 2, … J, where I (arg) =1 if arg is true and zero if arg is false

Step 4: Perform the score test or partial likelihood ratio test for the addition z_1 and z_2 to the model

Application of the four step procedure given by Stukel (1988) to the fitted model in Table 1 yields a value for the partial likelihood ratio test of 0.061 which, with two degrees of freedom, yields p = 0.97. Again large p-value signifies we cannot reject the null hypothesis that the logistic regression model is the correct model.

An intuitively appealing way to summarize the results of a fitted logistic regression model is via a classification table. This table is the result of cross-classifying the outcome variable with a dichotomous variable whose

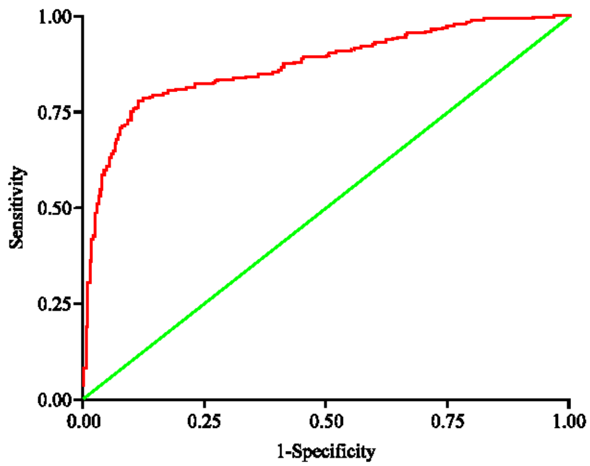

Fig. 1: Receiver operating characteristic curve for the fitted model

values are derived from the estimated logistic probabilities. To obtain the derived dichotomous variable we must define a cut point, c and compare each estimated probability to c. If the estimated probability exceeds c then we let the derived variable be equal to 1 otherwise it is equal to 0. The most commonly used value for c is 0.5. The terms sensitivity and specificity come from the classification table. Sensitivity is the ability of the model to predict an event correctly and specificity is the ability of the model to predict a nonevent correctly. Sensitivity and specificity rely on a single cut point to classify a test result as positive. A more complete description of classification accuracy is given by the area under the ROC (Receiver Operating Characteristic) curve. This curve, originating from signal detection theory, shows how the receiver operates the existence of signal in the presence of noise. But ROC curve analysis has more recently been used as a test of model adequacy in Medicine, Psychology, Demography and other areas like data mining. So, it has been considered as a statistical tool to evaluate the performance of a model adequacy. In the social sciences, ROC curve analysis is often called ROC accuracy ratio, a common technique for judging the accuracy of fitted binary logistic regression model. It plots the probability of detecting true signal (sensitivity) and false signal (1-specificity) for an entire range of possible cut points.

The area under the ROC curve gives a quantitative indication of how good the test is. The ideal curve has an area of 1, the worst case scenario is 0.5, with most practical test giving values of somewhere in between. This area provides a measure of the models ability to discriminate between those subjects who experience the outcome of interest versus those who do not. In this study, for the given model (Table 1), a plot of sensitivity versus 1-specificity over all possible cut points is shown (Fig. 1). The curve generated by these points is called the ROC curve and the area under this curve is determined by Mann-Whitney U statistic and is 0.87.

The accuracy of the test depends on how well the test separates the group being tested into those with or without the criteria in the model. Accuracy is measured by the area under the ROC curve. If the area lies $0.8 \leq ROC < 0.9$, it is considered the model performance is excellent. So, on the basis of the area under the curve indicates our model performance is excellent.

DISCUSSION

There has been a striking increase in the use of the logistic regression, but no increase in the frequency of assessment of the adequacy of the model even though the techniques are now readily available in many software packages. Because much of the lack of efforts in assessing the fit of any statistical model and logistic regression in particular, may be traced to a general misunderstanding and confusion by many users of regression methods as to what goodness-of-fit is and what its role is in the modeling process. Thus, inferences from analysis where there has been no assessment of goodness-of-fit should be viewed with some skepticism.

Despite the frequent use of logistic regression in the social sciences, considerable confusion exists about its use and interpretation in the modeling process. This conclusion is attributed to a lack of fit, inadequate teaching materials and to unfamiliarity with logistic regression by many instructors (Lottes *et al*., 1996). Some of the earlier study illustrated the basic concept of binary logistic regression, analogies between ordinary least square regression and logistic regression but the research did not quantifying the predictive ability of the fitted model. But the present study illustrated clearly the fitting process of binary logistic regression model under standard assumptions indicating evaluation process of optimum summary measures of goodness-of-fit to determine its predictive ability.

Tsiatis (1980) had been used maximum partial likelihood test or score test to assess the goodness-of-fit but the disadvantage of this test is that actual values of the observed and estimated expected frequencies need not be obtained. Thus, the lack of fit was remained hidden there. On the contrary, the advantages of different summary measures in the present study provide observed and expected frequencies that can be easily interpretable and adequate to assess the fit of the model.

The test proposed by Su and Wei (1991), based on cumulative sums of residuals whose p-value must be determined by complicated and time consuming simulations. But the proposed Stukel (1988) test in the study is similar but more easily computed than the test proposed by Su and Wei (1991).

CONCLUSIONS

A complete assessment of fit of binary logistic regression is a multi-faceted investigation involving summary measures of goodness-of-fit. This is especially important to keep in mind when using overall goodness of-fit tests. The desired outcome for most investigators is the decision not to reject the null hypothesis that the model fits. With this decision one is subject to the possibility of the type II error and hence, the power of the test becomes an issue. The simulation results reported in Hosmer *et al.* (1997) indicated that none of the overall goodness-of-fit tests is especially powerful for small to moderate sample sizes n<400. In general, for large sample (n>400), the goodness of fit tests are powerful. This study was demonstrated the preferred pattern for the application of logistic methods with an illustration of logistic regression applied to a large data set in testing a research hypothesis. Recommendations are also offered in the study for appropriate modeling process and check the adequacy of the fitted model.

In the present study, our sample size is large enough and the different summary measures of goodness-of-fit suggest the model fits adequately. Failure to address model adequacy may lead to misleading or incorrect inferences. However, it is strongly recommended that the methods for assessing model adequacy in different aspects should be used before proceed to presenting the results from a fitted logistic regression model.

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