# Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 11 (11): 1945-1951, 2011
ISSN 1812-5654 / DOI: 10.3923/jas.2011.1945.1951


# A Multi-state Reliability Model for a Gas Fueled Cogenerated Power Plant

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**Abstract:** Cogeneration plant consists of some equipment and distinguished by their structural complexity. Failure of a component can cause failure of a sub-system or the entire system with various adverse consequences such as loss of power may result in loss of production, in damage of production equipment and it may cause accidents. In such cases, the system failure can direct to reduce ability to carry out the particular task, but not to complete failure. Moreover, each system element can also perform its task with some various levels. For example, the generating unit in power system has its maximum generating capacity, which is fully obtainable if there are no failures. Some types of failure can cause complete unit outage, whereas other types of failure can cause a unit to work with reduced capacity. Therefore, reliability and availability have to be considered in the operation of power systems using multi-state system theory. Cogeneration power plant can have an arbitrary finite number of different states (task performance levels) the system is termed a multi-state system (MSS). This study assesses the reliability and availability of cogeneration power plant using Markov model associated with universal generating function takes into account multistate models for all system components.

**Key words:** Multi-state system, reliability, cogeneration power plant

## INTRODUCTION

Cogeneration power plant produces power and chilled water to meet the customer requirement. In order to meet the customer need, the plant has to perform at certain level and the equipment should be reliable. Therefore reliability assessment of the overall plant system is required to deliver the expected output and to keep the equipment in good conditions. This study focuses on study of system reliability of the plant. A Multi-state system (MSS) reliability analysis is applied for this study.

The Multi-state system was introduced in the middle of the 1970s (Murchland, 1975; Modarres *et al*., 1999; Barlow and Wu, 1978). Griffith (1980) generalized the coherence definition and studied three types of coherence. The reliability importance was extended to MSSs by Griffith (1980) and Butler (1979). An asymptotic approach to MSS reliability evaluation was developed by Koloworcki (2000). An engineering method for MSS unavailability boundary point estimation based on binary model extension was formulated by Pourret *et al.* (1999). Practical methods of MSS reliability evaluation are based on three different approaches (Aven, 1993): the structure function approach, where Boolean models are extended for the multi-valued case; the stochastic process (mainly Markov) approach; and Monte Carlo simulation. Since the Markov modeling approach can generate all possible state of a system, the number of state can be extremely large even for a relatively small number of Markov elements. Thus, Markov modeling approach must become familiar with reduction techniques that reduce the number of states. Simulation can be performed in order to assess MSS reliability. The simulation technique is also very sensitive towards the number of state in the model. It has the same problems during the model construction stage and often requires enormous computational resources during the solution stage. Universal Generating Function (UGF) which is based on algebraic procedure can reduce the problem's dimension and extremely beneficial for reliability analysis. Ushakov (Griffith, 1980) introduced the basic ideas of the UGF method in the mid-1980s. Since then, the method has been considerably expanded (Lisnianski and Levitin, 2003). In the last ten years, the UGF approach was further developed and completed by Lisniaski and Levitin for evaluating and optimizing reliability indices of multi-state systems (Trivedi, 2002; Levitin *et al*., 1998; Lisnianski and Levitin, 2003; Gnedenko and Ushakov, 1995). Therefore, this study adopts combined random process and the universal generating function (UGF) so as drastically reduces the number of state in multi state model. The universal

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generating function procedure helps to find the entire MSS performance distribution based on the performance distribution of its elements by using algebraic procedures.

## MODEL DESCRIPTIONS

**Generic model for a multi-state system (MSS):** In order to determine and analyze MSS behavior one has to know the characteristics of its components. A functional and logical order of the blocks in Fig. 2 is described by the system structure function and each block's behavior is defined by the corresponding performance stochastic process (Ushakov, 1986; Aven and Jensen, 1999; Gnedenko and Ushakov, 1995). In a multi-state analysis of cogeneration power plant used in UTP, each block of the Reliability Block Diagram (RBD) as shown in Fig. 1 indicates one multi-state element of the system. The GDC, UTP plant is designed to provide 8.4 MW of electrical power and 5300 refrigeration tons (RT) of cooling capacity to UTP. The plant consists of two gas turbine generators, each rated at 4.20 MW. For chilled water production double effect steam absorption system each rated to produce 1250 RT of cooling capacity. In addition there are four electric chiller (EC) and one thermal energy storage (TES) each rated to produce 250 RT and 1000RT/hr respectively. The reliability of block diagram of GDC is as shown Fig. 1.

