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Stress Intensity Factor for Cracks Emanating from a Shaft

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Abstract: Shaft is a rotational body used to transmit power or motion. Due to cyclic loading conditions, surface cracks frequently grow in the shaft. Normally these cracks will propagate with a semi-circular shape and cause premature failure to the whole system. In order to prevent the catastrophic incident, there must be a monitoring system that provides an early warning during the operation of the shafts. The semi-analytical or experimental method applied previously had its own limitations and disadvantages. Hence, there is a great need to determine the stress intensity factor in cracked shafts by using finite element method. The objective of this study is to determine the stress intensity factor for cracks emanating from a shaft by using finite element method and also to verify the finite element results with those obtained semi-analytically. The scope of this study is focused on the semi-circular crack and the crack loadings considered are Mode I and Mode III. This study is divided into two phases. For the first phase, modeling of the cracked shaft was carried out in ANSYS software, while for the second phase; the results obtained from the finite element analysis were compared with those obtained semi-analytically. The relationship between dimensionless stress intensity factor and the normalized relative crack depth is presented in the results and discussion section. The results obtained numerically and semi-analytically had been compared and the deviation in term of percentage is relatively small. In conclusion, the stress intensity factor of a cracked shaft determined by the numerical method was verified to be accurate.

Key words: Stress intensity factor, finite element method, semi-circular crack, mode I and mode III

INTRODUCTION

Shaft is a rotational body used to transmit power or motion. It provides the axis of rotation for gears, pulleys, flywheels and etc. Due to cyclic loading conditions, i.e. axial, bending and torsional load, surface cracks or flaws frequently grow in the shaft. Normally these cracks will propagate with a semi-circular or semi-elliptical shape. If these surface cracks or flaws reached their critical stage, the cracks will expand at the speed of sound and cause undesirable catastrophic failures.

The crack in a body can be subjected to three different types of loading that indicate in Fig. 1. These load types can be categorized as Mode I, Mode II and Mode III. For mode I (opening), the load is applied normal to the crack plane and tends to open the crack. This alternation of shape is the most important mechanism which directly affects the failure of homogeneous or isotropic materials (Sanford, 2003). Mode II refers to in-plane shear loading or sliding. Mode III corresponds to out-of-plane loading or tearing (Dally and Riley, 1991). The failures in rotor shafts are mainly due to cyclic Mode I loading combined with steady Mode III loading.

Since the cracks propagate lightning fast when they reached critical stage, there must be a monitoring system that provides an early warning to the plant operator before catastrophic incident happen. In order to ensure

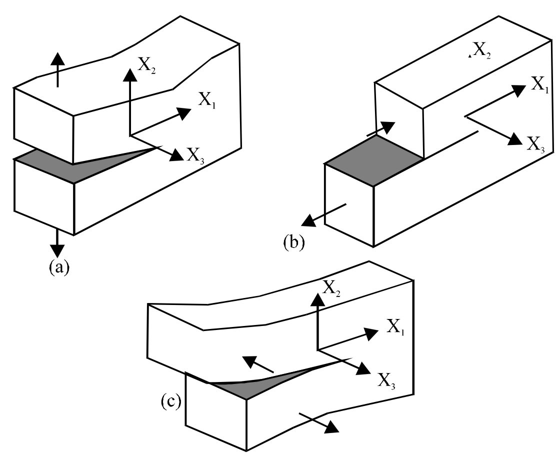

Fig. 1: Three types of loading on a cracked body (a) Mode I (b) Mode II (c) Mode III

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the safety of the shafts, engineers are always required to perform an assessment on the cracked shafts. This assessment was to calculate the stress intensity factor (SIF), K, and to check the safety level of the cracked shafts. SIF gives more accurately description of the stress state near the tip of a crack as compared to stress concentration factor.

Due to time constraint and economic purposes, the engineers always search for the fastest way to calculate the SIF. The semi-analytical or experimental method applied previously had its own limitations and disadvantages. For example, the semi-analytical method is only applicable to simple geometry. While for the experimental method, the cost to determine the SIF is considered far too expensive. At the same time, it is impossible to test all the cracked models experimentally since they require expensive set-up cost.

