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Non-commutative Geometry, Quantum Field Theory and Second-class Constraints

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Abstract: We review the non-commutative geometry in the second-class constraints. We show the non-commutativity in the space-time coordinates based on quantum field theory and matrix theory compactifications has correspondence to the non-commutativity in the space-time coordinates based on the second-class constraints.

Key words: Non-commutative geometry, second class constraints, anyons

INTRODUCTION

Even in the early days of quantum mechanics and quantum field theory, continuous space-time and Lorentz symmetry was considered inappropriate to describe the small-scale structure of the universe. It was also argued that one should introduce a fundamental length scale limiting the precision of position measurements. Snyder was the first to formulate these ideas mathematically. He introduced non-commutative coordinates. Therefore, a position uncertainty arises naturally. The success of the renormalization program made people forget about these ideas for some time. However, when the quantization of gravity was considered thoroughly, it became clear that the usual concepts of space-time are inadequate and the space-time has to be quantized or non-commutative (Connes, 1994; Witten, 1996). As a result, there is a deep conceptual difference between quantum field theory and gravity: In the former, space and time are considered as parameters, in the latter as dynamical entities. To resolve the problem, quantum mechanics in non-commutative space (NCQM) was developed. If NCQM is a realistic physics, all the low energy quantum phenomena should be reformulated in it. In literature, NCQM have been studied in detail (Chaichian *et al*., 2001; Gamboa *et al*., 2001; Hatzinikitas and Smyrnakis, 2002; Ho and Kao, 2002; Nair and Polychronakos, 2001; Zhang, 2004; Farmany, 2005, 2010; Farmany *et al*., 2009a-c). In this letter, we focus on the non-commutative geometry in the second-class constraints.

NON-COMMUTATIVE GEOMETRY

According to Dirac's procedure for dealing with constrained systems, Dirac brackets must replace those constraints, which do not commute with all other (the second-class constraints), must be solved explicitly or their Poisson brackets, but the first and second-class constraints are mixed up. Convincing ways have been found to disentangle them in a covariant manner by Ghosh (1994).

Let us begin with the Lagrangian of an anyon in the background gravity:

$$L = (M^2 u^\mu u_\mu + J^2 \Sigma^a \Sigma_a + 2MJu^\mu e^\mu{}_a \Sigma^a)^{\frac{1}{2}} \tag{1}$$

Where:

$$u^a = \frac{dx^a}{d\tau}, e^\mu{}_a e_{\mu b} = \eta_{ab}, e_\mu{}^a e_{vb} = g_{\mu v}$$

and $g_{\mu v}$ is the space-time metric.

In the non-Abelian gauge interactions the canonical momenta are:

$$P_\mu = \frac{\partial L}{\partial u^\mu} = (\frac{M}{J} e_\mu{}^a - w_\mu{}^a) J_a \tag{2}$$

$$\frac{\partial L}{\partial \bar{\sigma}^a} = 2MJu^\mu e_\mu{}^a + 2J^2 \Sigma_a = 2LJ_a \tag{3}$$

In our framework, from the four primary constraints we can write:

$$J^a J_a = J^2 \tag{4}$$

$$\Pi_\mu = \frac{\partial_L}{\partial u_\mu} - w_\mu^a J_a = \lambda e_\mu^a J_a \tag{5}$$

$$V_\mu = \varepsilon^{\mu v \lambda} \Pi v e_\lambda{}^a J_a \tag{6}$$

$$C = \Pi^\mu \Pi_\mu \text{-} M^2 \tag{7}$$

where, λ = M/J Using analytical methods, from four primary constraints we can construct further constraint set (Chou *et al*., 1993). We focus on the second-class constraint set (V^μ, χ^μ) (Dirac, 1964)) The Poisson bracket matrix of the constraint set (V^μ, χ^μ) is:

$$m = \{V^\mu, \chi^\nu\} \quad (8)$$

In addition, the inverse Poisson bracket is defined by:

$$m^{-1} = \frac{1}{M^2}\begin{pmatrix} \frac{-1}{N}[(F-\lambda T)]^{\mu\nu} & -g^{\mu\nu} \\ g^{\mu\nu} & S^{\mu\nu} \end{pmatrix} \quad (9)$$

Where:

$$m = \{V^\mu, \chi^\nu\} \quad N = M^2 - \frac{1}{2}\varepsilon^{\eta\nu\lambda}J_\mu(F-\lambda T)_{\nu\lambda}$$

and $T_{\mu\nu}$ is the torsion term and N, M and F are three dimensional matrixes.

According to Dirac's procedure for dealing with constrained systems, those constraints that do not commute with all other ones the second-class constraint must be solved explicitly or their Poisson brackets must be replaced by Dirac brackets. Let x^μ and x^ν be the coordinates. Using Eq. 8 we can write the generic Dirac bracket for coordinates as:

$$\{x^\mu, x^\nu\}^* = \{x^\mu, x^\nu\} - (\{x^\mu, V^\mu\}\{x^\mu, \chi^\nu\})m^{-1}\begin{pmatrix} \{V^\mu, x^\nu\} \\ \{\chi^\nu, x^\nu\} \end{pmatrix} \quad (10)$$

Equation 10 can be simplified to:

$$\{x^\mu, x^\nu\}^* = -\frac{1}{N}S^{\mu\nu} \quad (11)$$

where, $S^{\mu\nu}$ is a matrix and $\frac{1}{N}S^{\mu\nu}$ have dimension of x^2. Equation 11 shows the generic non-commutativity in the geometry of the space-time.

CONCLUSION

The non-commutative geometry in the second-class constraints is studied. It is shown that the non- commutativity in the space-time coordinates based on quantum field theory and matrix theory compactifications is correspondence to the non-commutativity in the space-time coordinates based on the second-class constraints.

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