The electric power production of cogeneration power plant which is currently working in universiti teknologi PPETRONAS highly depend on the performance of the gas turbines which are connected in parallel. Basically these gas turbines produced electric power directly to customer and exaust gas for chilled water production. This study focuses on the reliability and availability analysis of gas turbines for production of electricity. The functional relation and the corrsponding associated performance is shown in Fig. 2.

**States definition:** The state of each gas turbine is highly depending on the daily production performance. Subtractive clustering analysis is done to cluster the performance for each gas turbine to find the system state for 1400 operation days. The subtractive clustering method assumes each production performance data point is a potential cluster center and calculates a measure of the likelihood that each data point would define the cluster center, based on the density of surrounding data points. The algorithm of subtractive cluster (Romera *et al*., 2007) does the following:

- Selects the data point with the highest potential to be the first cluster center
- Removes all data points in the vicinity of the first cluster center (as determined by radii), in order to determine the next data cluster and its center location
- Iterates on this process until all of the data is within radii of a cluster center.

The daily production performance cluster points of gas turbines are show in Fig. 3 and Table 1.

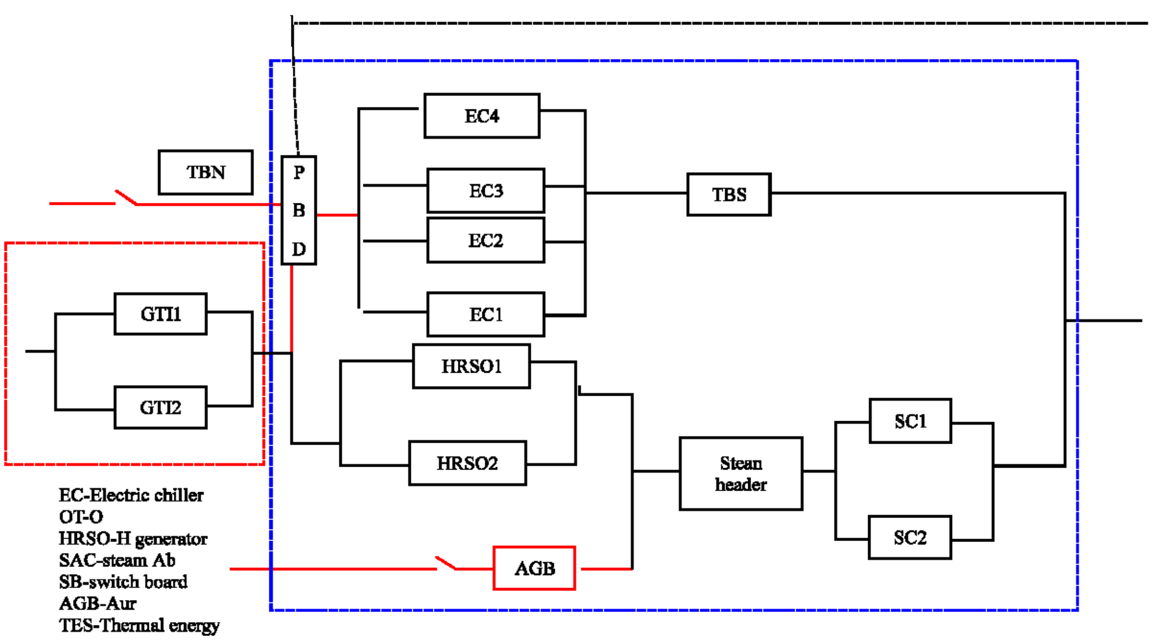


Fig. 1: System block diagram for GDC

Table 1: Performance data cluster for GT 1 and 2

| Time | Cluster 1 | Cluster 2 | Cluster3 | |
|---|---|---|---|---|
| 10 | 3188 | 3359 | 3509 | GT2 |
| 2 | 0 | 0 | 0 | |
| 11 | 3143 | 3272 | 3313 | GT1 |
| 23 | 2905 | 2248 | 2560 | |

