Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 11 (6): 1058-1061, 2011
ISSN 1812-5654 / DOI: 10.3923/jas.2011.1058.1061

Gup and Spectrum of Quantum Black Holes

[1]A. Farmany, [1]H. Noorizadeh and [2]S.S. Mortazavi
[1]Islamic Azad University, Ilam Branch, Ilam, Iran
[2]Islamic Azad University, Hamedan Branch, Young Researchers Club, Hamedan, Iran

Abstract: The evaporation of quantum black holes would leave very distinctive imprints on the detectors and spectrum of such black holes could be obtained. To study the quantum gravity effects on the black hole spectrum, one can take into account the generalized uncertainty principle. In this paper, employing the Bekenstein-Mukhanov approach, the spectrum of a quantum black hole is obtained. It is shown that the energy spacing between consecutive levels for MJ>>h is corresponding to a fundamental frequency.

Key words: Gravitational spectroscopy, Bekenstein-mukhanov approach, generalized uncertainty principle, quantum black hole, hawking radiation

INTRODUCTION

In canonical quantum gravity, the character of Hawking radiation is modified when quantum gravity effects are properly taking into account, even for non-rotating, neutral, and very massive black hole with respect to the Planck scale. To study the quantum gravity effects on a quantum black hole, Adler *et al.* (2001) used the generalized uncertainty principle. They showed that in the presence of generalized uncertainty principle, the radiation temperature of black hole is modified. Other approaches of Adler-Chen-Santiago proposal are obtained in Nouicer (2007), Myung *et al.* (2007), Farmany *et al.* (2008), Dehghani and Farmany (2009) and Farmany and Dehghani (2010). In this letter, we concentrate on the quantum gravity effects of a quantum black hole. First, we begin with a fundamental frequency from energy spacing between consecutive levels. Then we consider the relation between generalized uncertainty principle and energy-time uncertainty. Then, we calculate spectral lines and line-width of quantum black hole.

A fundamental frequency from energy spacing between consecutive levels: In canonical quantum gravity the area of a non-rotating neutral black hole is quantized as (with G = c = 1):

$$A = \alpha n h \tag{1}$$

where, n is the energy level. Thermal character of black hole radiation is entirely due to degeneracy of levels and same degeneracy becomes manifest as black hole entropy (Bekenstein, 2002; Jiang *et al.*, 2010; Majhi, 2010; Banerjee *et al.*, 2010; Jadhav and Burko, 2009; Drasco, 2009; Van Den Broeck and Sengupta, 2007; Dappiaggi and Raschi, 2006; Dreyer *et al.*, 2004; Setare, 2004a, b; Bekenstein and Mukhanov, 1995) Setting (n) as the multiplicity of degeneracy, Bekenstein and Mukhanov (1995) found that in the level n = 1, g (1) = 1, in this level (n = 1) the black hole entropy is zero. Here a general form of multiplicity degenerate (energy level) is g (n) = $e^{\alpha(n-1)/4}$ where, αInk and k = 2, 3, 4,... the energy spacing between consecutive levels for M>>h corresponds to a fundamental frequency (Bekenstein and Mukhanov, 1995):

$$\bar{w} = \frac{\ln 2}{8\pi M} \tag{2}$$

A quantum black hole can decays during interval of observer time Δl by a sequence of integers $\{n_1, n_2, ..., n_j\}$ of length j. During Δl, the black hole first jumped down to n_1 elementary levels in one ago, then n_2 level, etc. In this process, black hole emits a quantum of some species of energy $n_1\hbar\bar{w}$, then a quantum of energy , etc. Each one of j quanta carries the energy $n_2\hbar\bar{w}$. In average, during Δl, the mass of black hole decreases (Bekenstein and Mukhanov, 1995) by:

$$d\langle M\rangle/dt = -2\hbar\bar{w}\Delta t/\tau \tag{3}$$

Since the main value of j is Δt/lτ where, τ is a survival timescale, both could be determined. This decreasing of black hole mass is radically different from one obtained in the standard discussion of hawking radiance. Bekenstein and Mukhanov (1995) obtained the mean time γ between quantum leaps as:

Corresponding Author: S.S. Mortazavi, Islamic Azad University, Hamedan Branch, Young Researchers Club, Hamedan, Iran

$$\tau = 3840\gamma^{-1} <M> \ln 2 \qquad (4)$$

Bekenstein and Mukhanov argues that "this (spectroscopy of quantum black hole) to be possible to test quantum gravity with black hole well above Planck scale".

