Journal of Applied Sciences

ISSN 1812-5654

science alert

ANSInet
an open access publisher
http://ansinet.com

Journal of Applied Sciences 11 (7): 1255-1260, 2011
ISSN 1812-5654 / DOI: 10.3923/jas.2011.1255.1260

Development of Some Numerical Methods Applying Complexity Reduction Approach for Solving Scientific Problem

[1]M.K. Hasan, [2]J. Sulaiman, [3]S.A.A. Karim and [4]M. Othman
[1]Programme of Industrial Informatic, School of Information Technology,
Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia
[2]Programme of Mathematic with Economy, School of Science and Technology, Universiti Malaysia Sabah,
88999 Kota Kinabalu, Sabah, Malaysia
[3]Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS,
Bandar Seri Iskandar, 31750 Tronoh, Perak, Malaysia
[4]Department of Network Technology and Communication,
Faculty of Computer Science and Information Technology, Serdang, Selangor, Malaysia

Abstract: The objective of this study was to describe numerical methods that apply complexity reduction approach to solve various scientific problems. Due to their low in complexity, their computations are faster than their standard form. Some of the methods have even higher in accuracy compared to their standard methods. In this study, we will describe the development of some of the methods that have been recently used to solve various scientific problems.

Key words: Complexity reduction approach, one stage method, two stages method, scientific problem finite difference scheme

INTRODUCTION

Partial differential equations can simulate most scientific problems. Finite element, finite difference, boundary element, and finite volume methods have been used to solve such problems. All these methods can be categorized as mesh-based methods.

Various iterative methods have been proposed to solve any system of linear equations, which is generated by discretisation of differential equations (Young, 1971, 1972, 1976; Hackbusch, 1995). Abdullah (1991) has proposed a complexity reduction approach scheme via the Explicit Decoupled Group (EDG) method in order to speed-up the computational execution time. Initially, the EDG method has been presented to solve two-dimensional Poisson equations. The method has been utilized to solve various problems (Ibrahim and Abdullah, 1995). Previous studies have shown that EDG have reduced computational complexity of its full-sweep methods, the Explicit Group (EG). Motivated by the concept of half-sweep iterations, Othman and Abdullah (2000) have proposed the quarter-sweep iteration via the Modified Explicit Group (MEG) method.

MATERIALS AND METHODS

To illustrate the implementations of the full-, half-, and quarter-sweep iterative schemes for solving any boundary value problem, this study will restrict to describe in finding computational molecules for only one dimensional (only consider in x space dimension) and two dimensional (only consider in x and y space dimension) spatial problems.

ONE-STAGE METHODS

Computational molecule for one dimensional problem: Let us consider the following three equations:

- Poisson equation:

$$-\frac{\partial^2 c}{\partial x^2}=f(x),x\in[a,b] \quad (1)$$

- Water quality model:

$$\frac{\partial c}{\partial t}=-U\frac{\partial c}{\partial x}+E\frac{\partial^2 c}{\partial x^2},x\in[a,b],t\geq 0 \quad (2)$$

Corresponding Author: Mohammad Khatim Hasan, Programme of Industrial Informatic, School of Information Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia

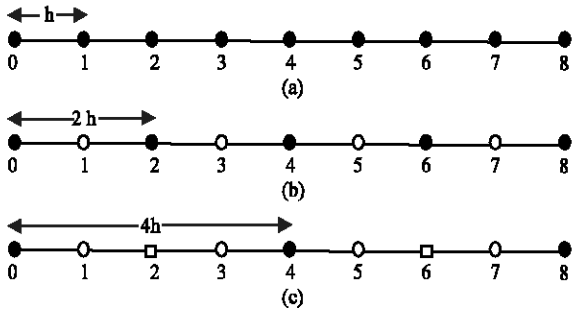

Fig. 1: Nodes for (a) full-sweep, (b) half-sweep and (c) quarter-sweep cases (steady problem)