In order to solve both problems above, SIF determination by using numerical method is preferred in the 21st century. This is because the analyzing time reduces dramatically without sacrificing the accuracy of the results. By using numerical method, engineers or designers can perform any kind of simulations on the cracked shaft with different crack parameters relatively easily. Meanwhile, a faster decision can be made when solving the cracked shaft's problem. Hence, the objective of this study is to determine the SIF for cracks emanating from a shaft by using finite element method and also to verify the finite element results with those obtained semi-analytically. Meanwhile, the scope of this study is focused on the semi-circular cracks on the shaft and the loading conditions considered are Mode I (opening) and Mode III (tearing/torsion) crack loading.

REVIEW OF NUMERICAL METHOD

A brief review of the work by a number of researchers in the field of fracture mechanics is presented here. In presented a paper (Fonte and Freitas, 1999) which is related to the SIF for semi-elliptical surface cracks in round bars under bending and torsional loading. The Stress Intensity Factor (SIF) for semi-elliptical surface cracks subjected to Mode I and Mode III loading was found by using a three dimensional finite element method. The relative difference for shorter cracks depths was between 2 to 5% if compared to the literature value. While for the larger crack depths, the difference was between 12 and 15% and this value was still considered acceptable. Besides, the results showed that at maximum crack depth, the pure Mode III exists and had the highest value. This result has significant impact on the crack growth rate predictions of the semi-elliptical surface cracks in round bars under bending and torsional loading.

Shih and Chen (2002) had carried out a study which was related to SIF of an elliptical cracked shaft The numerical model of a round bar was evaluated by collapsed singular element with detailed mesh on crack front. The mesh of the three-dimensional finite element model of cracked bar is constructed by employing 20-node regular and collapsed singular element as illustrated in Fig. 2. For this study, the ratio of crack depth to shaft diameter was considered in the range between 0.1 and 0.6. Besides, there were some assumptions being used in conducting this study. For example, only the Mode I (opening) fracture is considered and the round bar is made of homogeneous, isotropic and linear elastic material.

Vaziri and Nayeb-Hashemi (2005) studied the effect of crack surface interaction on the SIF in Mode III crack growth in round shafts. In this paper, the effective SIF in circumferentially cracked round shafts has been evaluated for a wide range of applied torsional load. Considering a pressure distribution between mating fracture surfaces did this evaluation. The results showed that the pressure profile not only depends on the fracture surface roughness, but also depends on the magnitude of the applied Mode III loading.

Fonte *et al.* (2006) carried out a research on the effect of steady torsional loading on fatigue crack growth in shafts. In this paper, long cracks growth tests have been carried out on cylindrical specimens made of DIN Ck45k steel and two types of testing was accomplished. The testing was rotary or alternating bending combined with steady torsion in order to simulate the real conditions on power rotor shafts. The cylindrical specimen surface was measured for several loading conditions to understand the growth and the shape evolution of semi-elliptical surface cracks. A three dimensional finite element analysis was used to obtain the Mode I and the Mode III SIFs along the front of semi-elliptical surface cracks in shafts. The surface crack in this study was on a normal plane to the axis of the shaft and the mesh explain in Fig. 3. The symmetry conditions are not valid since the presence of torsional loading. The SIFs results of pure bending were compared with the available results. While for the surface cracks in round bars subjected to torsion loading, no comparison was made since there were no available results.

