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A Semi-elliptical Crack Modeling and Fracture Constraint on Failure Diagram

J.B. Ariatedja and O. Mamat
Department of Mechanical Engineering, Universiti Teknologi PETRONAS,
Bandar Seri Iskandar, 31750 Tronoh, Perak, Malaysia

Abstract: Modeling the semi-elliptical crack with the 'in-house' capability of finite element tool is the first challenge of this work. In spite many effective schemes had been found their systematic processes are still not openly published. Beside, almost all the schemes were using the complicated-external grid generator. On the other hand, predicting the fracture toughness under constraint condition is another challenging subject. Failure assessment based on constraint parameter is also needed to further study particularly in semi-elliptical crack problem. In this study, the effective meshing and modeling scheme of semi-elliptical crack was firstly developed using simple sweep strategy. Convergence study and evaluation of the scheme are conducted. Then, specific modeling scheme is finally proposed. The constraint parameter fracture mechanic was reviewed in detail. The construction of failure assessment diagram based on this constraint parameter is then reconsidered. Both subjects are needed to produce the alternative mode in the designing and assessing the structural integrity.

Key words: Semi-elliptical crack, finite element modeling fracture constraint, failure assessment diagram

INTRODUCTION

Failure by fracture in high pressure structure such as pipeline and pressure vessel; can lead to greatly economic losses and, moreover, human lives impairment (McHenry *et al.*, 1986; NTSB, 2004; CEPPO, 1997). Accessing fracture based failure criterion in design process of the high pressure structures is a mandatory. One method to assess this criterion in design code and standard for pressurized equipment (such as PD 6493) (BSI PD 6493 1980) is Failure Assessment Diagram (FAD). It is based on comparison of the location of geometry-dependent assessment point to the Failure Assessment Line (FAL). To construct the both parameters, the elastic-plastic fracture parameter, like J-integral, need to be calculated and predicted carefully. Mainly, these calculations are based on Finite Element Analysis (FEA) and are particularly depend on modeling approach of the crack geometry and material property.

Finite element modeling scheme for semi-elliptical crack problem is neither an instant nor a direct process with certain human intervention. In spite some effective schemes had been found (such as: Anderson with spider-web arrangement and singular tip element Shih and Shahani with sweep scheme Cao and Branco that combine above schemes) their systematic processes are still not openly published. Beside, almost all the schemes were using the complicated-external grid generator, like Nasgro, Wrap3d, Zencrack, Feacrack. This limitation led to the first part of this work, which is to explore and proposed in-house effective modeling scheme for particular semi-elliptical crack problem.

Many researchers currently found that fracture parameter is finely affected by the constraint condition (the triaxiality stress state). In the low constraint condition, the fracture toughness prediction based on single-parameter (such as J-integral) is always underestimated. It implies that, in such condition, overly conservative assessment results are obtained Ainsworth, and O'Dowd, 1995. Addition of the second parameter that accounts the affect of low constraint maximizes the prediction of load carrying capacity of the cracked structure and at the end, provides engineers better information to design structure (Yee and Kapper, 2006; Ferreno *et al.*, 2010; Gutierrezsolana and Cicero, 2009; Ainsworth and Hooton, 2008; Flewitt, 2008; Ainsworth, 2000). Some theories have been proposed to predict this fracture toughness dependence and to quantify the crack-front constraint against the plastic flow. Among them are the J-T theory, J-Q theory and the J-A2 three-term solution (Wang, 2009; Zhao *et al.*, 2008; Zhu and Leis, 2006; Kim *et al.*, 2001; Yuh-Jin and Poh-Sang, 1998). The second subject of this study is to review briefly these current two-parameter constraint-based fracture mechanics and their applications in construction of FAD.

In this study, as mention before, two subjects will be briefly discussed; the modeling approach and the two-parameter fracture mechanics. First, a simple modeling approach of the surface crack in plate is developed and reviewed. Singular element, spider-web pattern and

Corresponding Author: J.B. Ariatedja, Department of Mechanical Engineering, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 31750 Tronoh, Perak, Malaysia

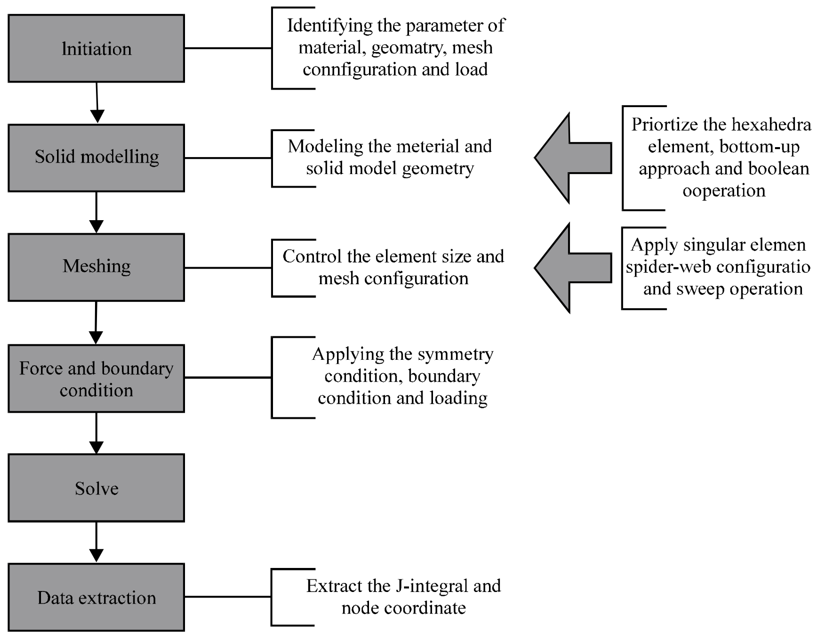


Fig. 1: Main algorithm of crack modelling

sweep-mesh scheme is implemented cautiously, along with several original strategies on free-mesh size control. The J-integral using energy domain integral is calculated and compared to Raju closed-form equation of fracture parameter. The convergence study and the numerical result are reviewed. The proposed modeling approach is then summarized. Second, the current theories of two-parameter fracture mechanics were reviewed shortly. The characteristics of these two-parameters are discussed. The implementation of these two-parameters on FAD is summarized.