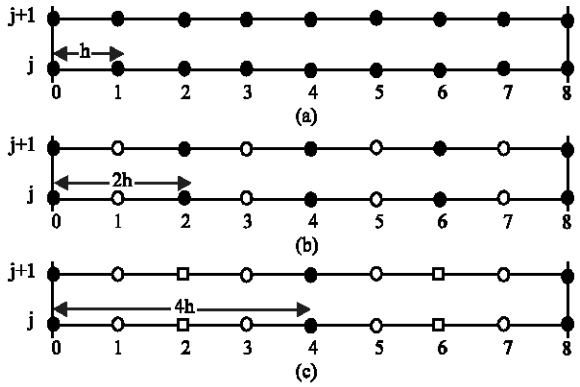

Fig. 2: Nodes for (a) full-sweep, (b) half-sweep and (c) quarter-sweep cases (unsteady problem)

- First-Order hyperbolic equation:

$$\frac{\partial c}{\partial t}+\frac{\partial c}{\partial x}=f(x,t),x\in[a,b]t\geq 0 \tag{3}$$

The nodes distributions for steady and unsteady problems are given by Fig. 1 and 2, respectively.

To simplify the discussion, we consider uniform node points only as shown in Fig. 1 and 2. For this purpose, we assume the solution domain can be uniformly divided into $m = 2^p$, $p\geq 2$ and R subintervals in x and t directions respectively. The subintervals in the x and t directions are denoted by Δx and Δt respectively, which are defined as:

$$\left.\begin{aligned}\Delta x&=\frac{(b-a)}{m}=h,\quad m=n+1\\ \Delta t&=\frac{(T-0)}{R}\end{aligned}\right\} \tag{4}$$

Generally, the second-order scheme for the full-, half-, and quarter-sweep finite difference approximation equations can be easily shown as:

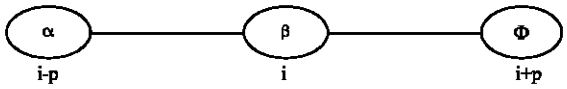

Fig. 3: Computational molecule of the second-order full-, half- and quarter-sweep finite difference schemes

$$\alpha C_{i-p}+\beta C_i+\varphi C_{i+p}=Y_i \tag{5}$$

for $i = 1_p, 2_p, 3_p, \ldots, m-p$, where α, β, and Φ are coefficients and y_i, $i = 1_p, 2_p, 3_{p,m}, \ldots, m-p$ are known values. The values of p, which corresponds to 1, 2 and 4, represents the full-, half-, and quarter-sweep cases respectively. Thus, the computational molecule for Eq. 5 is shown in Fig. 3.

The corresponding system of linear equations generated by Eq. 5 can be stated as:

$$\underset{\sim}{A}\underset{\sim}{C}=\underset{\sim}{Y} \tag{6}$$

where:

$$A=\begin{bmatrix}\beta & \varphi & & & & \\ \alpha & \beta & \varphi & & & \\ & \ddots & \ddots & \ddots & & \\ & & \alpha & \beta & \varphi & \\ & & & \alpha & \beta & \end{bmatrix}_{\left(\left(\frac{m}{p}\right)-p\right)\times\left(\left(\frac{m}{p}\right)-p\right)}$$

$$\underset{\sim}{C}=\begin{bmatrix}C_{1p} & C_{2p} & \cdots & C_{m-p}\end{bmatrix}^T,\underset{\sim}{Y}=\begin{bmatrix}Y_{1p} & Y_{2p} & \cdots & Y_{m-p}\end{bmatrix}^T$$

