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Reconfigurable Stewart Platform for Spiral Contours

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Abstract: Cutting with machine tools is an expensive activity, in which even little progress achieved in technology can produce great results. The problem of low stiffness to weight ratio of the rectangular-box-like machine tools has been partially solved with the introduction of Stewart platform. Having six degrees of freedom it executes the positioning of the end-effector in space. The objective of this study is to analyze stiffness of 3-3 Stewart platform using the mathematical model developed based on inverse kinematics for Spiral contours. The set of positions and orientations to be followed by the tool for different contours in terms of the moving platform for maximum stiffness are synthesized. Two planar contours are taken for the purpose of the analysis. The results obtained would serve as a tool in the hands of a design engineer to model the often ignored parameters. This method provides us with set of positions and orientations of the moving platform that should be executed throughout the machining process for a given contour in order to have maximum stiffness of the Stewart platform. The results obtained are to be validated for singularity-free path planning and attempt has to be made to estimate the computational efficiency of the algorithm.

Key words: Spiral contours, stewart platform, reconfiguration, logarithmic spiral, archimedian spiral, maximum stiffness trajectory

INTRODUCTION

The reason why a re-configurable Stewart platform with a variable geometry has not as yet been successfully implemented, is the lack of an efficient methodology for determining the optimum geometry for the prescribed task at hand (Du-Plessis and Snyman, 2006; Wavering, 1998; Zhang and Bi, 2006). The original problem being the development of a reconfigurable Stewart platform suitable for various applications; contour generation and vibration isolation are the two applications chosen in this study.

From the time the potential of Stewart platform as a six DOF mechanism was identified by Stewart (1965) for flight simulation, researchers have constantly tried the Parallel Kinematic Machines (PKM) in various other fields. Combining the advantages of PKM and industrial robots to solve the problems of Machine tool applications, a new concept for machine tool application in the name of Stewart platform was identified (Dasgupta and Mruthyunjaya, 2000).

Presently commercial hexapods, such as Octahedral Hexapod from Ingersoll Milling Machine, Geodetic from Geodetic Technology Ltd., are mission specific and there is not much modular capabilities allowed for the end-user to choose between structural rigidity and dexterity (Ji and Song, 1998; Ji and Li, 1999; Xi, 2001). Modular systems will enable technology obsolescence to be countered, new technologies to be exploited and changing operational demand to be supported, in a cost-effective way. The added advantages of having modular configurations are (I-Ming, 2001):

- Shortening of system development cycle
- Rapid design, fabrication and low cost
- Meeting the changing operational demand and less maintenance

The need now is a reconfigurable configuration of Stewart platform developed with at least two applications in mind. While facing the demand for reconfigurability much of research needs to be done. This study presents the initial investigations done for the contour generation application only, from the ongoing research on the problems mentioned beforehand.

Stewart platform has been studied for possible use in multi-axis machine tools for quite some time

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(Du-Plessis and Snyman, 2006; Dasgupta and Mruthyunjaya, 2000; Gosselin and Angeles, 1990). Figure 2 shows Stewart platform as a machine tool. Later it was proposed to place the Hexapod in the design office along with the plotter (Matar, 1999) which would reinforce the ethos of a responsive and robust machine. It would enable us to adapt to a variety of modeling demands without incorporating large set-up overloads. A multi-axis structure enables the tool cutting path to take the most efficient route for maximum clearance which would equate in real time saving up to 20% in machining.

A lot of research work (El-Khasawneh and Ferreira, 1999; Tsai, 1999; Svinin *et al*., 2001; Huang and Schimmels, 1998) has taken place in order to model the stiffness of the Stewart platform. The stiffness variation for the straight line and circular contours (Pugazhenthi *et al*., 2002) was studied and was found to yield good results based on the dynamics of the Stewart platform. The stiffness increases as the height of the tool path increases and is independent of its location in the x-y plane. For complex tool paths involving non-planar cuts this is not valid. The model needs to be amended to suit the demands of the complex tool paths. Apart from that the ill-posed nature of inverse problem the noise of the observed data leads to deviations in solutions (Jing-Lei and Zhi-jian, 2011). So a thorough study on the stiffness of the Stewart platform is performed for different contours including space contours like Logarithmic spiral. The developed mathematical model for the maximum stiffness of Stewart platform is implemented for different trajectories within the workspace and the trajectory with maximum stiffness for different contours are identified. The leg length variations for each trajectory were found to validate the exactness of the algorithm in determining the maximum stiffness trajectory. With a continuously data moving object management systems have been attempted, though precisely mot applicable to this research problem (Ghajary and Alesheikh, 2008).