Relationship between generalized uncertainty principle and energy-time uncertainty: Generalized uncertainty principle has been the subject of interesting works over the years. In these works, modification of usual uncertainty relation at microphysics is obtained (Amelino-Camelia *et al*., 2006; Adler and Santiago, 1999; Adler *et al*., 2001; Farmany and Dehghani, 2010; Farmany, 2010; Farmany *et al*., 2008; Farmany *et al*., 2007; Hossenfelder *et al*., 2003):

$$\Delta x \geq \frac{\hbar}{\Delta p} + l'\frac{\Delta p}{\hbar} \qquad (5)$$

where, $\sqrt{l'}$ is the Planck length. Using relation (5) it is easy to obtain a similar relation between time-energy. Dividing both side of relation (5) by c (speed of light) reads (Farmany *et al*., 2007):

$$\frac{\Delta x}{c} \geq \frac{\hbar}{\Delta pc} + l'\frac{\Delta p}{\hbar c} \qquad (6)$$

Relation (6) reads:

$$\tau \geq \frac{\hbar}{\Delta E} + t'\frac{\Delta E}{\hbar} \qquad (7)$$

where, $\sqrt{t'}$ is the Planck time. We use the natural units $l_p, c, \hbar = 1$, however, we restore occasionally t', in important formulae (7) for the sake of clarity. Using this approximation, the uncertainty in time-energy reads (Farmany *et al*., 2007):

$$\tau \geq \frac{1}{\Delta E} + t'\Delta E \qquad (8)$$

Frequency of spectra: Observation of spectrum of any quantum black hole would immediately make quantum gravity effects well above the Planck scale. Relation (8) is quadratic in ΔE:

$$t'\,\Delta E^2 - \tau\Delta E + 1 = 0 \qquad (9)$$

This leads to an uncertainty in the energy as follow:

$$\Delta E_{min} = \frac{\tau}{2t'}\left(1 - \sqrt{1 - \frac{4t'}{\tau^2}}\right) \qquad (10)$$

The eigen-functions and eigen-values of energy operator play an important role in our calculations. The physical measurements often involve determination of energy (or radiation frequency) emitted or absorbed by system that makes a transition from one energy eigen-state to another. Here we calculate the modified energy levels by solution of the time-dependent Schrödinger equation. The time-dependent Schrödinger equation is:

$$H\psi = i\hbar\frac{\partial}{\partial t}\psi \qquad (11)$$

In a special case when H doesn't depend explicitly on time, general solution of Eq. 11 is:

$$\psi(r,t) = \sum_n a_n u_n(r) e^{-iE_n t/\hbar} \qquad (12)$$

where, u_n is the eigen-function of H with energy E_n,

$$Hu_n(r) = E_n u_n(r) \qquad (13)$$

Note that a_n doesn't depend on time. The equation of motion for the Schrödinger-wave function reads:

$$i\partial_t |\psi> = \hat{W}(\hat{E})|\psi> \qquad (14)$$

or,

$$\tilde{E}_n|\psi> = \left(\frac{P^2}{2M} + V(\hat{x})\right)|\psi> \qquad (15)$$

In harmonic oscillator, the potential energy is $M\Omega^2\hat{x}^2/2$, we can write:

$$\tilde{E}_n|\psi> = \left(\frac{\hat{P}^2}{2M} + \frac{M\Omega^2\hat{x}^2}{2}\right|\psi> \qquad (16)$$

From Eq. 5 we can obtain a relation between momentum and coordinate as (Hossenfelder *et al*., 2003; Kempf *et al*., 1995; Dadic *et al*., 2003):

$$\hat{x} = i\hbar\left(1 + \frac{P^2}{m_p^{\,2}}\right)\partial_p \qquad (17)$$

Combining Eq. 16 and 17 we obtain the modified energy levels $\tilde{E}_n$ of system based on the generalized uncertainty principle. To obtain the eigen-value and eigen-functions of the energy we solve the modified Schrödinger equation:

$$\tilde{E}_n|\psi>=(\frac{p^2}{2M}-\frac{M\hbar^2\Omega^2}{2}((1+p^2/m^2_p)\partial p)^2)|\psi> \qquad (18)$$

The solution of Eq. 18 is obtained in (Hossenfelder *et al*., 2003; Kempf *et al*., 1995; Dadic *et al*., 2003) in:

$$\tilde{E}_n|\psi>=\frac{\hbar^2}{2M}[(1-\frac{l^2_p}{3}\partial^2x\partial x]^2+\frac{Mw_n^2}{2}x^2|\psi> \qquad (19)$$

Equation 19 shows the energy levels of quantum black hole. Using $w_n = w(E_n)$ and comparing Eq. 2 with 19 we can write:

$$\bar{w}_n=\sqrt{\frac{nhc}{\pi}}(\frac{1}{Mx})[(\frac{\ln 2}{4}+\frac{\hbar}{2C})(1-\frac{l^2_p}{3}\partial^2x)\partial x] \qquad (20)$$

Equation 20 is the frequency of the spectral lines for the n-th level energy of the quantum black hole. Let we calculate the line-with of the quantum black hole. The total uncertainty in the frequency (half-intensity line width or just half-width) is due to lifetime effects. $\Delta\bar{w}_1$ is the sum of upper and lower state of energy:

$$\Delta\bar{w}_1=\frac{\Delta E_2+\Delta E_1}{2\pi} \qquad (21)$$

Note that we used the natural unite $\hbar=1$, so $h = 2\pi$. To calculate the line-width of the quantum black hole spectral line, one can take into account $\bar{w}=1/\lambda$, so, $d\bar{w}=-d\lambda/\lambda^2$.