Lissenden *et al.* (2007) studied the relationship between cracks, which were propagated due to bending loads, and the torsional stiffness of the shaft. They assumed that these cracked stainless steel shafts are susceptible to fatigue cracking when run under near-continuous operation. A 3-D finite element model of a shaft section with a crack has been used to predict the effect of a crack on the shaft's stiffness. For the finite

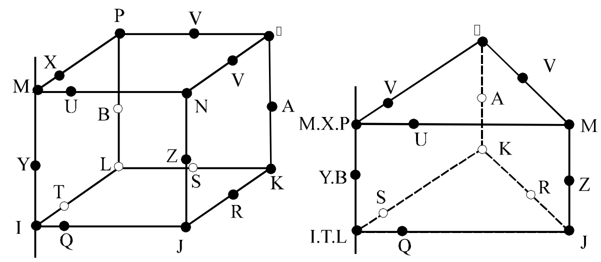

Fig. 2: Three-dimensional crack front element

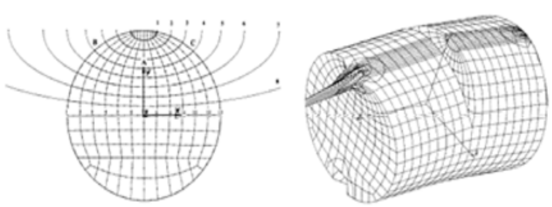

Fig. 3: Finite element mesh

element analysis, thirteen different cracks depths ranging from 0 to 1.3R were analyzed. One end of the shaft is restrained in the z-axis while the other end is constrained except for rotation about z-axis. This model's crack size was varied with crack depth and had approximately 4220 elements and 16850 degrees of freedom. A mesh convergence study on the model having the deepest crack indicated that this model is sufficiently refined.

Most recently in Toribio *et al.* (2009) presented a study on the numerical modelling of crack shape evolution for surface flaws in round bars under tensile loading. They have studied how the aspect ratio (relation between the semi-axes of the ellipse) changes with the relative crack depth. The model used in the study explain in Fig. 4. A computer program in Java Programming language was developed to determine iteratively the geometric evolution of the crack front when the round bars were subjected to tensile loading. Few assumptions were made for this study, amongst others were the basic hypothesis of the modelling that consisted a crack shape of an ellipse which centre is located at the bar surface.

GENERAL EQUATIONS

The three relevant general equations used to calculate the Stress Intensity Factor (SIF) of a cracked shaft under axial, bending and torsional load were obtained from the stress and strain handbook (Pilkey, 2005). Since the modelling of a cracked shaft by finite element method is of the same conditions as in the handbook, it is reasonable to assume that the results obtained from both methods should be the same.

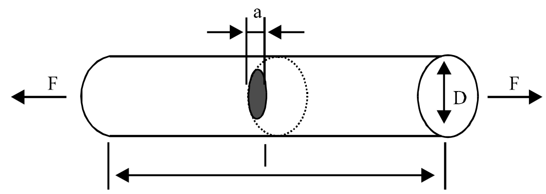

Fig. 4: Semi-elliptical crack in round bars under tensile loading

Figure 5 shows a cracked shaft under (a) axial load, (b) bending and (c) torsional load. The SIF, K_I, of a cracked shaft under axial load using our nomenclature is:

$$K_I = \sigma_N \sqrt{\pi c} F_{In}(a/b) \tag{1}$$

where, σ_N is the normal stress, a is the radius of uncracked region, b is the shaft's diameter, c is the crack width (c = b − 2a) and F_{In} (a/b) is the boundary correction factor (refer Appendix 1).

Whilst the SIF of a cracked shaft under bending load and torsional load, is given by:

$$K_I = \sigma_b \sqrt{\pi c} F_{Ib}(a/b) \tag{2}$$

$$K_{III} = \tau \sqrt{\pi c} F_{III}(a/b) \tag{3}$$

where σ_b is the bending stress, τ is the shear stress, c is the crack width, F_{Ib} (a/b) and F_{III} (a/b) are the boundary correction factors for cracked shaft under bending load and torsional load respectively (refer Appendix 1). The formulation for the normal stress, σ_N, bending stress, σ_b and the shear stress, τ_is also included in Appendix 1.

MODELLING PROCESSES

Three sets of models were created in ANSYS to simulate the cracked shaft under axial, bending and torsional load. These finite element models have been assigned the same material properties, i.e. the material used was steel which have the modulus of elasticity of 206 GPa and Poisson's ratio of 0.3. Solid95 was assigned for the elements of the three sets of models. For each loading condition, a total of 9 models had been created, with c/b ratios varying from 0.1-0.9 with the increment of 0.1. The shaft length and diameter were fixed at 100 mm and 10 mm respectively.

