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Identification of Modal Series Model of Nonlinear Systems Based on Subspace Algorithms

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Abstract: In this study, we expressed an algorithm for nonlinear system identification based on modal series. Modal series is basically a method for solving nonlinear differential equations. Modal Series analysis of nonlinear systems could have found several new applications. The ability of this approach in using of linear system analysis rules for nonlinear systems attracts researchers to this method. In this study, a new form of modal series which is suitable for identification purposes has been presented and then an algorithm which uses this representation of modal series and a subspace identification method has been illustrated for identification of a modal series model of nonlinear systems. Simulation results expressed the efficient ability of this approach in identification of nonlinear systems.

Key words: Nonlinear systems, modal series, subspace identification

INTRODUCTION

Subspace identification is by now a well-accepted method for the identification of multivariable Linear Time-Invariant (LTI) systems. In many cases these methods provide a good alternative to the classical nonlinear optimization-based prediction-error methods (Ljung, 1999). Subspace methods do not require a particular parameterization of the system; this makes them numerically attractive and especially suitable for multivariable systems. Subspace methods can also be used to generate an initial starting point for the iterative prediction-error methods. This combination of subspace and prediction-error methods is a powerful tool for determining an LTI system from input and output measurements. Unfortunately, in many applications LTI systems do not provide an accurate description of the underlying real system. Therefore, identification methods for other descriptions, like time-varying and nonlinear systems, are needed. In recent years, subspace identification methods have been developed for certain nonlinear systems: Wiener systems (David and Verhaegen, 1996; Chou and Verhaegen, 1999), Hammerstein systems (Michel and Westwick, 1996), bilinear systems (Huixin and Maciejowski, 2000) and LPV systems (Vincent and Verhaegen, 2001).

Subspace identification methods for LTI systems can basically be classified into two different groups. The first group consists of methods that aim at recovering the column space of the extended observability matrix and use the shift-invariant structure of this matrix to estimate the matrices A and C; this group consists of the so-called MOESP methods (Verhaegen and Dewilde, 1992; Michel, 1994).

The methods in the second group aim at approximating the state sequence of the system and use this approximate state in a second step to estimate the system matrices; the methods that constitute this group are the N4SID methods (Moonen *et al*., 1989; Van Overschee and de Moor, 1994, 1996), the CVA methods (Larimore, 1983; Peternell *et al*., 1996) and the orthogonal decomposition methods (Katayama and Picci, 1999).

Pariz and Vaahedi (2003) has presented a new approach for analysis and modeling of nonlinear systems. Abdollahi (2002) expanded this approach for analyzing and modeling of continuous and discrete nonlinear systems. Modal series can expressed a nonlinear system in a new form which is more accurate than the linearized model of system and expresses many nonlinear effects of main system (Chen *et al*., 2010). This new modeling structure of nonlinear systems can be used to identify a nonlinear system effectively.

In this study, we present a new method to determine a modal series state space model from a finite number of measurements of the inputs and outputs. The method was inspired by the subspace identification method for linear systems (Katayama and Picci, 1999; Moonen *et al*., 1989) and is based on modal series (Abdollahi, 2002).

In this study, a short and simplified summary of subspace identification method for linear systems is first illustrated. Then, the modal series presentation of nonlinear systems is summarized. We then introduce a

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new representation of modal series which is more suitable for identification purposes. Based on the results presented, an algorithm for identification of nonlinear state space systems is then expressed. The presented technique is illustrated for a simple example of a nonlinear system using computer simulations.

REVIEW OF LINEAR SUBSPACE IDENTIFICATION

Consider an observable linear state space system:

$$x_{k+1} = Ax_k + Bu_k \tag{1}$$

$$y = Cx_k \tag{2}$$

Subspace identification (Michel and Westwick, 1996; Moonen *et al*., 1989) is a computationally efficient method to determine from input and output measurements a linear state space system up to a similarity transformation; it provides estimates of the matrices $A_T = TAT^{-1}$, $B_T = TB$ and $C_T = CT^{-1}$ where, T is a square nonsingular matrix. In a nutshell, subspace identification consists of three steps:

Step 1: Remove the influence of future inputs

We want to reconstruct the state sequence x_k. It is easy to see that the following equation holds.