Analogously, fourth-order finite difference approximation equations can also be performed on (1-3) and resulted in:

$$\underset{\sim}{A}\underset{\sim}{C}=\underset{\sim}{Y}^* \tag{7}$$

where:

$$A=\begin{bmatrix}a_{1p} & b_{1p} & c_{1p} & & & & \\ d_{2p} & a_{2p} & b_{2p} & c_{2p} & & & \\ e_{3p} & d_{3p} & a_{3p} & b_{3p} & c_{3p} & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & e_{m-3p} & d_{m-3p} & a_{m-3p} & b_{m-3p} & c_{m-3p} \\ & & & e_{m-2p} & d_{m-2p} & a_{m-2p} & b_{m-2p} \\ & & & & e_{m-p} & d_{m-p} & a_{m-p}\end{bmatrix}_{\left(\left(\frac{m}{p}\right)-p\right)\times\left(\left(\frac{m}{p}\right)-p\right)}$$

$$\underset{\sim}{C}=\begin{bmatrix}C_{1p} & C_{2p} & \cdots & C_{m-p}\end{bmatrix}^T,\underset{\sim}{Y}^*=\begin{bmatrix}Y_{1p}^* & Y_{2p}^* & \cdots & Y_{m-p}^*\end{bmatrix}^T$$

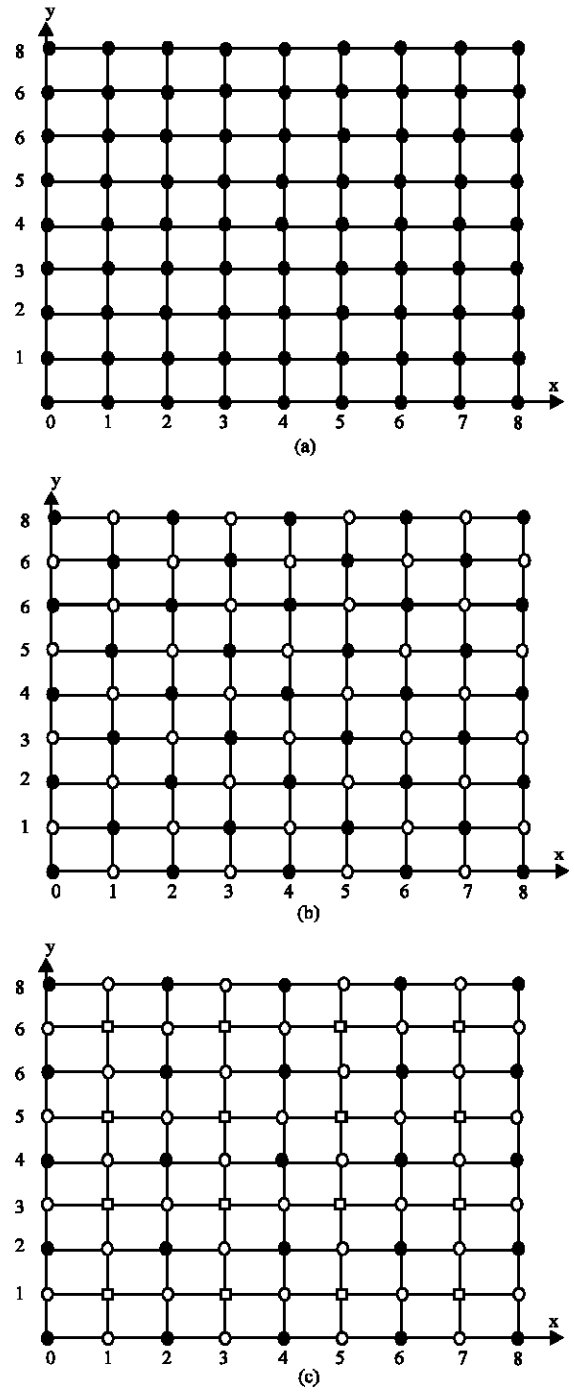

Fig. 4: Nodes for 2D problems for the (a) full-sweep, (b) half-sweep and (c) quarter-sweep cases respectively

Computational molecule for two dimensional problems: Let us consider 2D unsteady and steady problems given as:

- Poisson equation:

$$\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}=f(x,y),\quad (x,y)\in[a,b]\times[a,b] \tag{8}$$

- Convection-Diffusion equation:

$$\frac{\partial C}{\partial t}=-U\left(\frac{\partial C}{\partial x}+\frac{\partial C}{\partial y}\right)+E\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right),\quad (x,y)\in[a,b]\times[a,b],\ t\geq 0 \tag{9}$$