Pre-processing phase: A solid shaft was created and followed by a crack which was located on the surface of

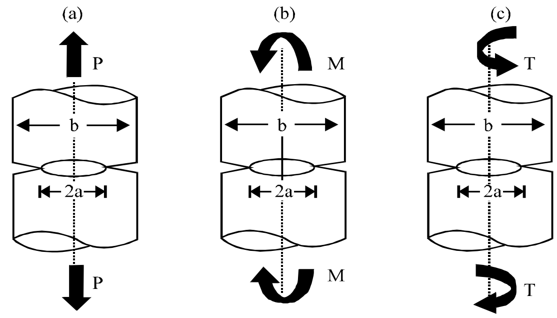

Fig. 5: Cracked shaft subjected to (a) axial load (b) bending load (c) torsional load

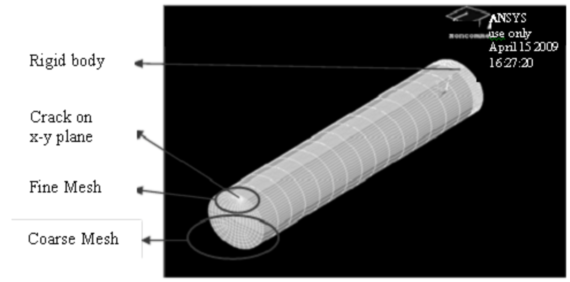

Fig. 6: Cracked shaft under axial load

the shaft. For the modelling of a cracked shaft under axial and bending load, only half of the solid shaft was modelled due to symmetrical shape and loading in the z-direction. While for the cracked shaft under torsional load, modelling of the whole shaft was necessary due to unsymmetrical loading condition. In addition, a rigid body was created at the end of the cracked shaft to distribute the loads evenly on the cracked shaft.

Extremely fine mesh was employed near the crack tip since the study of stress intensity factor was focused on it, whilst coarse mesh was employed for the part far away from the crack tip explain in Fig. 6. These meshing criterions were applied for all loading conditions.

The next step was the application of correct boundary conditions for the models. For the cracked shaft under axial load, the displacement in the z-direction must be set to zero since the shaft is symmetrical in z-direction and the force was applied in z-direction. The rotation in x and y-axis must be set to zero to prevent any rotation in x and y-axis. Fig. 7 explain the force being applied in negative z-direction at the end of the shaft in order to create an axial load.

For the cracked shaft under bending load, the displacement in x and z-direction must be set to zero since the shaft is symmetrical in z-direction and the force was applied in x-direction. At the same time, the rotation in x and y-axis must be set to zero to prevent any rotation in x and y-axis. Figure 8 indicates the distributed forces being applied in negative x-direction at the end of the shaft in order to create a bending moment.

For the cracked shaft under torsional load, the displacement in x, y and z-direction at one end were set to zero together with the rotational constraint in x, y and z-axis. In order to create the torque, forces at four different locations at the other end of the cracked shaft were applied explain in Fig. 9.