CONCLUSION

The character of Hawking radiation is modified when quantum gravity effects are properly taking into account, even for very massive black hole. In this viewpoint, decreasing of black hole mass is radically different from one entertained in the standard discussion. Thermal character of radiation is entirely due to degeneracy of levels. Same degeneracy becomes manifest as black hole entropy. Bekenstein and Mukhanov calculated that, in the level n = 1, the black hole entropy is zero. The energy spacing between consecutive levels fo MJ>>h is corresponding to a fundamental frequency.

REFERENCES

Adler, R.J. and D.I. Santiago, 1999. On gravity and the uncertainty principle. Mod. Phys. Lett. A, 14: 1371-1381.

Adler, R.J., P. Chen and D.I. Santiago, 2001. The generalized uncertainty principle and black hole remnants. Gen. Rel. Grav., 33: 2101-2108.

Amelino-Camelia, G., M. Arzano, Y. Ling and G. Mandanici, 2006. Black-hole thermodynamics with modified dispersion relations and generalized uncertainty principles. Class. Quan. Grav., 23: 2585-2606.

Banerjee, R., B.R. Majhi and E.C. Vagenas, 2010. Quantum tunneling and black hole spectroscopy. Phys. Lett. B, 686: 279-282.

Bekenstein, J.D. and V.F. Mukhanov, 1995. Spectroscopy of the quantum black hole. Phys. Lett. B, 360: 7-12.

Bekenstein, J.D., 2002. The case for discrete energy levels of a black hole. Int. J. Mod. Phys. A, 17S1: 21-31.

Dadic, I., L. Jonke and S. Melyana, 2003. Harmonic oscillator with minimal length uncertainty relations and ladder operators. Phys. Rev. D, 67: 087701-087701.

Dappiaggi, C. and S. Raschi, 2006. Spectroscopy of an AdS Reissner-Nordstrom black hole. Int. J. Mod. Phys. D, 15: 439-457.

Dehghani, M. and A. Farmany, 2009. Higher dimensional black hole radiation and a generalized uncertainty principle. Phys. Lett., 675: 460-462.

Drasco, S., 2009. Verifying black hole orbits with gravitational spectroscopy. Phys. Rev. D, 79: 104016-104016.

Dreyer, O., B. Kelly, B. Krishnan, L.S. Finn, D. Garrison and R. Lopez-Aleman, 2004. Black hole spectroscopy: Testing general relativity through gravitational wave observations. Classical Quantum Gravity, 21: 787-804.

Farmany, A., S. Abbasi and A. Naghipour, 2007. Probing the natural broadening of hydrogen atom spectrum based on the minimal length uncertainty. Phys. Lett. B, 650: 33-35.

Farmany, A., A. Abbasi and A. Naghipour, 2008. Correction to the higher dimensional black hole entropy. Acta Phys. Polon., 114: 651-655.

Farmany, A. and M. Dehghani, 2010. Thermodynamics of LHC-black holes: A review. J. Applied Sci., 10: 2491-2492.

Farmany, A., 2010. A proposal for spectral line profile of hydrogen atom spectrum in the sub-nano-meter space time. J. Applied Sci., 10: 784-785.

Hossenfelder, S., M. Bleicher, S. Hofmannb, J. Rupperta, S. Scherera and H. Stocker, 2003. Signatures in the Planck regime. Phys. Lett. B, 575: 85-99.

Jadhav, M.M. and L.M. Burko, 2009. Black hole quasinormal mode spectroscopy with LISA. Astrophysical J., 698: 562-562.

Jiang, Q.Q., Y. Han and X. Cai, 2010. Quantum corrections and black hole spectroscopy. J. High Energy Phys., 2010: 1-15.

Kempf, A., G. Managano and R.B. Mann, 1995. Hilbert space representation of the minimal length uncertainty relation. Phys. Rev. D, 52: 1108-1118.

Majhi, B.R., 2010. Hawking radiation and black hole spectroscopy in *Horava–lifshitz* gravity. Phys. Lett. B., 686: 49-54.

Myung, Y.S., Y.W. Kim and Y.J. Park, 2007. Black hole thermodynamics with generalized uncertainty principle. Phys. Lett. B, 645: 393-397.

Nouicer, K., 2007. Quantum-corrected black hole thermodynamics to all orders in the Planck length. Phys. Lett. B, 646: 63-71.

Setare, M.R., 2004a. Area spectrum of extremal Reissner-Nordstroem black holes from quasi-normal modes. Phys. Rev. D, 69: 044016-044018.

Setare, M.R., 2004b. Non-rotating BTZ black hole area spectrum from quasi-normal modes. Class. Quan. Grav., 21: 1453-1453.

Van Den Broeck, C. and A.S. Sengupta, 2007. Binary black hole spectroscopy. Class. Quan. Grav., 24: 1089-1114.