$$z_k = \underbrace{\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{d-1} \end{bmatrix}}_{G_d} x_k + \underbrace{\begin{bmatrix} 0 & 0 & \cdots & 0 \\ CB & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{d-2}B & CA^{d-3}B & \cdots & CB \end{bmatrix}}_{H_d} \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+d-1} \end{bmatrix} \tag{3}$$

where, d≥n+1. The first part, $\Gamma_d x_k$, is the response of the system from time k to time k+d-1 due to the initial state x_k. The second part is the response due to the future inputs u_k, u_{k+1}, ..., u_{k+d-1}. To reconstruct the state x_k we have to remove the influence of the future inputs. If the Markov parameters of the system are known and hence the matrix H_d is known, we can simply do this by subtraction:

$$\tilde{z}_k \triangleq z_k - H_d \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+d-1} \end{bmatrix}^T = G_d x_k \tag{4}$$

The vector $\tilde{z}_k$ can be viewed as the response of the system due to the initial state x_k with the input switched off. Note that there exists a clever way to remove the influence of the future inputs without the need to know the matrix H_d. This is done by using a linear projection as described by Katayama (2005).

Step 2: Reconstruct the state sequence

Let us store the vectors $\tilde{z}_k$ constructed in the first step into following matrix.

$$\tilde{Z}_k = \begin{bmatrix} \tilde{z}_k & \tilde{z}_{k+1} & \cdots & \tilde{z}_{k+N} \end{bmatrix}, \quad N \gg d \tag{5}$$

By computing Singular Value Decomposition (SVD) of this matrix, we can reconstruct the state sequence:

$$X_k = \begin{bmatrix} x_k & x_{k+1} & \cdots & x_{k+N} \end{bmatrix}, \quad N \gg d \tag{6}$$

up to a linear state transformation T. Let the SVD of $\tilde{z}_k$ be given by:

$$\tilde{Z}_k = USV^T \tag{7}$$

Then the reconstructed state is given by:

$$\hat{X}_k = S^{1/2}V^T = TX_k \tag{8}$$

Note that the number of singular values in S determines the dimension of the state vector. In general the dimension of the state vector x_k will be less than the dimension of the delay vector $\tilde{z}_k$.

Step 3: Estimate the model

We use the time sequences y_k, u_k and $\hat{x}_k$ to determine the matrices A_T, B_T and C_T. It is easy to see from the Eq. 1 and 2 that this boils down to solving a linear least squares problem.

MODAL SERIES

As expressed by Pariz and Vaahedi (2003) Abdollahi (2002) and Shanechi and Vaahedi (2003) any nonlinear system which is in the form of:

$$\dot{x} = g(x,u) \tag{9}$$

where, $x = [x_1, x_2, \ldots, x_n]^T$ is the state vector, $u = [u_1, u_2, \ldots, u_m]^T$ is the input vector and $g: R^n \times R^m \rightarrow R^n$ is a smooth vector function which g(0,0) = 0, can be modelled by Eq. 10 and 11 called modal series.

$$x(t)=\sum_{i=1}^{\infty}v_i(t)+\sum_{j=1}^{\infty}w_j(t)+\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}z_{ij}(t) \quad (10)$$

$$\begin{cases} v_1(t+1)=B_{10}v_1(t) \\ v_2(t+1)=B_{10}v_2(t)+\frac{1}{2}\begin{bmatrix} v_1(t)^T B_{20}^1 v_1(t) \\ \vdots \\ v_1(t)^T B_{20}^n v_1(t) \end{bmatrix} \\ \vdots \end{cases} \quad (10a)$$

$$\begin{cases} w_1(t+1)=B_1 w_1(t)+B_1 u(t) \\ w_2(t+1)=B_1 w_2(t)+\frac{1}{2}\begin{bmatrix} w_1(t)^T B_{20}^1 w_1(t) \\ \vdots \\ w_1(t)^T B_{20}^n w_1(t) \end{bmatrix}+\begin{bmatrix} w_1(t)^T B_{11}^1 u(t) \\ \vdots \\ w_1(t)^T B_{11}^n u(t) \end{bmatrix}+\frac{1}{2}\begin{bmatrix} u(t)^T B_{02}^1 u(t) \\ \vdots \\ u(t)^T B_{02}^n u(t) \end{bmatrix} \\ \vdots \end{cases} \quad (10b)$$