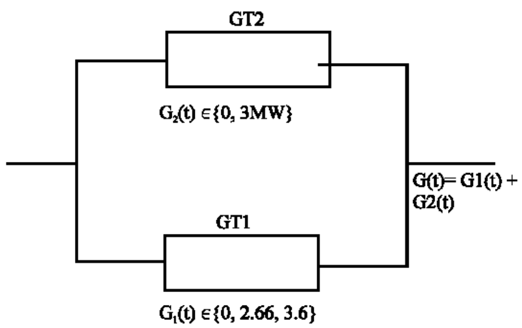


Fig. 2: RBD of gas turbine

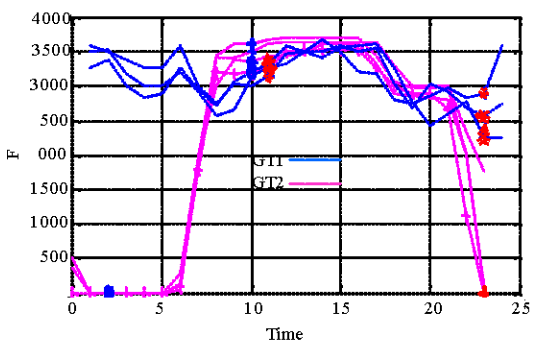


Fig. 3: Clustering production performance of Gts

Therefore based on the data as shown in Table 1, using subtractive cluster method the performance the state of the two gas turbines are determined. Gas turbine GT1 has contained three states which are complete failure, partial failure and zero failure at maximum performance level and GT2 contains only two states which are complete failure and zero failure. A state of total failure for both turbines corresponds to a capacity of 0 and the maximum operational states 3.6 MW and 3.0 Mw. GT1 has partial failure, which is the capacity of 2.6MW. The state space diagram and system state are described in the next section.

**State space diagram and determination of state probabilities:** Multi-state system was considered to have constant demand. In practice, it is often not so. A multi-state element can fall into a set of unacceptable states in two ways: either through performance degradation because of failures or through an increase in demand. If all failures and repair times are distributed exponentially then the performance stochastic process will have a Markov property and can be described by a Markov model (Trivedi, 2002). The state space diagram of the system developed as follows;

Every element state there is associated performance of the element. Minor failure and repairs cause element transition from one state to only adjacent state. As can be seen in the Fig. 4. with assumption state 1 is the best state of the system, there is transition to the state 2 from the state 1, if failure ($\lambda_1$) occurs in the state 2, and there is in transition to the state 1 ($\mu_1$), if the repair will be completed. Similarly, there will be transition from state 3 and state 5 to state 4 and state 6 respectively with failure rate of $\lambda_1$ if there is performance degradation. If state 1 and 2 fail and goes to state 3 and 4 respectively, there will be failure rate give by $\lambda_2$. Analogously if state 3 and 4 fail and go to state 5 and 6 respectively, there will be failure rate give by $\lambda_3$. If state 5 and 6 getting minor repair $\mu_3$, the states will be up in to state 3 and 4. The state of the system and state space diagram is defined. The corresponding performance $g_s$ is associated with each state s. Let $P_S(t)$, s={1,2........$k_j$} is the state probabilities of the element's performance process G(t) at time t:

$$P_S(t)=\Pr\{G_j(t)=g_{ji}\},$$
$$s=\{1,2.........K_j\};t\geq 0$$

Then the probability of each state has to be defined using Eq. 1. For a Markov process, each transition from the states to any state m (s, m=1; . . . ; k) has its own associated transition intensity designated as $a_{sm}$. In this study, any transition is caused by the element's failure or repair. If m<s, then $a_{sm}=\lambda_{sm}$, where $\lambda_{sm}$ is a failure rate for the failures that cause the element transition from state s to state m. If m<s, then $a_{sm} = \mu_{sm}$, where $\mu_{sm}$ is a corresponding repair rate. System of differential equations for finding the state probabilities $P_S(t)$, s={1,2........$k_j$} for the homogeneous Markov process is defined (3) as follows:

$$\frac{dP_{(t)}}{dt}=[\sum_{\substack{i=1\\ i\neq s}}^{k} P_i(t)a_{is}]-P_i(t)\sum_{\substack{i=1\\ i\neq s}}^{k} a_{si} \quad (1)$$

In this case, all transitions are caused by the element's failures and repairs corresponding to the transition intensities $a_{is}$ and are expressed by the element's failure and repair rates. Therefore, the corresponding system of differential equations for the power system as shown in Fig. 4 are written as:

