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## Predicting Flow Rate of V-shape Custom Tank using Derivative Free Recursive Algorithm

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**Abstract:** One of the basic elements that any plant process facility consists of its control tank system. Custom tank is one type of these control tank systems. Proper controlling of this element will help the facility to work smoothly and it will increase the reliability of the whole system. This study is looking into predicting the state of variables that completely represent the dynamics the custom tank (height of fluid or output flow rate). This prediction can be used in controlling the custom tank (predictive control). The study involve MATLAB SIMULINK simulation program for the custom tank along with different prediction models. The obtained results showed that introducing Multilayer Perceptron (MLP) Neural Network architecture improve the prediction significantly where different algorithms, Recursive Kalman Filter (RKF) and Extended Kalman Filter (EKF) have been used simultaneously to estimate fluid height and output flow. It further shows that introducing centered finite difference or derivative free with EKF improve the performance of the network.

**Key words:** Custom tank, system identification, neural network, extended kalman filter, black box modeling

### INTRODUCTION

Custom tank is one of the common vessels being used in chemical or plant process industry (Prakash, 2007). The study aims to build a simulation program to predict the flow rate and level of the fluid inside the custom tank which will enhance the controllability of the system. System identification (Ljung, 1987) will be carried out on a v-shaped custom tank as a nonlinear benchmark problem with the implementation of different prediction algorithms.

The estimation of state variables (fluid level and flow rate) is very essential to produce a reliable controlling system for the v-shape custom tank. In order to control such a process and to keep level and flow rate at desired set point, an efficient control strategy is required. Also the suggested techniques such as Recursive Least Square, Recursive Kalman Filter (Haykin, 2001) and Multilayer Neural Network (Norgaard *et al.*, 2000) need to be implemented under specific conditions. All the above will help in building a reliable system that can be used in the real application. The objectives of the research are to do system identification model for the variables that describes the dynamics of the v-shaped custom tank. The

model obtained can predict the flow rate and level of the fluid inside the tank using several linear and nonlinear training algorithms.

Many criteria such as computation time, memory usage and the prediction error can be used to compare between the results obtained by the used algorithms. The obtained results showed a comparison between the training algorithms taking the obtained percentage mean of squared error (%MSE) into consideration. The result obtained shows the improvement in the optimization performance by using the new derivative free approach in comparison with the traditional way of the finding the derivative of the error function using back propagation (Simandl and Dunik, 2009).

### MATERIALS AND METHODS

Figure 1 shows a custom tank block with two input flow and one output flow. It is generally used for mixing fluids; however it may has other functions depending on the purpose of facility (Prakash, 2007).

$$\frac{dH1}{dt} = \frac{Q1 - k1\sqrt{H1 - H2}}{A1} \quad (1)$$

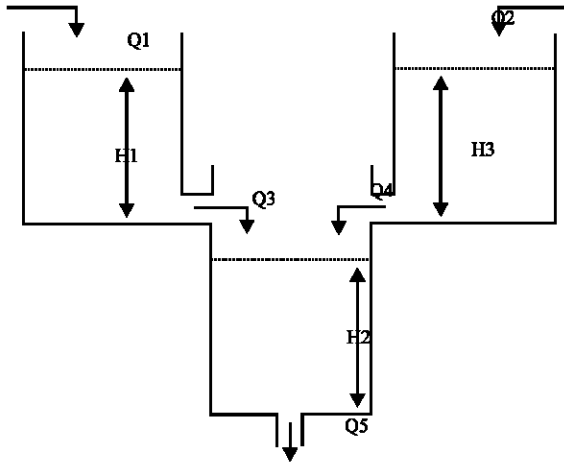


Fig. 1: Custom tank block diagram

$$\frac{dH3}{dt} = \frac{Q2 - K3\sqrt{H3 - H2}}{A3} \quad (2)$$

$$\frac{dH2}{dt} = \frac{-k2\sqrt{H2} + K1\sqrt{H1 - H2} + K3\sqrt{H3 - H2}}{A2} \quad (3)$$

Eq. 1-3 have been derived from Fig. 1 which represents the custom tank or the v-shaped custom tank system. The equations are derived based on the total energy balanced in the system. The  $k1$ ,  $k2$  and  $k3$  are constants assumed to be equal to a value of one, and it represent all other variable that might affect the performance of the system except fluid level and flow rate outlet which both are investigated.  $A1$ ,  $A2$  and  $A3$  are the area cross-section which also assumed to be equal to  $1 \text{ m}^2$ . These equations have been used to build the tank into SIMULINK to simulate the benchmark problem.