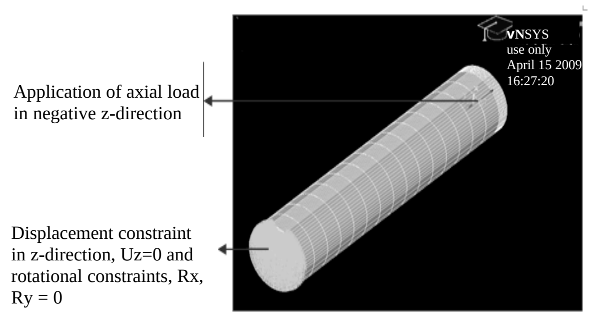

Fig. 7: Boundary conditions for a cracked shaft under axial load

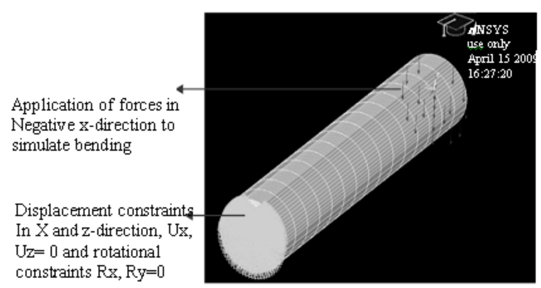

Fig. 8: Boundary conditions for a cracked shaft under bending load

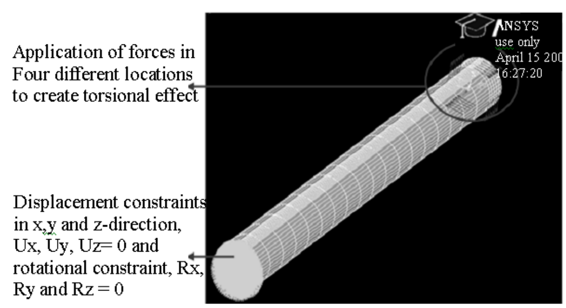

Fig. 9: Boundary conditions for a cracked shaft under torsional load

Processing and post-processing phase: During the Processing Phase, a series of calculations were performed by using the ANSYS software. In the Post-Processing Phase, the stress intensity factor (SIF) for the cracked shaft under axial, bending and torsional load was obtained by using the KCAL command in the ANSYS software. The path operation was used to select 3 nodes at the crack tip. These 3 nodes results were then extrapolated by ANSYS software and the SIF value was calculated. A convergence study was also carried out in order to show that the mesh was sufficiently refined and the results obtained from the numerical modelling are accurate.

RESULTS AND DISCUSSION

Figure 10 explain the comparison of the Mode I stress intensity factor obtained by using the numerical (dotted points) and semi-analytical (solid line) methods.

The formulation for K_0 is included in Appendix 1. For the cracked shaft under axial load, the deviation of results in term of percentage is in the range of minimum 0.4% to the maximum of 3.2%. These deviations are considered acceptable since the maximum difference is less than 5%.

Figure 11 shows the results comparison for the cracked shaft under bending load. The deviation of the results in term of percentage is in the range of minimum 0.1% to the maximum of 4.3%. These deviations are considered acceptable since the maximum difference is less than 5%.

The Mode III stress intensity factors comparison is as in Fig. 12. For this case, the deviation of the results in term of percentage is in the range of minimum 0.3% to the maximum of 2.7%. These deviations are considered acceptable since the maximum difference is less than 5%.

The graphs above show that the results obtained from the FEA and the semi-analytical methods are close to each other. In conclusion, the results obtained from the numerical method were verified to be correct.

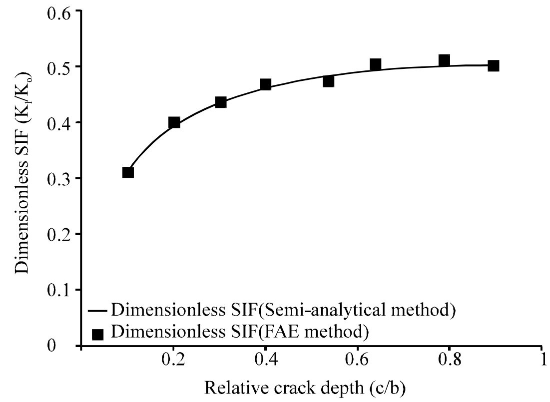

Fig. 10: Dimensionless SIF versus normalized relative crack depth under axial load

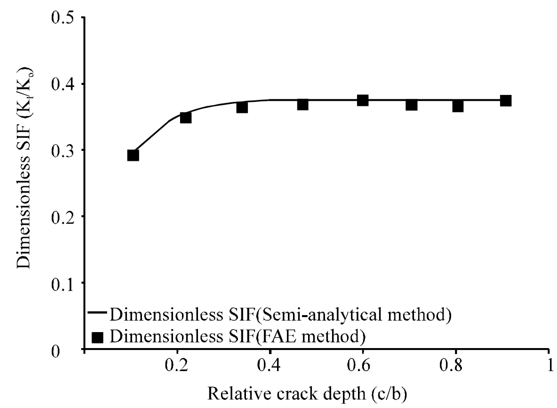

Fig. 11: Dimensionless SIF versus normalized relative crack depth under bending load

