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A Proposal for Spectral Line Profile of Hydrogen Atom Spectrum in the Sub-Nano-Meter Space Time

Abbas Farmany
Department of Physics, Azad University of Ilam, Iran

Abstract: Based on the minimal length uncertainty, both the spectral lines and natural broadening of spectral lines of hydrogen atom may be corrected in the sub-nano-meter space time. It is interesting to obtain the related spectral line profile of the corrected hydrogen atom spectrum, which is presented in a proposal for the spectral line profile of hydrogen atom spectrum based on the minimal length uncertainty. It is shown that, based on the minimal length uncertainty, the hydrogen atom spectrum is modified.

Key words: Quantum gravity, minimal length uncertainty, hydrogen atom spectrum, spectral lines, natural broadening, spectral profile

INTRODUCTION

Both the spectral lines (Brau, 1999); Akhoury and Yao, 2003; Kempf, 1997; Udem *et al.*, 1997; Hossenfelder *et al.*, 2003; Kempf and Managano, 1997) and the natural broadening of spectral lines (Farmany *et al.*, 2007, 2008) of hydrogen atom may be corrected based on the minimal length uncertainty relations. These corrections are based on the minimal length uncertainty analysis. In this study we obtain a proposal for the spectral line profile of hydrogen atom based on the minimal length uncertainty. Let us begin with the minimal length uncertainty. An exiting quantum mechanical implication of the microphysics is a modification of the uncertainty principle as (Das and Vagenas, 2008, 2009; Benczik *et al.*, 2005; Stetsko and Tkachuk, 2008):

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \frac{\Delta p}{\hbar} \qquad (1)$$

where, $\sqrt{\alpha'}$ is the Planck length. Based on the relation (1), we have obtained the corrected natural broadening of hydrogen atom spectrum as (Farmany *et al.*, 2007, 2008):

$$\Delta v_N = \frac{A_{ji}}{2\pi}(1 + t' A^2_{ji}) \qquad (2)$$

where, $\sqrt{t'}$ and A_{ji} are the Planck time and Einstein coefficient, respectively. When atomic emission, absorption or fluorescence spectra are recorded, narrow spectral lines are obtained. With ordinary spectrometer, the widths of the lines are determined not by atomic system but by properties of the spectrometer employed (slit function and spectral band pass). With very high resolution monochromators or with Fabry-Perot interferometer, the actual widths obtained are the result of a variety of line broadening phenomena. These processes give rise to a spectral distribution or spectral profile of photons which are called S_v or S_λ. The quantity $S_v dv$ and $S_\lambda d\lambda$ can be interpreted as the fraction of photons with frequencies in the interval v to v+dv or with wavelengths in the interval λ to λ+dλ. The spectral distribution function is normalized by:

$$\int_{line} S_v dv = 1 \quad or \quad \int_{line} S_\lambda d\lambda = 1 \qquad (3)$$

Because line broadening expression is simpler and easier to interpret than in wavelength units, we shall deal most often with the spectral distribution in the terms of frequency,. The distribution S_v has the units of time or Hz^{-1}. It can be converted to s_λ in length units (such as nm^{-1}) by the relation:

$$S_\lambda = S_v(\frac{c}{\lambda^2_m}) \qquad (4)$$

where, λ_m is the peak wavelength. In the thermal equilibrium, the forward rate of a microscopic process must be equal to the reverse rate of that same process which is known as the principle of detailed balancing. The detailed balancing principle allows to state that the emitted and observed photons from a continues radiation field in equilibrium have the same spectral distribution, namely S_v. Here, we consider the factors that contribute to the distribution and keep in mind that the results apply equally well to absorption and emission as long as equilibrium conditions prevail. Spontaneous emission of

photons leads to an exponential time decay of the excited state population. To determine the frequency distribution S_v of the emitted radiation, it is necessary to convert a time domain description to a frequency domain description through Fourier transformation. The Fourier transformation of an exponentially damped sine wave is a Lorentzian function. Since, the normalized spectral profile of the natural broadening is a Lorentzian dispersion function as:

$$S_{vN} = \frac{2 / (\pi \Delta v_N)}{1 + [2(v_m - v) / \Delta v_N]^2} \qquad (5)$$

where, v_m is the frequency at the line center. Note that the Lorentzian profile is symmetric with respect to the line center. Combing Eq. 5 with 2 we obtains:

$$S^{corrected}_{vN} = \frac{4A_{ji}(1 + t'A^2_{ji})}{[A_{ji}(1 + t'A^2_{ji})]^2 + [4\pi(v_m - v)]^2} \qquad (6)$$

Equation 6 is the corrected spectral line profile of natural broadening. Comparing Eq. 6 with the non-corrected spectral line profile of natural broadening we obtain:

$$\frac{S^{corrected}_{vN}}{S_{vN}} = \frac{(1 + t'A^2_{ji})(A^2_{ji} + [4\pi(v_m - v)]^2)}{[A_{ji}(1 + t'A^2_{ji})]^2 + [4\pi(v_m - v)]^2} \qquad (7)$$

In wavelength units the half intensity width is:

$$\Delta \lambda_N = \Delta v_N \lambda^2_m / c \qquad (8)$$

where, λ_m is the wavelength of maximum intensity. From Eq. 2 and 8 we obtain the corrected wavelength formula as:

$$\Delta \lambda^{corrected} = \frac{A_{ji}}{2\pi c}(1 + t'A_{ji}^2)\lambda^2_m \qquad (9)$$

And comparing Eq. 9 with 8 we can write:

$$\frac{\Delta \lambda^{corrected}}{\Delta \lambda} = (1 + t'A_{ji}^2) \qquad (10)$$

RESULTS

In this study we have obtained a proposal for spectral line profile of hydrogen atom spectrum based on the minimal length uncertainty. As the main result of the present study, it is interesting that the modified spectral line profile of the hydrogen atom spectrum is presented in a proposal for the spectral line profile of hydrogen atom spectrum based on the minimal length uncertainty. It is shown that, based on the minimal length uncertainty, the hydrogen atom spectrum is modified.

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