System identification steps have been carried out in part selection of prediction algorithm in the flow chart in Fig. 2.

Initial work involved linear estimation algorithms including Recursive Least Square and Recursive Kalman Filter. Both have been tested and showed good performance (low prediction error). Then the main focus was in implementing nonlinear estimation using MLP neural network with different training algorithms.

In both cases a black box modeling (Norgaard *et al.*, 2000) has been chosen, which mean that we derive the mathematical model for the prediction using only measurements taken from the system. These measurements are the applied inputs to the system and the corresponding outputs related using Autoregressive external input model (ARX) (Ljung, 1987; Norgaard *et al.*, 2000) as in Eq. 4.

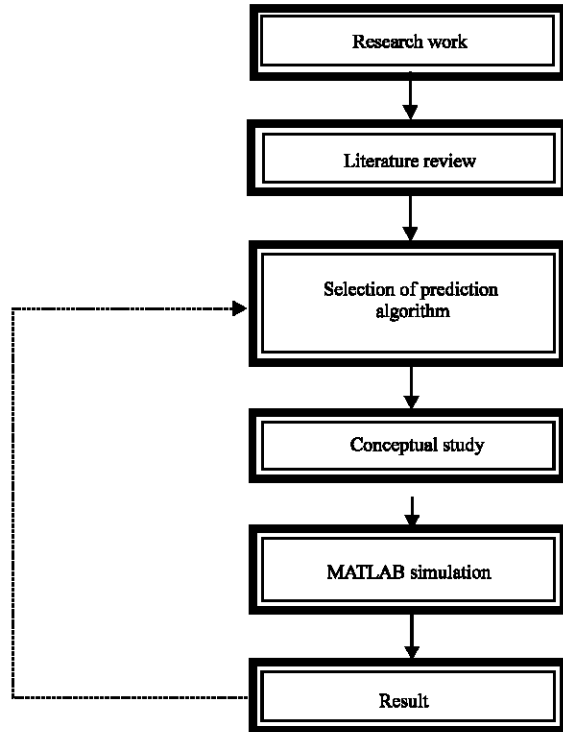


Fig. 2: Methodology flow chart

$$y(t) = a_1y(t-1) + a_2y(t-2) + \dots + b_1x(t-1) + b_2x(t-2) \quad (4)$$

The measurements are taken and applied in a form of pairs as an input to a MLP (4, 2, 1), which mean 4 inputs, two hidden neuron and one linear output.

One type of neural network architectures is the feedforward or multilayer perceptron (MLP). The general structure of an MLP is shown in Fig. 3 (Haykin, 2001). Basically neural network are meant to behave like human nervous system where the process is distributed among the small neuron that contains one type of activation function acting like a human biological neuron.

Figure 3 shows a general MLP structure. MLPs are usually used for prediction and classification using suitable training algorithms for the networks weights. The MLP output formula in Eq. 5 and the activation function is represented Eq. 6 used in the MLP neurons (Norgaard *et al.*, 2000; Alsaade, 2010).

$$y = \sum_{j=1}^{N_h} w_j^l f \left( \sum_{i=1}^{N_i} w_{ij}^{NL} u_i + b_j \right) + d \quad (5)$$

$$f(v) = \frac{1}{1 + \exp(-v)} \quad (6)$$

In SIMULINK, S-function (user defined function) block has been implemented in the design where measurements are taken and the specified algorithm is applied. Note that all predictions are done on-line or instantaneously.

The linear weights are the weights connecting the output neuron and the hidden neurons while the weights connecting the input with hidden neurons are the nonlinear weights. The aim is train the MLP network with suitable learning algorithm to produce an estimation of the height of the fluid and flow rate in the tank recursively.

Back propagation (BP) algorithm is the classical way of training the MLP network weights with strong stability. In approaching any nonlinear dynamic system BP can be used to train MLP weights in a stochastic or on-line way by feeding the network with measurements of input-output data taken from the tank model (Shi *et al.*, 2009). Recent approach is the hybrid training where two different algorithms are combined to training the network weights. In this work the linear weights are trained using RKF where the nonlinear weights are trained using EKF (Mao *et al.*, 2009; Asseu *et al.*, 2010).

This method is used to predict the fluid height H2 and the outlet flow Q5 as shown in Fig. 1 (Kadirgama *et al.*, 2006).

The recursive algorithms are used for both linear and nonlinear networks (Ljung, 1987; Andryani *et al.*, 2009). The recursive Kalman filter (RKF) is described in Eq. 7-10 (Haykin, 2001).