A mathematical model was also constructed to realize the automatic positioning of the workpiece (Wang *et al*., 2001). This solves the problem of identifying the pose with which a workpiece should be presented however, if stiffness is considered, the problem still remains in the form of the machine tool posture. So, an attempt is made to develop a stiffness model based on the kinematics of the Stewart platform. This brings down the complexity of the stiffness analysis and saves considerable amount of time from being built into the algorithm. Kumar *et al*. (2005, 2009) have presented a similar analysis for various contours. The developed model is now used to study the parameters affecting reconfigurability of the Stewart platform. Of the various parameters studied the effect of 'configuration' is presented in this study.

Spiral trajectory: Archimedes studied the first and simplest spiral and it bears his name: the Archimedean spiral. An Archimedean spiral trajectory is shown in the Fig. 1. It is generated when a traveling object P moves at constant speed v on a pole that in turn, rotates uniformly around one of its points, at angular speed w. The importance of the spiral trajectory in machine tool applications is predominant in the case of pocket machining (Bieterman and Sandstrom, 2003).

The problems faced with the use of conventional machining tool path or trajectory, to mention a few:

- Machine axis drive capabilities get concentrated near corners and other high-curvature path segments
- This results in more required machining time, unnecessary cutting tool wear and wear and tear on the whole machine

If the initial position of the point on the pole (measured from rotation center is indicated with r_0 and the initial angle with q, the expressions become (Matar, 1999):

$$q = q_0+wt \qquad (1)$$

$$r = r_0+vt \qquad (2)$$

From Eq.1 and 2, deriving t from the second equation and replacing it in the first one, the equation of the curve described by the object becomes

$$r = r_0+v/w\ (q-q_0) \qquad (3)$$

where, r represents the distance from the rotation centre and q the angle counted starting from the initial position.

Figure 2a shows the conventional parallel-offset tool path. Using smooth low-curvature contours for the tool path solves this problem. Spiral trajectory stands as the best candidate in this case. Lower curvature distributes available acceleration along the path and decreases machining time. Figure 2b shows the Spiral tool path in pocket machining. Saving wear and tear on machine and tool is the additional benefit obtained which gives extended tool life when cutting hard metals.

This novel method of introducing the low-curvature contours is made practical only with the help of six Degrees of Freedom (DOF) mechanisms like Stewart platform. Following a similar analysis for the simulation as done for other trajectories, the moving platform is allowed to follow a spiral path represented by Eq. 4. Since the radius n (θ) and the angle θ are proportional for the simplest spiral (Qualls and Pimentel, 1999), the Spiral of Archimedes, the expression developed for displacement along the spiral trajectory is given by:

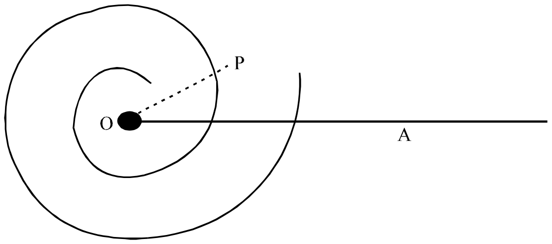
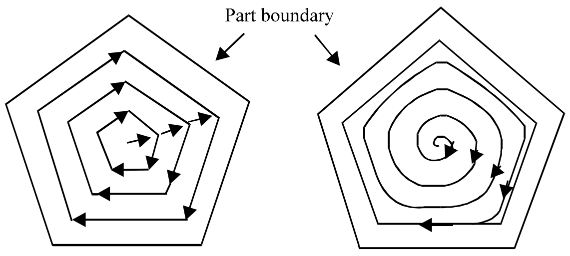


Fig. 1: Spiral of archimedes



(a)

(b)

Fig. 2: Conventional and curvilinear tool paths

$$\Delta x = (p_x + n^* \cos \beta,\ p_y + n^* \cos \beta,\ p_z,\ \psi,\ \theta,\ \phi) \qquad (4)$$

where, n - a * β (a - constant) β - the cutter angle position with respect to the tool cutter (i.e., β = (feed rate/radius) * time).

