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On Integer Sequences Related to a Memorization Technique

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Abstract: This research studies the properties of integer sequences related to a memorization technique. The fractal property of the main sequence is investigated as well as the formula for the related sequences. Using a simple description, the self similarity of the main sequence is proved and its relation with known sequences is also shown.

Key words: Integer sequences, fractal sequences

INTRODUCTION

Suppose that we want to memorize a poem composed of several verses. The following technique can be used:

- Memorize verse 1
- Memorize verse 2
- Memorize (revise) verse 1 and then verse 2
- Memorize verse 3
- Memorize (revise) verse 2 and then verse 3
- Memorize (revise) verse 1, then verse 2 and then verse 3
- Memorize verse 4
- Memorize (revise) verse 3 and then verse 4
- Memorize (revise) verse 2, then verse 3 and then verse 4
- Memorize (revise) verse 1, then verse 2, then verse 3 and then verse 4
- Etc

Listing down the verses in the order they are memorized or revised gives the following sequence that we will refer as M.

$$1, 2, 1, 2, 3, 2, 1, 2, 3, 4, 2, 3, 4, 1, 2, 3, 4, 5...$$

The fascinating property of self-containment characterizes fractal sequences. The interest of Mathematicians to this property is probably because of the similarity with the more known geometric fractals. Fractal sequences are also related to interspersions and dispersions (Kimberling, 1997). The first definition of fractal sequence was probably given by Kimberling (1995). Proving that a given sequence is fractal requires exhibiting a fractal decimation rule. In other words, how to extract the initial sequence inside itself? Some examples of fractal integer sequences are described by Gilleland (2010) with their decimation rule. In this study we give a description of the sequence M that allows us to prove its fractal property.

With the above described method how many steps are added to memorize the nth verse after having memorized the n-1 first verses? This question gives a second sequence that we will denote by A. n = 1corresponds to memorizing the first verse from 0 verse known, A (1) = 1. When n = 2 the following steps have to be added to memorize verse 2 after having memorized the first verse: 2, 1, 2, that means A (2) = 3. When n = 3 the sequence to be added is: 3, 2, 3, 1, 2, 3, that means A (3) = 6.

The third sequence associated to the given method consists on counting the total number of steps to memorize the verses. We will denote it by S . When, n = 1, there is just one step which consists on memorizing the first verse, i.e., S (1) = 1. When n = 2, the method gives the following steps , which means When, the method gives the following sequence 1, 2, 1, 2, 3, 2, 3, 1, 2, 3. The total number of steps is S (3) = 10.

PROPERTIES

Sequence M: To describe the sequence, we will introduce few definitions.

Definition: The span of n (that we will denote by Sp (n) is the following sequence of numbers:

$$n, n-1, n-2, n-1, n, n-3, n-1, n, ..., 1, 2, 3, ..., n.$$

Example: Sp (1) = 1, Sp (2), 1, 2 Sp(3) = 3, 2, 3, 1, 2, 3 and Sp (4) = 4, 3, 4, 2, 3, 4, 1, 2, 3, 4.

Remark 1: M = Sp (1), Sp (2),..., Sp (n)...

Remark 2: Sp (n) is the subsequence of M starting at the first appearance of n until the last appearance of n before the first appearance of n+1.

The span of n can be described as a sequence of arithmetic subsequences of the form:

n-k, n-k+1, n-k+2,..., n where k = 0, 1,.., n-1.

When k = 0, the subsequence is just n, its cardinal is 1. When k = 1, the subsequence is n-1, n, its cardinal is 2. Etc. The last subsequence corresponds to k = n-1 and is given by 1, 2, 3,.., n. Its cardinal is . Finally we have the following:

Lemma 1:

$$\text{Card}(\text{Sp}(n)) = 1 + 2 + ... + n = \frac{n(n+1)}{2}$$

In each of the arithmetic subsequences n-k, n-k+1, n-k+2,.., n where k = 0, 1,.., n-1, the integer appears only once. We can then write.

Lemma 2: The number appears n times in the sequence Sp (n). Striking out the integer in the arithmetic subsequence will yield an empty subsequence (when) or a subsequence . That remark establishes the following.

Lemma 3: Striking out all occurrences of n in Sp (n) yields the sequence Sp (n-1).

Definition: An infinite sequence is said to be a fractal sequence if it contains an infinite number of copies of itself.