- Diffusion equation:

$$\frac{\partial C}{\partial t}=E\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+f(x,y,t),\quad (x,y)\in[a,b]\times[a,b],\ t\geq 0 \tag{10}$$

For simplicity, the problems in Eq. 8-10 and the rectangular solution domain is uniformly divided into subintervals in the x and y directions (as in one dimensional case) and R subintervals in the *t* direction. Figure 4a-c show the finite grid networks used to derive the full-, half-, and quarter-sweep finite difference equations for problems (8-10).

Using the fourth order approximation, it can be easily shown that the five point finite difference approximation equations for the full-, half-, and quarter-sweep finite difference approximation equations are given as:

$$\alpha_1 C_{i-1,j}+\alpha_2 C_{i+1,j}+\alpha_3 C_{i,j-1}+\alpha_4 C_{i,j+1}+\alpha_5 C_{i,j}=Y^{\alpha}_{i,j} \tag{11}$$

$$\beta_1 C_{i-1,j-1}+\beta_2 C_{i+1,j-1}+\beta_3 C_{i-1,j+1}+\beta_4 C_{i+1,j+1}+\beta_5 C_{i,j}=Y^{\beta}_{i,j} \tag{12}$$

$$\varphi_1 C_{i-2,j}+\varphi_2 C_{i+2,j}+\varphi_3 C_{i,j-2}+\varphi_4 C_{i,j+2}+\varphi_5 C_{i,j}=Y^{\varphi}_{i,j} \tag{13}$$

where, α_k, β_k and φ_k, k = 1, 2, ..., 5 are constants. Thus, the computational molecule for each of (11-13) is as shown in Fig. 5a-c.

The corresponding system of linear equations generated by Eq. 11-13 can be stated in a matrix form as:

$$\underset{\sim}{A}\underset{\sim}{C}=\underset{\sim}{Y} \tag{14}$$

where, its coefficient matrix, A is a penta-diagonal matrix. Using higher order approximation, we can develop some nine point finite difference stencils that have been proposed such as:

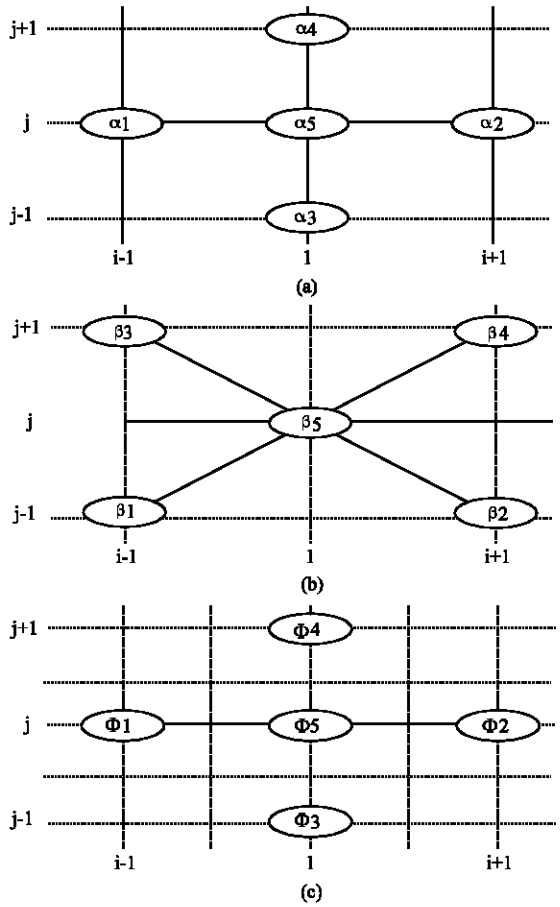

Fig. 5: Computational molecules for five points finite difference schemes: (a) full-sweep, (b) half-sweep, and (c) quarter-sweep