$$\begin{cases} z_{11}(t+1)=B_{10}z_{11}(t)+\frac{1}{2}\begin{bmatrix} v_1(t)^T B_{20}^1 w_1(t)+w_1(t)^T B_{20}^1 v_1(t) \\ \vdots \\ v_1(t)^T B_{20}^n w_1(t)+w_1(t)^T B_{20}^n v_1(t) \end{bmatrix}+\begin{bmatrix} v_1(t)^T B_{11}^1 u(t) \\ \vdots \\ v_1(t)^T B_{11}^n u(t) \end{bmatrix} \\ \vdots \end{cases} \quad (10c)$$

$$\begin{cases} v_1(t=0)=x(0) \\ v_i(t=0)=0 \quad i=2,3,... \\ w_j(t=0)=0 \quad j=1,2,3,... \\ z_{ij}(t=0)=0 \quad i=1,2,3,...,\ j=1,2,3,... \end{cases} \quad (11)$$

where,

$$B_{10}=\frac{\partial g}{\partial x}\Big|_{\substack{x=0\\u=0}},\ B_{01}=\frac{\partial g}{\partial u}\Big|_{\substack{x=0\\u=0}},\ B_{20}^j=\frac{\partial^2 g_j}{\partial x\partial x}\Big|_{\substack{x=0\\u=0}},$$
$$B_{11}^j=\frac{\partial^2 g_j}{\partial x\partial u}\Big|_{\substack{x=0\\u=0}},\ B_{02}^j=\frac{\partial^2 g_j}{\partial u\partial u}\Big|_{\substack{x=0\\u=0}}$$

and so on.

Remarks:

- Equations 10 are categorized in three classes in Eq. 10a-c
- Class in Eq. 10a is affected by the initial condition and is the zero input response of the system
- Class in Eq. 10b is affected by the input and is the zero state response of the system
- Class in Eq. 10c is affected by both initial condition and input. It is the interaction between initial condition and input and differs from zero when both of them do exist
- In linear systems the complete response of a system is equal to sum of its zero input and zero state responses, but this is not the case for nonlinear systems, because of the existence of equations class in Eq. 10c
- Modal series method provides a solution for the system in terms of the modes of the system and the input. This can be better seen if we apply the transformation x = Ty, where T is the matrix of the right eigenvectors of B_{10}, use modal series approach to yield the solution and use back transformation $y = T^{-1}x$ to obtain the solution of Eq. 10

Extension to discrete modal series is straight and it will bring us to similar equations (Abdollahi, 2002).

A MODIFIED REPRESENTATION OF MODAL SERIES MODEL OF NONLINEAR SYSTEMS

Definition: For matrices P and Q with dimensions $n_p\times m_p$ and $n_q\times m_q$, respectively, the Kronecker product is defined as a $(n_pn_q)\times(m_pm_q)$ matrix:

$$P\otimes Q=\begin{bmatrix} P_{11}Q & \cdots & P_{1m_p}Q \\ \vdots & \vdots & \vdots \\ P_{n_p1}Q & \cdots & P_{n_pm_p}Q \end{bmatrix} \quad (12)$$

We define the superscript notation ⊗ and (p) for referring to Kronecker product and the repetitive application of the Kronecker product, respectively.

Now we can express that the class a deals with transient states of nonlinear system which depends on initial conditions. We can neglect transient effects and assume zero initial condition for class v when we want to identify nonlinear system. Since, class in Eq. 10c depends on class in Eq. 10a and b and class in Eq. 10a states are assumed zero, so class in Eq. 10c are zero, too. Then we can rewrite the discrete modal series in the form of Eq. 13. Where A, B_1, B_2 ... and u_1, u_2 ... are defined by Eq. 14 and 15.

$$x(t)=\sum_{j=1}^{\infty}w_j(t) \quad (13)$$

$$\begin{cases} w_1(t+1)=Aw_1(t)+B_1u_1(t) & (13a) \\ w_2(t+1)=Aw_2(t)+B_2u_2(t) & (13b) \\ \vdots \end{cases}$$

$$u_1(t)=u(t) \quad (14a)$$

$$\zeta_2(t) = \begin{bmatrix} u(t) \\ w_1(t) \end{bmatrix}$$
$$u_2(t) = \zeta_2(t) \otimes \zeta_2(t) = \zeta_2^{(2)}(t) \tag{14b}$$
$$\vdots$$