A dataset of the positions and orientations of the centroid of the moving platform is obtained as a result of simulation. It should be executed throughout the machining process for a given contour, in order to have maximum stiffness for the Stewart platform. It also provides the trajectory for the spiral contour avoiding singular positions. The position and orientation vector of the start point for maximum stiffness trajectory is:

$$p_x = 0.5000;\ p_y = 0.8660;\ p_z = 1.0000;$$
$$\psi = -15.0000;\ \theta = 10.0000;\ \phi = 5.0000;$$

Maximum stiffness value obtained is = 3.0048e+004 N m^{-1} for Tsai 6-6 platform. Since this is an offline process the machining trajectory with maximum stiffness could be acquired from the database of trajectory parameters. An analysis of the starting points of the maximum stiffness trajectory would provide us an idea of the relation between the starting points and the maximum stiffness. This would help us to solve any positioning problems within the workspace. The notations used in this section are tabulated in Table 1.

Influence of configuration: Configuration is chosen as one of the parameter to study the reconfigurability of the Stewart platform. Four different platforms are chosen for the analysis: Tsai 6-6, Tsai 3-3, PeilPOD 6-6 and PeilPOD 3-3. Figure 3 and 4 give the stiffness variation of the

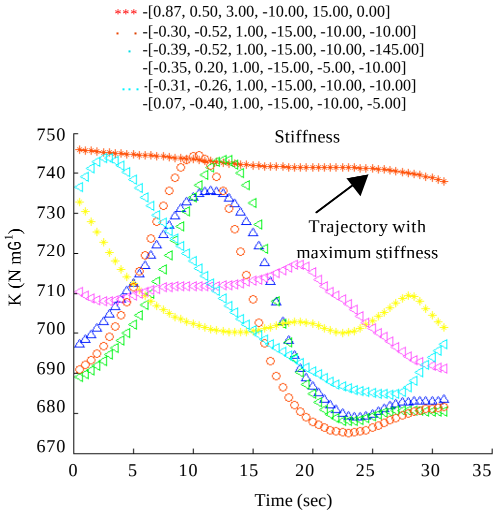


Fig. 3: Stiffness variation for spiral trajectory (Tsai 6-6)

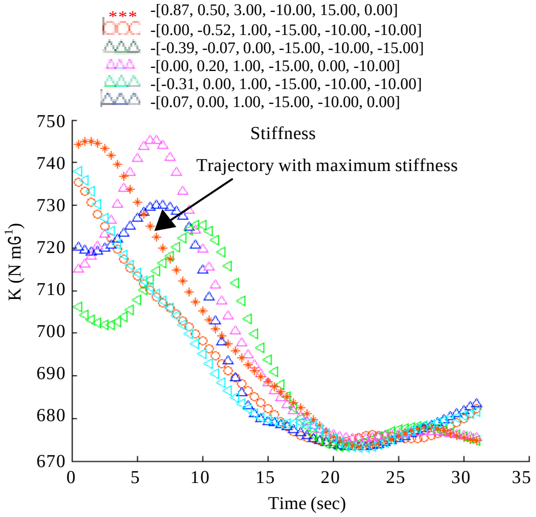


Fig. 4: Stiffness variation for spiral trajectory (Tsai 3-3)

Table 1: Notations

Symbol	Description
p_x	Linear displacement in x direction
p_y	Linear displacement in y direction
p_z	Linear displacement in z direction
ψ	Angular displacement in x direction
θ	Angular displacement in y direction
ϕ	Angular displacement in z direction
Δx	Displacement of the centroid of the moving platform
R	Horizontal distance between the centroids of the top and bottom platforms

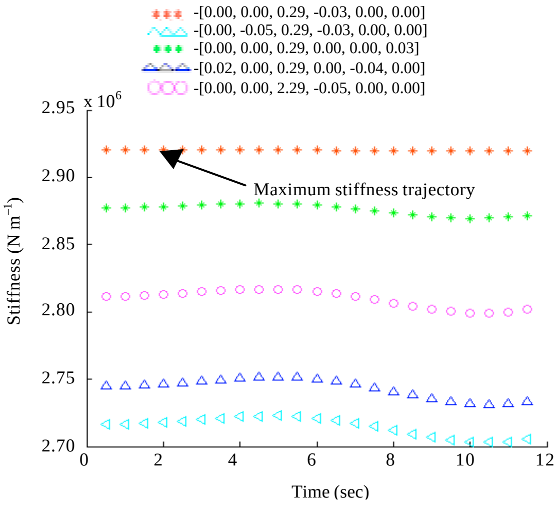


Fig. 5: Stiffness variation for spiral trajectory (PeilPOD 3-3)