- Fourth order standard nine point stencil (Ali, 1998):

$$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} C_{i,j} = Y_{i,j}$$

- Fourth order rotated nine point stencil (Ali *et al*., 2002):

$$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} C_{i,j} = Y_{i,j} \begin{bmatrix} & * & \\ * & & * \\ * & & * \\ & * & \end{bmatrix} C_{i,j} = Y_{i,j}$$

- Fourth order spark nine point stencil (Shariffudin and Abdullah, 2001).

$$\begin{bmatrix} * & & * \\ & * & * \\ & * & \\ * & & * \end{bmatrix} C_{i,j} = Y_{i,j}$$

TWO-STAGES TECHNIQUES

Apart from the one-stage iterative methods, the concept of the two-stages iterative methods has been proposed widely to be one of the efficient methods for solving any system of linear equations. These methods involve two levels of virtual time such as $C^{(1)}$ and $C^{(2)}$. In developing two-stage iterative schemes, usually, the coefficient matrix, A will be decomposed by using several splitting techniques. The following is a list of two-stage iterative methods that can be considered.

Iterative Alternating Decomposition Explicit (IADE) Method: The Iterative Alternating Decomposition Explicit (IADE) method proposed by Sahimi *et al*. (1993) is an extension of the Alternating Direction Implicit (ADI) method. Instead of using the Douglass-Rachford and Peacemen-Rachford variants, the general scheme of this method using the Mitchell-Fairweather variant can be expressed as:

$$\begin{aligned} (rI + G_1)\underset{\sim}{C}^{(k+1/2)} &= (rI - gG_2)\underset{\sim}{C}^{(k)} + \underset{\sim}{Y} \\ (rI + G_2)\underset{\sim}{C}^{(k+1)} &= (rI - gG_1)\underset{\sim}{C}^{(k+1/2)} + g\underset{\sim}{Y} \end{aligned} \quad (15)$$

where, r, k and I represent as an acceleration parameter, the k iteration and an identity matrix respectively and the relation of g and r is given by G=(6+r) / 6 The formulation of the IADE methods is to assume that the coefficient matrix, A of the system (6) can be written generally as:

$$A = G_1 + G_2 - \frac{1}{6}G_1G_2 \quad (16)$$

Motivated by the concept of the complexity reduction techniques, the standard IADE method has been modified and now there are several various IADE called as the Reduced Iterative Alternating Decomposition Explicit (RIADE) proposed by Sahimi and Khatim (2001), the Half-Sweep Iterative Alternating Decomposition Explicit (HSIADE) proposed by Sulaiman *et al*. (2004a), and the Quarter-Sweep Iterative Alternating Decomposition Explicit (QSIADE) proposed by Sulaiman *et al*. (2004b).

Various mean schemes: The Arithmetic Mean (AM) iterative method (Ruggiero and Galligani, 1990) can be named as the Full-Sweep Arithmetic Mean (FSAM). In 2004, this standard AM has been modified via implementing the concept of the half-sweep iteration and called as the Half-Sweep Arithmetic Mean (HSAM) method (Sulaiman *et al*., 2004c). Further studies of the HSAM method have been conducted by Sulaiman *et al*. (2005). Let the coefficient matrix, A is represented as the sum of three matrices:

$$A = L + D + T \tag{17}$$

where, L, D and T are strictly lower triangular, diagonal and strictly upper triangular matrices respectively. The general scheme for the FSAM, HSAM and QSAM methods can be represented as:

$$\begin{aligned}(D + rL)\underset{\sim}{C}^{(1)} &= ((1-r)D - rT)\underset{\sim}{C}^{(k)} + r\underset{\sim}{Y}\\ (D + rT)\underset{\sim}{C}^{(2)} &= ((1-r)D - rL)\underset{\sim}{U}^{(k)} + r\underset{\sim}{Y}\ Z\\ \underset{\sim}{C}^{(k+1)} &= \frac{1}{2}\left(\underset{\sim}{C}^{(1)} + \underset{\sim}{C}^{(2)}\right)\end{aligned} \tag{18}$$

where, r and $C^{(k)}$ represent as an acceleration parameter and an unknown vector at the k^{th} iteration respectively.