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_k \epsilon_k \tag{7}$$

$$K_k = \left( \frac{P_{k-1} X_k}{R_2 + X_k^T P_{k-1} X_k} \right) \tag{8}$$

$$\epsilon_k = y_k - x_i^T \hat{\theta}_{k-1} \tag{9}$$

$$P_k = \left( P_{k-1} \frac{P_{k-1} X_k X_k^T P_{k-1}}{R_2 + X_k^T P_{k-1} X_k} + R_1 \right) \tag{10}$$

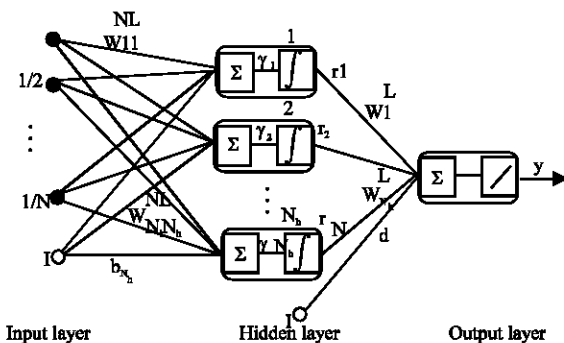


Fig. 3: MLP (Ni, Nh, 1)

where,  $\hat{\theta}_k$  is the regressor vector  $K_k$  is Kalman gain,  $\epsilon_k$  is the error,  $P_k$  is the covariance matrix,  $R_2$  is set to 1.45 and  $R_1$  to zero. RKF is used to training the linear weights while nonlinear weights have been fixed to the initialized value, this process is known as extreme learning machine.

Extended Kalman Filter (EKF) is the nonlinear version of RKF (Ljung, 1987; Andryani *et al.*, 2009; Haykin, 2001). The equations are the same as RKF except input  $x$  is replaced by the change in the output to the change in the gradient of network weight (Eq. 11-15). EKF is used for the nonlinear weights while the linear weights are still trained using RKF (Hybrid Training). The gradient in Eq. 11 is either computed analytically or using derivative free as in Eq. 12 (Freitas *et al.*, 2000; Simandl and Dunik, 2009).

$$\nabla y(w) = \frac{\nabla E(w)}{e} \tag{11}$$

$$\nabla y(w) = \frac{f(v+d) - f(v-d)}{2h} \tag{12}$$

$$\hat{\theta}_k = \hat{\theta}_{k-1} + P_k \nabla y(w) \epsilon_k \tag{13}$$

$$\epsilon_k = y_k - x_i^T \hat{\theta}_{k-1} \tag{14}$$

$$P_k = \frac{1}{\lambda} \left( P_{k-1} + \frac{P_{k-1} \nabla y(w) \nabla y^T(w) P_{k-1}}{\lambda + \nabla y^T(w) P_{k-1} \nabla y(w)} \right) \tag{15}$$

## RESULTS AND DISCUSSION

The simulation is performed on v-shaped custom tank to evaluate the performance of derivative free technique with recursive training algorithms.

Figure 4 shows the prediction for height of the fluid in the v-shaped custom tank with the settings mentioned in the previous section. The result in Fig. 6 has been obtained using MLP with RKF training for the linear weights while the nonlinear weights are fixed (extreme learning machine). The simulation was for 5000 iteration, the graph shows the measured output with the predicted output from iteration 4500 until 5000 only for demonstration purpose.

Figure 5 shows a comparison between MLP with derivative free approach and analytical approach for prediction of fluid height in the tank. Table 1 shows error

Error	RKF	Hybrid (Analytical)	Hybrid (Derivative free)
Mean	0.004053	0.006306	0.003892
Std.	0.04912	0.02443	0.01788
Min.	0	0	0
Max.	3.453	0.36	0.3062