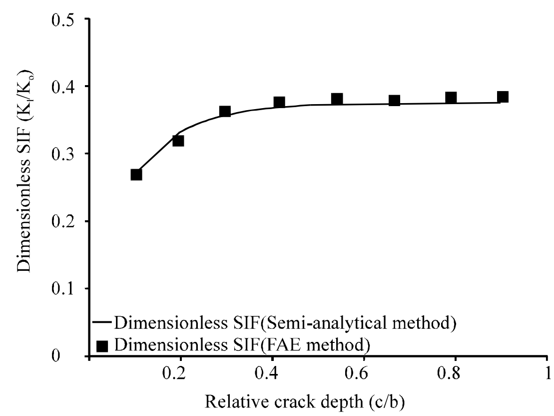

Fig. 12: Dimensionless SIF versus normalized relative crack depth under torsional load

CONCLUSIONS AND RECOMMENDATIONS

For this study, the cracked shafts were subjected to axial, bending and torsional load respectively. These three sets of calculations were performed by using semi-analytical and numerical method respectively. For the semi-analytical method, the stress intensity factor (SIF) of a cracked shaft was calculated by using the general equations obtained from the stress handbook. While for the numerical method, three sets of modelling under axial, bending and torsional load were modelled successfully using FEA software, ANSYS. By comparing the results obtained semi-analytically and numerically, the deviation in term of percentage had been found to be relatively small. In conclusion, the SIF of a cracked shaft determined by using the numerical method were verified to be accurate. By using the numerical method, it significantly shortens the SIF determining time and save money for complex geometry.

There are several pieces of work which merit further study in order to get a better understanding about a cracked shaft condition. The recommendations prompted by this study are to analyze the cracked shaft under dynamic load. In a real-world condition, shafts are normally operated under dynamic load instead of static load which was applied in this study. Hence, it is necessary to analyze a cracked shaft under dynamic load in order to get a better understanding or perspective of it.

APPENDIX 1

$$F_{In}(a/b)=0.5\sqrt{1-\frac{2a}{b}}\left[1+\frac{1}{2}\frac{2a}{b}+\frac{3}{8}\left(\frac{2a}{b}\right)^2-0.363\left(\frac{2a}{b}\right)^3+0.731\left(\frac{2a}{b}\right)^4\right]$$

$$F_{Ib}(a/b)=\frac{3}{8}\sqrt{1-\frac{2a}{b}}\left[1+\frac{1}{2}\frac{2a}{b}+\frac{3}{8}\left(\frac{2a}{b}\right)^2+\frac{5}{16}\left(\frac{2a}{b}\right)^3+\frac{35}{128}\left(\frac{2a}{b}\right)^4+0.537\left(\frac{2a}{b}\right)^5\right]$$

$$F_{III}(a/b)=\frac{3}{8}\sqrt{1-\frac{2a}{b}}\left[1+\frac{1}{2}\frac{2a}{b}+\frac{3}{8}\left(\frac{2a}{b}\right)^2+\frac{5}{16}\left(\frac{2a}{b}\right)^3+\frac{35}{128}\left(\frac{2a}{b}\right)^4+0.208\left(\frac{2a}{b}\right)^5\right]$$

$$\sigma_N=\frac{P}{\pi r^2}$$

$$\sigma_b=\frac{4M}{\pi r^3}$$

$$\tau=\frac{2T}{\pi r^3}$$

$K_o=\sigma_N\sqrt{\pi c}$ (Axial Load)

$K_o=\sigma_b\sqrt{\pi c}$ (Bending Load)

$K_o=\sigma_\tau\sqrt{\pi c}$ (Torsional Load)

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