Apart from the average mean, geometric mean have also be utilized with the complexity reduction approach to form the Half-Sweep Geometic Mean (HSGM) and Quarter-Sweep Geometric Mean (QSGM) for solving a two-point boundary value problem (Sulaiman *et al*., 2008).

The general scheme for the FSGM, HSGM and QSGM methods can be represented as:

$$\begin{aligned}(D + rL)\underset{\sim}{C}^{(1)} &= ((1-r)D - rT)\underset{\sim}{C}^{(k)} + r\underset{\sim}{Y}\\ (D + rT)\underset{\sim}{C}^{(2)} &= ((1-r)D - rL)\underset{\sim}{U}^{(k)} + r\underset{\sim}{Y}\\ \underset{\sim}{C}^{(k+1)} &= \sqrt{\underset{\sim}{C}^{(1)}\ \underset{\sim}{C}^{(2)}}\end{aligned} \tag{19}$$

where, r and $C^{(k)}$ represent as an acceleration parameter and an unknown vector at the k^{th} iteration respectively.

APPLICATIONS IN WAVE PROPAGATION PROBLEM

Inspired by the concept of the half- and quarter-sweep iteration, a method called High Speed Low Order Finite Difference Time Domain (HSLO-FDTD) have been proposed to solve free space wave propagation problems (Hasan *et al*., 2006, 2007a). Through numerical experiments in the previous studies, it has shown that the HSLO-FDTD method has successfully reduce 67% for 1D and 85.2% for 2D for solving free space wave propagation compared to conventional FDTD. Further studies of the HSLO-FDTD have also been conducted by Hasan *et al*. (2007b). An extension to fourth order approximation with complexity reduction approach called High Speed High Order Finite Difference Time Domain (HSHO-FDTD) to solve 1D and 2D free space wave propagation problems have also being proposed (Hasan *et al*., 2009). Research conducted shows that HSHO-FDTD have improve not only the speed of solving 2D wave propagation problem (compared to conventional FDTD method), but also improved the accuracy (compared to HSLO-FDTD method).

CURRENT WORK AND FUTURE RESEARCH

As mentioned in the second, third and fourth sections, we, actually, can reduce the complexity of existing method via complexity reduction techniques. by considering the same order of the approximations. Obviously, the advantage of the complexity reduction approach is to reduce the complexity of the original method. As a result, the methods have successfully speed-up the computational time.

Beside the finite difference methods, we have also applied the same concept of the complexity reduction approach to galerkin type of finite element methods (Sulaiman and Abdullah, 2001; Sulaiman *et al*., 2005). Overall, the complexity reduction approach have successfully speed-up the computational execution time. This is because of the computational complexity of the method applying complexity reduction approach is lower than the conventional methods.

ACKNOWLEDGMENTS

Special thanks to fundamental research grant UKM-TT-02-FRGS0139-2009 for financial support. We also appreciate the anonymous referees for the constructive comments that improved the study.

REFERENCES

Abdullah, A.R., 1991. The four point Explicit Decoupled Group (EDG) method: A fast poisson solver. Int. J. Comput. Mathe., 38: 61-70.

Ali, N.M., 1998. The design and analysis of some parallel algorithms for the iterative solution of partial differential equations. Ph.D. Thesis, Department of Industrial Computing, Universiti Kebangsaan Malaysia.

Ali, N.M., Y. Yunus and M.Othman, 2002. A new nine-point multigrid V-Cycle algorithm. Sains Malaysiana, 31: 135-147.

Hackbusch, W., 1995. Iterative Solution of Large Sparse Systems of Equations. Springer-Verlag, New York.

Hasan, M.K., M.Othman, Z.Abbas, J.Sulaiman and J.M.Desa, 2006. New algorithms for time-domain maxwell equations. Int. J. Mathe. Comput. Sci., 1: 271-288.