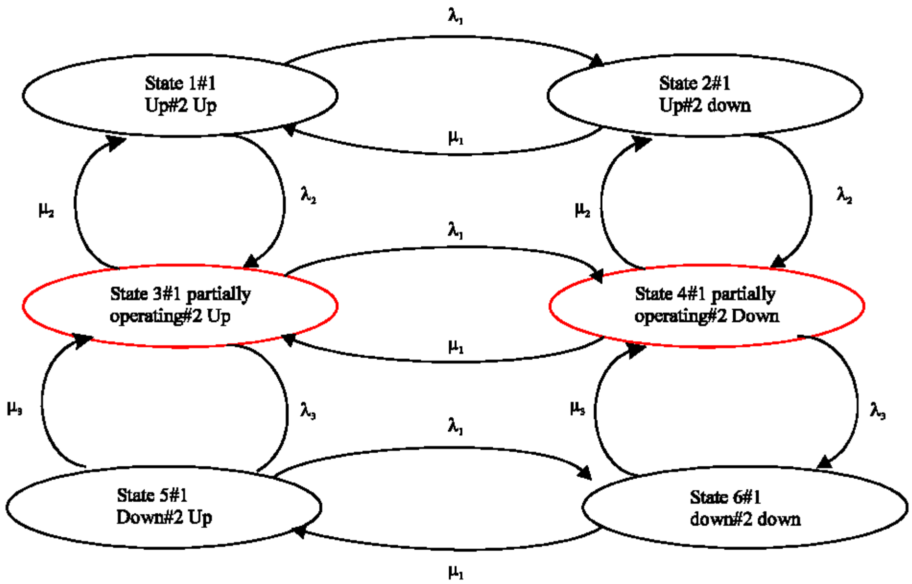


Fig. 4: Sate space diagram of the system

$$\frac{dP_1(t)}{dt}=-(\lambda_1+\lambda_2)P_1(t)+\mu_1P_2(t)+\mu_2P_3(t), \tag{2}$$

$$\frac{dP_2(t)}{dt}=-\lambda_1\ P_1(t)+\mu_2P_4(t)-(\mu_1\lambda_2)P_2(t), \tag{3}$$

$$\frac{dP_3(t)}{dt}=-\lambda_2\ P_1(t)+\mu_1P_4(t)-(\mu_2+\lambda_2+\lambda_1)P_2(t)+\mu_3P_{5(t),} \tag{4}$$

$$\frac{dP_4(t)}{dt}=\lambda_2P_2(t)+\lambda_1P_3(t)-(\mu_2+\mu_1+\lambda_3)P_4(t)+\mu_3P_6(t), \tag{5}$$

$$\frac{dP_5(t)}{dt}=-(\lambda_1+\mu_3)P_5(t)+\mu_1P_6(t)+\lambda_3P_3(t), \tag{6}$$

$$\frac{dP_6(t)}{dt}=-(\lambda_1)P_5(t)-(\mu_1+\mu_3)P_6(t)+\lambda_3P_4(t), \tag{7}$$

Assume that the initial state is the state k with the best performance. Therefore , by solving system (2-7) of differential equations under the initial condition $P_k(0)=1$, $P_{k-1}(0)=\dots\dots=P_2(0)=P_1(0)=0$, the state probabilities $P_s(t)$, s= 1,………k is obtained.

Moreover, the power output of each state is the sum of the output of each turbine which is connected in parallel as shown Fig. 2. The system state performances of the six states are indicated in Table 2.

**Model for Multi-state system reliability and its demand:** Based on state probabilities which are determined in Markov model for all elements, reliability can, in the general sense, be defined as a measure which depicts the probability of maintaining normal working of systems/components under determinate time and task conditions. Reliability is a result of the interaction between the task (demand) and the performance (capacity) in the time-varying probability space. It can be describes task and performance random variables by the UGFs firstly, enumerates state combinations by composition operators step by step, and then obtains the reliability of systems/ components finally.

Table 2: System state and performance

| System state | State of the elements | Systemperformance $\phi(G_1,G_2)=G_1+G_2$ |
|---|---|---|
| 1 | $\{g_{13},g_{22}\}=\{3.6,3\}$ | $g_1= g_{13}+ g_{22}=6.6$ MW |
| 2 | $\{g_{13},g_{21}\}=\{3.6,0\}$ | $g_2= g_{13}+g21=3.6$ MW |
| 3 | $\{g_{12},g_{22}\}=\{2.66,3\}$ | $g_3=g_{12}+g_{22}=5.66$ MW |
| 4 | $\{g_{12},g_{21}\}=\{2.66,0\}$ | $g_4=g_{12}+g_{21}=2.66$ MW |
| 5 | $\{g_{11},g_{22}\}=\{0.,3\}$ | $g_5= g_{11}+g_{22}=3$ MW |
| 6 | $\{g_{11},g_{21}\}=\{0,0\}$ | $g_6=g_{11}+g_{21}=0$ MW |