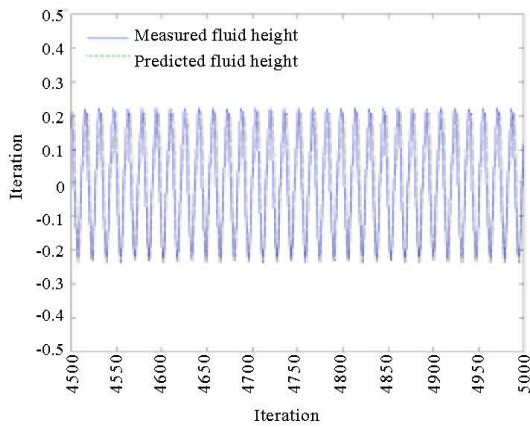


Fig. 4: MLP with RKF prediction for height

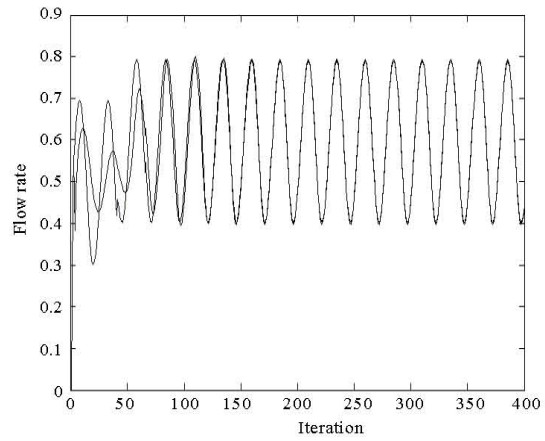


Fig. 7: Flow rate estimation

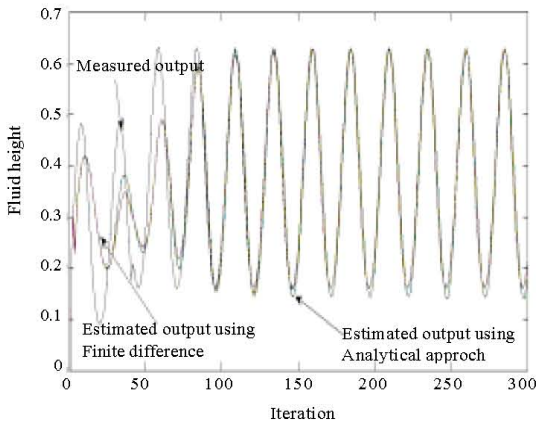


Fig. 5: Comparison using derivative free and analytical approach

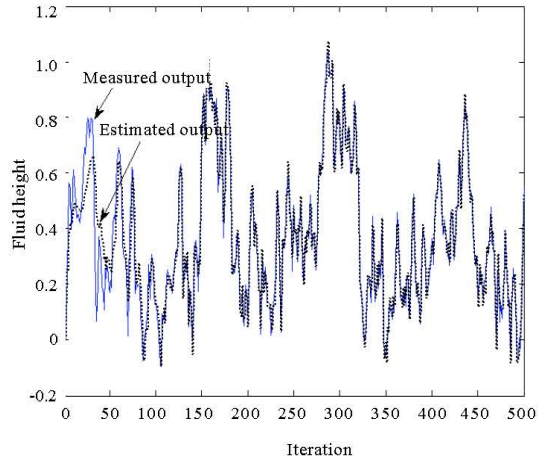


Fig. 8: Estimation MLP (Analytical) for noisy data

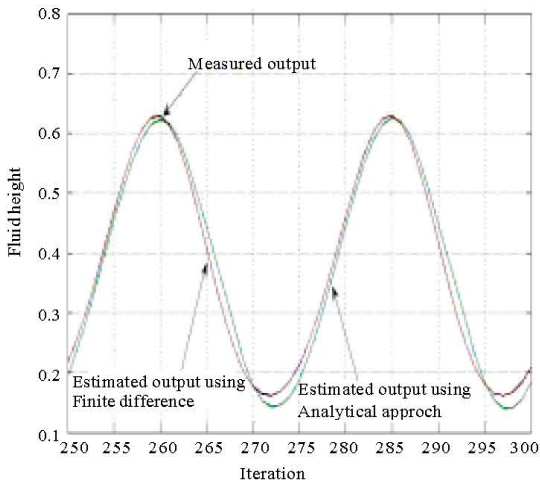


Fig. 6: Improvement using derivative free

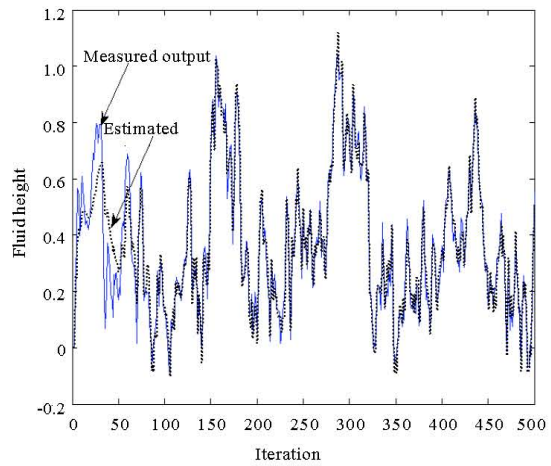


Fig. 9: MLP derivative free estimation for noisy data

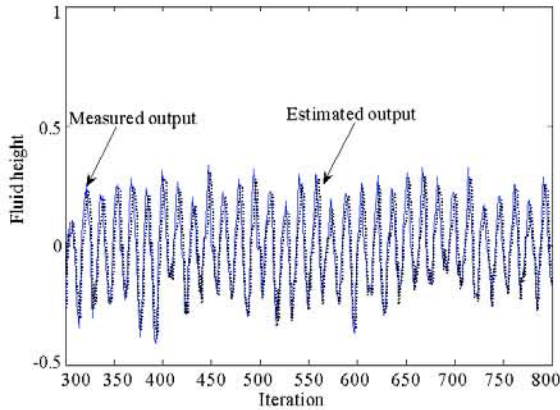


Fig. 10: MLP with RKF estimation for noisy data

comparison between the algorithms applied with 5000 samples (iteration) in the simulation which depicts the derivative free approximation overall performance compared to other techniques which includes the real gradient estimate.

Figure 6 illustrates the performance of extended Kalman filtering technique iteration using derivative free approach for 50 iterations. The central difference technique uses derivative free methods which bypass the need for the knowledge of the physics of nonlinear process plant.

Figure 7 shows the overall flow rate estimation for v-Shape custom tank. Using recursive training technique the neural network modeler show small level of difference at initial stage and started to fit well at the later stage.

Figure 8, 9 and 10 shows the estimation results for each algorithm where a noisy input were applied to the v-shaped custom tank block. Figure 11 illustrate the behavior of the squared error between the estimated and the true output between the 2700 and the 3000 iteration.

For more comparison Fig. 12 depicts a zoom in to compare the smoothness of both estimation using free derivative and analytical method. From the figure it is clearly shown that free derivative has better smoothness.