Hasan, M.K., M. Othman, Z. Abbas, J. Sulaiman and F. Ahmad, 2007a. Parallel solution of high speed low order fdtd on 2d free space wave propagation. Lecture Notes Comput. Sci., 4706: 13-24.

Hasan, M.K., M.Othman, Z.Abbas, J.Sulaiman and J. Md-Desa, 2007b. A new numerical algorithm for antenna analysis. Borneo Sci., 20: 71-77.

Hasan, M.K., M.Othman, Z.Abbas, J.Sulaiman and F. Ahmad, 2009. Fourth order approximation with complexity reduction approach for the solution of time domain maxwell equations in free space. Lecture Notes Comput. Sci., 5592: 751-763.

Ibrahim, A. and A.R.Abdullah, 1995. Solving The two-dimensional diffusion equation by the four point Explicit Decoupled Group (EDG) iterative method. Int. J. Comput. Mathe., 58: 253-256.

Othman, M. and A.R. Abdullah, 2000. An efficient four points modified explicit group poisson solver. Int. J. Comput. Mathe., 76: 203-217.

Ruggiero, V. and E. Galligani, 1990. An iterative method for large sparse systems on a vector computer. Comput. Mathematics Appl., 20: 25-28.

Sahimi, M.S. and M. Khatim, 2001. The reduced iterative alternating decomposition explicit (riade) method for the diffusion equation. Pertanika J. Sci. Technol., 9: 13-20.

Sahimi, M.S., A. Ahmad and A.A. Bakar, 1993. The iterative alternating decomposition explicit method to solve the heat conduction equation. Int. J. Comput. Math., 47: 219-229.

Shariffudin, R.H. and A.R. Abdullah, 2001. Hamiltonian circuited simulations of elliptic partial differential equations using a spark. Applied Mathe. Lett., 14: 413-418.

Sulaiman, J. and A.R. Abdullah, 2001. Finite element method using the explicit group iterative methods for 1d diffusion problems. J. Kejuruteraan, 13: 41-49.

Sulaiman, J., M.K. Hasan and M. Othman, 2004a. The half-sweep iterative alternating decomposition explicit (HSIADE) method for diffusion equations. Lecture Note Comput. Sci., 3314: 57-63.

Sulaiman, J., M.K. Hasan and M. Othman, 2004b. Quarter-sweep iterative alternating decomposition explicit algorithm applied to diffusion equations. Int. J. Comput. Mathematics, 81: 1559-1565.

Sulaiman, J., M. Othman and M.K. Hasan, 2004c. A new half-sweep arithmetic mean (hsam) algorithm for two-point boundary value problems. Proceedings of the International Conference on Statistics and Mathematics and its Application in the Development of Science and Technology, Oct. 4-6, Bandung, Indonesia, pp: 169-173.

Sulaiman, J., M. Othman and M.K. Hasan, 2005. Half-sweep arithmetic mean method using finite element approximation for poisson's equation. Proceedings of the International Conference on Applied Mathematics, Bandung Institute of Technology, Indonesia. CD-ROM.

Sulaiman, J., M. Othman and M.K. Hasan, 2008. Half-Sweep Algebraic Multigrid (HSAMG) Method Applied to Diffusion Equations in Modeling, Simulation and Optimization of Complex Processes. Springer, Heidelberg, pp: 547-556.

Sulaiman, J., M. Othman and M.K. Hasan, 2008. Half-Sweep Algebraic Multigrid (HSAMG) Method Applied to Diffusion Equations in Modeling, Simulation and Optimization of Complex Processes. Springer, Heidelberg, pp: 547-556.

Young, D.M., 1971. Iterative Solution of Large Linear Systems. Academic Press, London.

Young, D.M., 1972. Second-degree iterative methods for the solution of large linear systems. J. Approximation Theory, 5: 137-148.

Young, D.M., 1976. Iterative Solution of Linear Systems Arising from Finite Element Techniques in The Mathematics of Finite Elements and Applications II. Academic Press, London.