$$A = B_{10} \tag{15a}$$

$$B_1 = B_{01} \tag{15b}$$

$$B_2 = \text{Coefficients Matrix of } \zeta_2^{(2)} \tag{15c}$$
$$\vdots$$

It is supposed that the nonlinear system is linear in output equations. It means:

$$\begin{aligned} y(t) &= Cx(t) + Du(t) \\ &= C\sum_{k=1}^{K_m} w_k(t) + Du(t) \\ &= \underbrace{Cw_1(t) + Du(t)}_{y_1(t)} + \sum_{k=2}^{K_m} \underbrace{Cw_k(t)}_{y_k(t)} \end{aligned} \tag{16}$$

where, K_m is the maximum number of modal series terms. So, output of each w_k equation is as follows:

$$y_1(t) = Cw_1(t) + Du(t) \tag{17a}$$

$$y_k(t) = Cw_k(t) \qquad k = 2,3,... \tag{17b}$$

Looking at Eq. 13a, b, … implies that inputs ($u_k(t)$, k=2, 3,…) for $w_k(t)$ (k=2, 3,…) state equations, rely on $w_j(t-1)$(j=k-1, k-2,…1) and $u_1(t-1)$.

Since, it is possible to construct $\zeta_k(t)$ and then the input vector u_k of every w_k equation for every sample time, it is clear that Eq. 13a, b … are linear. w_k is in fact the linear model of nonlinear system and all other terms try to model nonlinear dynamics of the main nonlinear system.

Now we can express that there are a set of equations in the form of Eq. 18 which can approximate the main nonlinear system.

$$w_k(t+1) = Aw_k(t) + B_k u_k(t) \tag{18a}$$

$$y_k(t) = Cw_k(t) + D_k u(t) \tag{18b}$$

where, D_1 = D and D_k = 0 for k = 2,3,… . And the main state vector can be computed by Eq. 13a.

In an identification problem for nonlinear systems, we can identify Eq. 18a,b as an approximation of nonlinear systems. This would be very flexible method, since we can choose arbitrary number of modal series terms (w_k) to be identified.

Because of the state space form of this model of nonlinear systems, we proposed to use a modified version of a subspace algorithm.

MODAL SERIES IDENTIFICATION BASED ON SUBSPACE ALGORITHMS

The objective is to estimate, from measured input/output data sequences ({u(t)} and {y(t)}, respectively), a series of systems described by:

$$w_k(t+1) = Aw_k(t) + B_k u_k(t) + \eta(t) \tag{19a}$$

$$y_k(t) = Cw_k(t) + D_k u_k(t) + \varepsilon(t) \tag{19b}$$

Which u_k is defiened by Eq. 14. And:

$$E\left[\begin{pmatrix} \eta \\ \varepsilon \end{pmatrix}\begin{pmatrix} \eta^T & \varepsilon^T \end{pmatrix}\right] = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \geq 0 \tag{20}$$

Using one of subspace identification Algorithms (N4SID, MOESP, CVA), we can suggest the following algorithm to identify a modal series model of a nonlinear system.

Algorithm:

Step 1: Set $y_1(t)$ = y(t), $u_1(t)$ = u(t), k = 1 and choose an arbitrary number of modal series terms (K_m)

Step 2: Apply one of the subspace identification algorithms to the following linear system and identify system matrices (A, B_1, C and D_1)

$$w_1(t+1) = Aw_1(t) + B_1 u_1(t) + \eta(t) \tag{21a}$$

$$y_1(t) = Cw_1(t) + D_1 u_1(t) + \varepsilon(t) \tag{21b}$$

Step 3: Produce w_k a nd y_k using identified linear model

Step 4: Set k = k+1 and produce $y_k(t)$ and $u_k(t)$ using Eq. 22 and 14, respectively

$$y_k(t) = y(t) - \sum_{j=1}^{k-1} y_j(t) \tag{22}$$

Step 5: Using A and C estimated in step 2, estimate B_k and D_k of the following linear system

$$w_k(t+1) = Aw_k(t) + B_k u_k(t) + \eta(t) \tag{23a}$$

$$y_k(t) = Cw_k(t) + D_k u_k(t) + \varepsilon(t) \quad (23b)$$

Step 6: If $k<K_m$ (number of modal series terms) go to step 3 else go to step 7

Step 7: End of algorithm

SIMULATIONS

The example system was described by Narendra and Parthasarathy (1990) as an example for the use of neural networks to model dynamical systems. Junhong (1994) used this example to demonstrate the dynamic modeling capabilities of neuro-fuzzy networks and Anass *et al.* (1999) used it with local linear fuzzy models. The input-output description of the system is described by Eq. 24.