From the results obtained we observed the differences between different training algorithms for the MLP training. Generally the MLP architecture has shown great ability to predict the desired output with good accuracy. The variance R2 has been set to 1.45 and R1 to zero, however R2 can be calculated from offline data and inserted to the system. From the graphs and the error table it can be shown that MLP with hybrid training has demonstrated better performance. However MLP architecture requires longer time to learn the function especially in the case of derivative free.

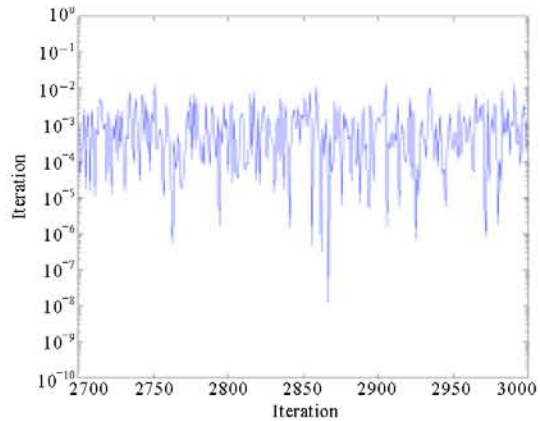


Fig. 11: Squared error (2700-3000 iteration)

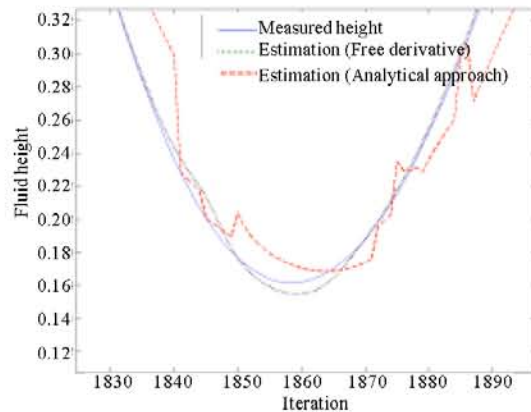


Fig. 12: Smoothness comparison

The evaluation of the different algorithms was based on the error between the estimated and measured outputs. MLP with Hybrid training with centered derivative free approach showed better perform in terms error is a new approach which can form basic understanding of particle filter.

## CONCLUSIONS

In conclusion, the model is able to predict the fluid heights and outlet flow using the derivative free approach with less error comparing to analytical approach due nonlinearity of the system. The dynamics of custom tank have been predicted using the multilayer perceptron (MLP) with different training algorithms derived based on derivative free gradient estimate. MLP has shown better performance especially with centered derivative free gradient estimate compared to analytical which is cumbersome to derive in real world. The stochastic behavior of derivative free estimates method can be

compared with particle free techniques which will be subject to investigation Simandl and Dunik (2009). Regularized form of recursive learning (Asirvadam, 2008) will be something interesting to look into for the case of derivatives gradient estimate.

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