By applying composition operators over UGF of individual elements and their combinations in the entire MSS structure, the resulting UGF for the entire MSS is obtained by using simple algebraic operations. UGF characterizes the output performance distribution for the entire MSS at each time instant t. MSS reliability indices easily derived from this output performance distribution. The following steps are executed:

- Having performances $g_{ji}$ and corresponding probabilities $P_{ji}$ (t)for each element j∈{1,2.......n};

$i \in \{1,2.........K_j\}$ UGF for this element is defined in the following form:

$$U_j(z)=P_{j1}(t)z^{g_{1z}}+...+P_{jkl}(t)z^{g_{lk_1}} \quad (8)$$

- The composition operators $\Omega_{\phi s}$ (for elements connected in a series), $\Omega_{\phi p}$ (for elements connected in parallel) and $\Omega_{\phi B}$ (for elements connected in a bridge structure) should be applied over the UGF of individual elements and their combinations. These operators were described in (Lisnianski and Levitin 2003), where corresponding recursive procedures for their computation were introduced for different types of systems. Based on the above procedures, the resulting UGF for the entire MSS is obtained:

$$U_j(z,t)=\Sigma_{i=1}^{k}P_i(t)z^{g_i} \quad (9)$$

where, K is the number of the entire system states and gji is the entire system performance in the corresponding state I, $I \in \{1,2..........K_j\}$

- Applying the operator's $\delta_A$, $\delta_E$, $\delta_D$ introduced in (Aven and Jensen 1999) over the resulting UGF of the entire MSS, the reliability indices of MSS is obtained
- MSS availability A(t, w) at instant t>0 for random constant demand w:

$$A(t,w)=\delta_A(U(Z,T),w)=\delta_A(\Sigma_{i=1}^{k}P_i(t)z^{gi}w)$$

$$A(t,w)=\sum_{i=1}^{k}P_i(t)1(gi-w\geq 0) \quad (10)$$

- MSS expected output performance at instant t > 0 for arbitrary constant demand w:

$$E(t)=\delta_E(\Sigma_{i=1}^{k}P_i(t)z^{gi})=\Sigma_{i=1}^{k}P_i(t)gi \quad (11)$$

## RESULTS AND DISCUSSION

**Estimation of probabilities:** In order to evaluate the performance distribution of the entire system, it is necessary to determine the probability of each system states with corresponding system performance. Using Eq. 2-7 with initial conditions $p_i=0$, for all $i\neq 1$ and $P_1(0)=1$ and based on the failure and repair data for 1400 operation days given in the Table 3, the sate probability defined as show in Fig. 5.

Table 3: Transition intensity rate for failure (λ) and repair (μ)

| Factor | Transition rate |
|---|---|
| $\lambda_1$ | 0.00284 |
| $\lambda_2$ | 0.011959 |
| $\lambda_3$ | 0.003119 |
| $\mu_1$ | 0.053096 |
| $\mu_2$ | 0.053479 |
| $\mu_3$ | 0.055836 |

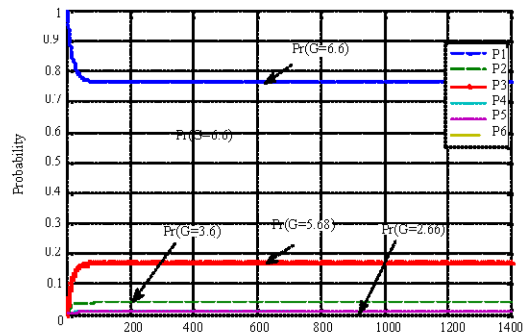


Fig. 5: Probability different performance level

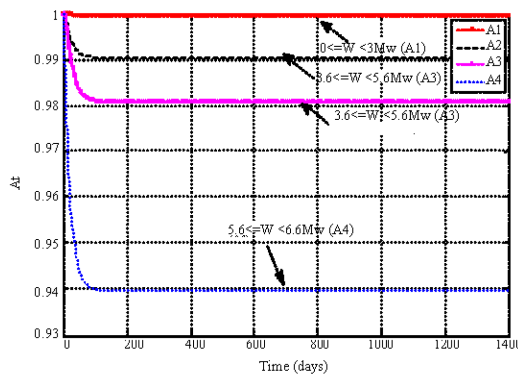


Fig. 6: Availability of the system

In this Fig. 5, each value of performance corresponds to the probability that the element provides a performance rate. As can be observed from Fig. 5, state 4 and 6 do not occur in the system. Whenever these states occurred, the plant uses electric power from Tenaga Nasional Berhad (TNB) to meet the required demand. The probability that the plant runs under state 4 and 6 conditions are almost negligible or zero. In the other way the plant run under state 1 over 75% to satisfy the requirement of high demand.