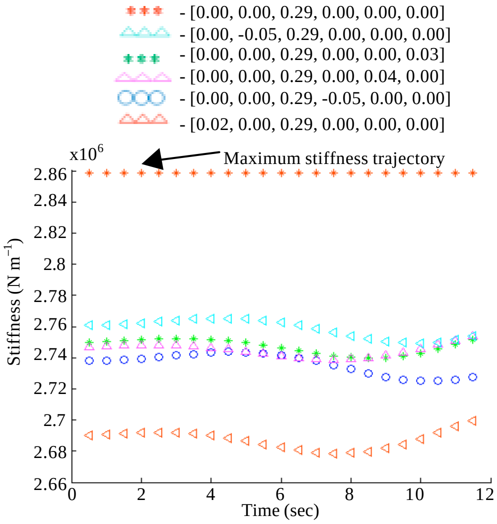


Fig. 6: Stiffness variation for spiral trajectory (PeilPOD 6-6)

maximum stiffness trajectories for Tsai 6-6 and Tsai 3-3 platforms, respectively. The variation of stiffness within the trajectory of maximum stiffness is more for Tsai 3-3 platform.

Similar analysis is done for the PeilPOD configurations also and the results are presented in Fig. 5 and 6. The parallel nature of the curves in the plots for the stiffness and leg length displacement is the characteristic of the 3-3 platform for trajectories based on space curves. Table 2 provides a comparison for the stiffness of the maximum stiffness trajectory, for different configurations. It is observed that the 6-6 configuration performs better than the 3-3 configuration in both cases, though the difference is not more than 4%.

Table 2: Comparison of maximum stiffness for archimedian spiral contour

Platforms	Maximum stiffness (kN m^{-1})
Tsai 6-6	30.048
Tsai 3-3	29.409
PeilPOD 6-6	137210
PeilPOD 3-3	130830

Table 3: Comparison of maximum stiffness for the logarithmic spiral contour

Platforms	Maximum stiffness (kN m^{-1})
Tsai 6-6	46.777
Tsai 3-3	41.275
PeilPOD 6-6	145810
PeilPOD 3-3	136100

Logarithmic spiral: Logarithmic spiral, also known as equiangular spiral, is taken for analysis as the next case. The characteristic feature of this curve is that each line starting in the origin cuts the spiral with the same angle. In parametric form, provided by Koller (2002), it is represented by $x(\theta) = \exp(\theta)\cos(\theta)$, $y(\theta) = \exp(\theta)\sin(\theta)$. For logarithmic spiral trajectory the displacement of the centroid of the moving platform is obtained as:

$$\Delta x = (p_x + \exp(r)\cos(r),\ p_y + \exp(r)\sin(r),\ p_z,\ \psi,\ \theta,\ \phi) \quad (5)$$

Where:

- r -$a * \beta$
- β - cutter angle position with respect to the tool cutter (i.e., β = (feed rate/ radius) * time).
- α -a constant

Table 3 provides a comparison for the stiffness of the maximum stiffness trajectory, for logarithmic spiral contour. Upon comparison with this result obtained for Archimedian spiral it is observed that the logarithmic spiral performs better than Archimedian spiral.

It is observed that 6-6 configuration performs better than 3-3 configuration in both cases, though the difference is not more than 4%. So the designer can prefer 3-3 configuration unless the loss of 4% stiffness over the 6-6 configuration is accountable, for a gain on the reconfigurable capability of a developed Stewart platform and increased workspace.

The length of the spiral trajectory taken for simulation is 66 mm at a feed rate of 1.33 mm sec^{-1} for Tsai platforms. The length of the trajectory for PeilPOD platforms is taken as 25 mm at the same feed rate. The simulation time for Tsai and PeilPOD platforms are

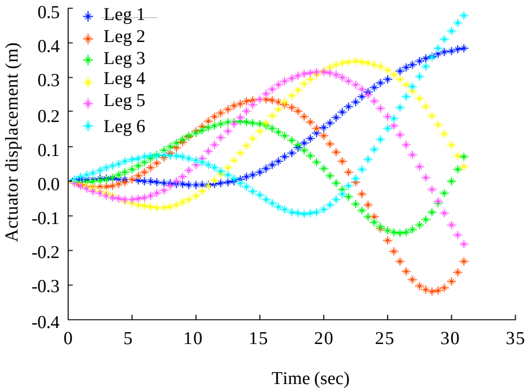


Fig. 7: Actuator displacement for spiral trajectory (Tsai 6-6)