For identification, a multistep input signal shown in Fig. 1 was used; the steps in this signal had a fixed length of 10 samples and a random magnitude between -1 and +1, determined by a uniform distribution.

To assess the quality of the model, a validation data set was generated using the input signal described by Eq. 25.

In Fig. 2, simulation results of a linear model of example system has been illustrated. This linear model has been estimated by an implementation of MOESP subspace algorithm in MATLAB7.

In Fig. 3, there are simulations for identified two term modal series model using the proposed algorithm. The suggested algorithm has also been implemented in MATLAB7 and makes use of MOESP identification algorithm.

The mean square error for linear model is 0.0077 and for modal series model is 0.0012. Therefore, Simulation results express that the proposed algorithm can be used for identification of every smooth nonlinear system. We can improve identification of nonlinear system by choosing sufficient number of modal series terms and more effective subspace identification algorithms.

$$x_1(t+1) = x_2(t) \quad (24a)$$

$$x_2(t+1) = x_3(t) \quad (24b)$$

$$x_3(t+1) = \frac{x_3(t)x_2(t)x_1(t)(1-x_1(t))u_1(t) + u_2(t)}{1 + x_1^2(t) + x_2^2(t)} \quad (24c)$$

$$y(t) = x_3(t) \quad (24d)$$

$$u_1(t) = u_2(t-1) \quad (25a)$$

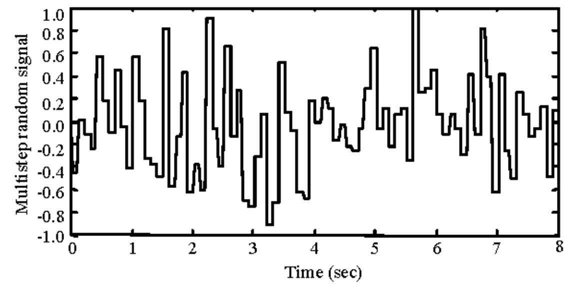

Fig. 1: Multistep random signal used for identification

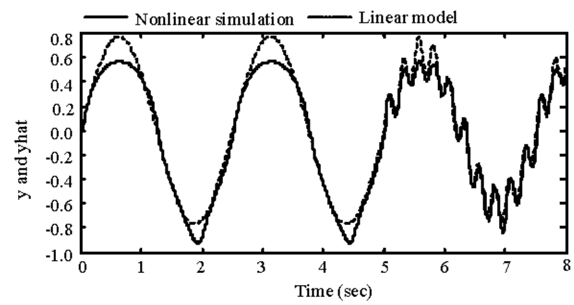

Fig. 2: Validation of identified linear state space model using subspace algorithm

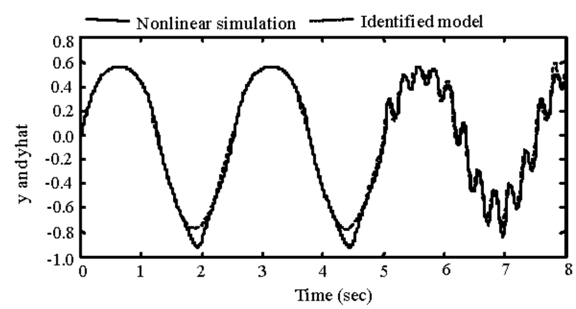

Fig. 3: Validation of identified two term modal series model using proposed algorithm

$$u_2(t) = \begin{cases} \sin(\frac{2\pi t}{250}) & 1 \le t \le 500 \\ 0.8\sin(\frac{2\pi t}{250}) + 0.2\sin(\frac{2\pi t}{25}) & 501 \le t \le 800 \end{cases} \quad (25b)$$

CONCLUSION

A new algorithm based on modal series and subspace identification algorithms has been illustrated in this paper. Actually, a novel approach for nonlinear system modeling and identification has been investigated. In attention to simulation results and theoretical analysis, we can introduce this approach as an effective and strong method for identification of nonlinear systems. Future works may consider using of this method for control purposes.

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