Proposition: M is a fractal sequence.

Proof: To establish the proposition the following fractal decimation rule can be applied to M: strike out the first n occurrences of n . From the remarks 1 and 2 and lemma 2, the fractal decimation rule corresponds to striking out the occurrences of n in the sequence Sp (n), for n = 1, 2, 3...i.e. striking out the occurrences of 1 in the sequence Sp (1), striking out the occurrences of 2 in the sequence Sp (2), striking out the occurrences of 3 in the sequence Sp (3), etc. Using then Lemma 3, the remaining sequence is just M.

1, 2, 1, 2, 3, 2, 3, 1, 2, 3, 4, 3, 4, 2, 3, 4, 1, 2, 3, 4,...

1, 2, 1, 2, 3, 2, 3, 1, 2, 3, ...

How many times each verse will appear in the process of memorizing n verses?

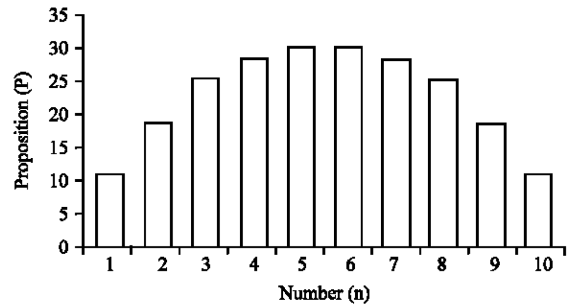

Fig. 1: Occurrences when n=10

An integer p≤ n will appear only in the following spans: Sp (p), Sp (p+1),...,Sp (n). In each of the spans Sp (k) (p ≤k ≤ n), using the description with arithmetic subsequences, the integer p will appear p times (only in subsequences starting by 1, 2, ..., p.) Finally, we have the

Proposition: The integer p will appear p (n-p+1) times in the process of memorizing n verses by the given method. The function f (x) = x (n+1 -x) is a reverse parabola getting its maximum value at $(n+1)/2$. That means by the given method, the verses around the middle will get maximum emphasize.

Sequences A and S: From Remark 2, memorizing the nth verse after having memorized the n-1 previous verses, corresponds to adding the subsequence Sp (n). Using then Lemma1, we can write the following:

Proposition: $A(n) = \frac{n(n+1)}{2}$.

A corresponds to the sequence A000217 of Sloane (2010).

For the sequence S, the term S (n) corresponds to the total number of steps by the given method to memorize a poem with n verses. It can be calculated from the terms A (k), k = 1, 2,..., n.

$$S(n) = \sum_{k=1}^{k=n} A(k) = \sum_{k=1}^{k=n} \frac{k(k+1)}{2}$$

$$= \frac{1}{2}\sum_{k=1}^{k=n} k^2 + \frac{1}{2}\sum_{k=1}^{k=n} k$$

$$= \frac{1}{2}\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}\frac{n(n+1)}{2}$$

$$=\frac{n(n+1)(2n+4)}{12}=\frac{n(n+1)(n+2)}{6}$$

We have the following:

Proposition: $S(n)=\frac{n(n+1)(n+2)}{6}$

S corresponds to the sequence A000292 of Sloane (2010). Using the previous result, the overall occurrences of each verse can be calculated by:

$$\frac{S(n)}{n}=\frac{(n+1)(n+2)}{6}.$$

CONCLUSION

In this study, we presented a memorization method and integer sequences derived from it. We described the main sequence in a way to prove easily its self similarity. The way the method uses the different verses is described as well. For the sequence A corresponding to the number of steps to add from n-1 to n verses, and the sequence S corresponding to the total number of steps, the formulas are given. They coincide respectively with the sequences A000217 and A000292 of Sloane (2010). The remaining question is: what is the formula for the main sequence M? The description of by the spans defined in this study might be useful for answering that question.

REFERENCES

Gilleland, M., 2010. Some self-similar integer sequences. http://www.research.att.com/~njas/sequences/selfsimilar.html.

Kimberling, C., 1995. Numeration systems and fractal sequences. Acta Arith., 73: 103-117.

Kimberling, C., 1997. Fractal sequences and interspersions. Ars Combinatoria, 45: 157-168.

Sloane, N.J.A., 2010. The on-line encyclopedia of integer sequences. http:// www.research. att.com/~njas/ sequences/Seis.html.