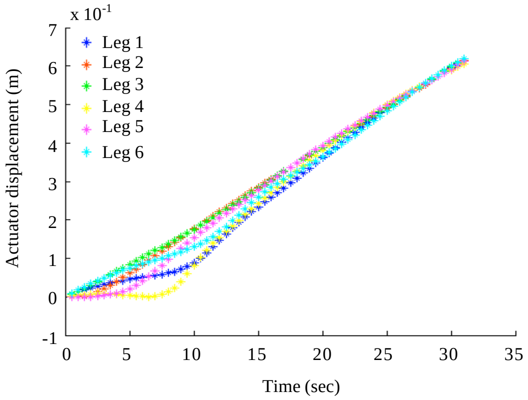


Fig. 8: Actuator displacement for spiral trajectory (Tsai 3-3)

31 and 12 sec, respectively. It could be seen that the maximum stiffness trajectories have minimum stiffness variation along the trajectory, except in Fig. 4. These variations are attributed to the occurrence of singular configurations along the trajectories. This also shows the importance of the stiffness analysis of different contours to be treated individually for different platforms, rather than generalizing the results. A matching trend is observed in the actuator displacement plot for Tsai 3-3 configuration shown in Fig. 8 which is unique in comparison to the plots of other platforms. This happens in order to compensate for the loss of stiffness when singularity is encountered.

Similarly Fig. 7 and 8 provides actuator displacements of Tsai 6-6 and Tsai 3-3 platforms, respectively, for the maximum stiffness trajectory. The actuator force requirements for the maximum stiffness trajectory could be deducted from it using the force-displacement correlation.

Fig. 9: Developed stewart platform (PeilPOD 3-3)

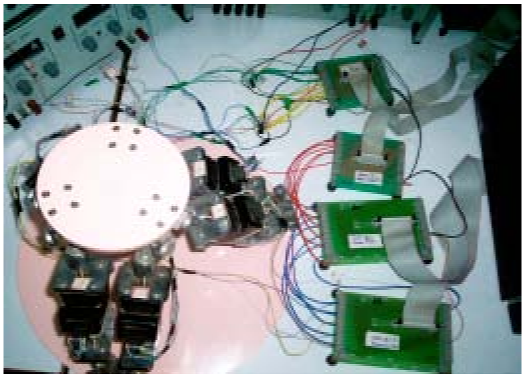


Fig. 10: Connections for PeilPOD 3-3

All the plots follow the same trend for the spiral trajectory with exception of Fig. 8 although with varying magnitudes for all the legs. The actuator displacement plot for Tsai 3-3 shown by Fig. 8 following the corresponding stiffness plot shows continuous increase in the displacement. This happens in order to compensate for the reduction in stiffness within its workspace. Similar results are obtained for the PeilPOD configurations too.

The effect of second chosen parameter (angle-between-legs) is currently being simulated to improvise the modular capabilities of the Stewart platform. The dataset obtained offline from simulation needs to be analyzed for the workspace demand it places on the platform. More information on the reconfiguration parameters could be obtained from Kumar *et al*. (2009).

Experimental setup: A Stewart platform (PeilPOD3-3) was developed at IIT Madras to conduct initial studies on the parameters affecting the modularity. Figure 9 shows the developed Stewart platform, a 3-3 configuration with spherical joints at the top and the bottom platform. The actuators used are micro-stepper linear actuators with micro-stepping drive and onboard electronics. Figure 10 shows the connections for the experimental investigations to be conducted. The drive's integrated electronics eliminates the need to run the motor cabling through the machine, reducing the potential for problems due to electrical noise.

CONCLUSION

A thorough study on the stiffness of Stewart platform is performed for different contours. The developed mathematical model for maximum stiffness of Stewart platform is implemented for different trajectories within the workspace and the trajectory with maximum stiffness for different contours are identified. Leg length variations for each trajectory were found to validate the exactness of the algorithm in determining the maximum stiffness trajectory.

This method provides us with set of positions and orientations of the moving platform that should be executed throughout the machining process for a given contour in order to have maximum stiffness. The results obtained are to be validated for singularity-free path planning and an attempt has to be made to estimate the computational efficiency of the algorithm.

In general the Tsai platform has relatively lesser stiffness when compared to the PeilPOD owing to its longer leg lengths. For most of the contours the 6-6 configuration fares better than the 3-3 configuration. The spiral trajectory which has influenced the manufacturing industries for the benefits, mentioned before is given special treatment in this research work. Archimedian and Logarithmic spirals are compared for stiffness performance. Logarithmic spiral performs better than Archimedian spiral. It is observed that 6-6 configuration performs better than 3-3 configuration in both cases, though the difference is not more than 4%. So the designer can prefer 3-3 configuration unless the loss of 4% stiffness over the 6-6 configuration is accountable, for a gain on the reconfigurable capability of a developed Stewart platform and increased workspace.

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