**Availability of the system:** Depend on the demand required, each state constitute the set of acceptable states. The states which have the output performance lower than the demand required will be combined in one state called absorbing state (unacceptable states). Therefore the instantaneous availability (10) is defined by the sum of probability of only acceptable state. The

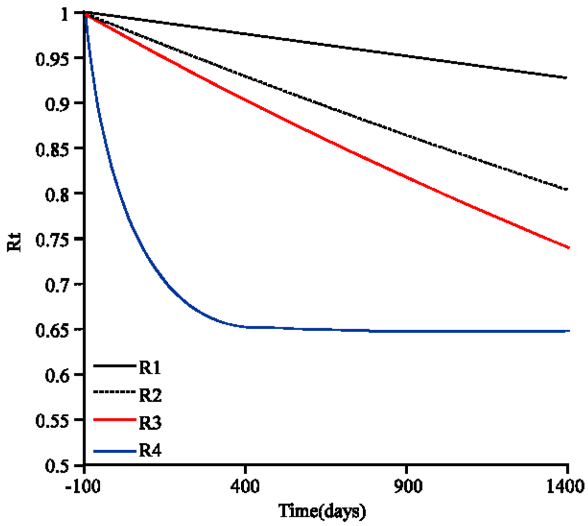


Fig. 7: Reliability of the system for different level demands

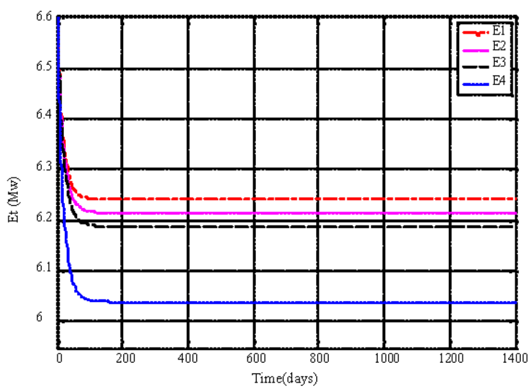


Fig. 8: Instantaneous mean performance

availability of the system for different demand level is shown in Fig. 6.

Figure 7 shows that the availability of the system with respect to time between 0 to 1400 days. As an overall trend it is clear that the availability of the system went down through time due to either the performance degradation or high demand need. If the required demand between 0 and 3 MW, the system delivers almost 99% of availability. When the demand between 3.6 and 5.6MW, the system availability become 98% and above. If the demand goes to 6.6 MW, the availability went down to around 94%. Therefore increasing of demand has an impact on the availability of the system.

**Reliability of the system:** The reliability function R(t, W) is defined by combing all unacceptable state into absorbing state, forbid repairs that return the MSS from this state to the acceptable states and replace the failure rate from each acceptable state to the absorbing state 0 by the sum of the failure rates from acceptable states to all unacceptable states. Therefore the reliability of the GTG system is the sum of the probability of absorbing state:

$$R(t, W)= 1-P_0(t),\ P_0(t)= \text{probability of absorbing state}$$

As can be seen from the graph, the reliability of the system went down when the load increased. In reliability 1, state 4 and 6 are absorbing state because the plant demand requires is not less than 3 MW and the reliability is greater than 93%. Analogously, the demand requirement greater than 5.6 MW, all state except state one will be absorbing state and the reliability is going to be 65%. Therefore the system reliability will be highly affected by high demand requirement. This will bring the performance degradation.

**Expected output performance:** The expected output performance is defined in Fig. 8 using Eq. 11. The expected output performance is decreasing through time. The efficiency of the GTG is also reduced due to frequent failure or over load.

## CONCLUSION

This study predicts the availability and reliability of the power generated from gas turbines which are connected in parallel using universal generating function and the random process method and takes into account multistate models. The result indicates that the availability and reliability of the system at each different state and performance level. As can be seen in the above graphs the reliability and availability of the system went down through time due to performance degradation and overload.

## ACKNOWLEDGMENT

I would like to thank University Technology PETRONAS for providing grant and facilities for the research.

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