MODELING APPROACH AND ITS RESULT

Careful attention is needed when building the crack model. The mesh has to be light and simple to reduce the computation time, meanwhile the accuracy has not to be much sacrificed. Simple development is performed, which is mainly based on the following BS4 approach (Brick, Spider-web, Singular, Sweep and Solid; Fig. 2).

- Brick (3D hexahedral) and the higher order 3D (20-node) element that exhibits quadratic displacement behavior is prioritized
- Spider-web configuration (concentric rings of four sided elements that are focused towards the crack tip) is utilized. The innermost elements are degenerated to wedges. Since the crack tip region contains steep stress and strain gradients, the mesh refinement should be greatest at the crack tip. The spider-web approach facilitates a smooth transition from a fine mesh at the tip to a coarse mesh remote to the tip
- The wedges element around the crack tip is transformed to (collapsed) singular element by move the midside node to a quarter point. It compensates the stress singularity ($1/\sqrt{r}$) in elastic problem
- Sweep mesh scheme is employed to extend the two-dimension (2D) mesh to 3D mesh. It provides meshing that fit to semi-elliptical crack front geometry
- All above approach impose meshing that is based on solid (geometric) modeling instead of free (automatic) meshing or direct modeling.

In the previous study Ariatedja (2009) employed BS4 modeling approach into the crack modeling algorithm Fig. 1 and developed the APDL (ANSYS Parametric Design Language) code. It presented simple study on comparison between linear-elastic stress intensity factor, K and elastic-plastic J-integral in order to divined their character and accuracy. Using simple case semi-circular surface crack on flat plate under very low uniaxial remote applied stress (<5% of yield stress), it showed that the J-integral provides reliable accuracy than stress intensity factor, K. This parameter was also more independent from the affect of element size and elastic singularity zone parameters.

The BS4 modeling approach were succeeded to reduce the calculation time from 7 to half minutes (from

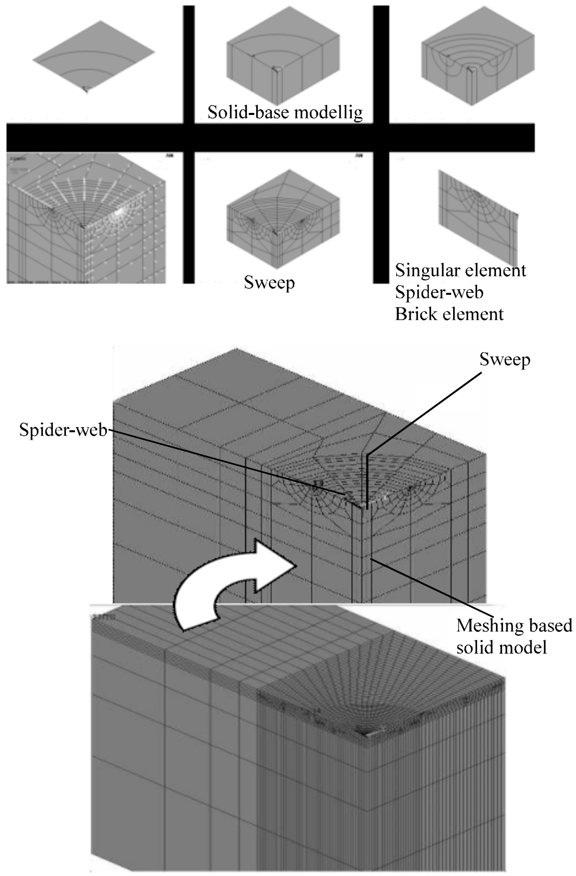


Fig. 2: Development of crack modeling: (a) step of process; (b) comparison: default and using BS4 approach

14200 to only 960 elements), meanwhile, it is also reducing significantly the difference of J-integral (from 1.1 to 0.7%) and K (from 8 to 0.1%) to Raju closed-form equation (compared to default meshing from ANSYS help, Fig. 2).

Current extend work Ariatedja (2010) had been done to clarify the numerical convergence for the proposed model. The non-dimensional J-integral of a semi-elliptical crack on flat plate under uniaxial loading and linear-elastic material had been evaluated. All BS4 approach was obeyed. The energy domain integral was used to obtain up to twenty contours of J-integral of each crack front nodes. The crack baseline model is described in Table 1. Two different sweep schemes were employed, which were undistorted and distorted sweep scheme. The ratio of crack depth to element size was considered in the range between 15 and 80, division of crack tip area was in range between 3 and 12, division of sweep was in range between 4 and 15 and the ratio of plate width to crack depth was in the range between 5 and 20. The non-dimensional J-integral from Raju closed-form equation was used as reference.

Table 1: Model parameters of semi-elliptical crack in plate

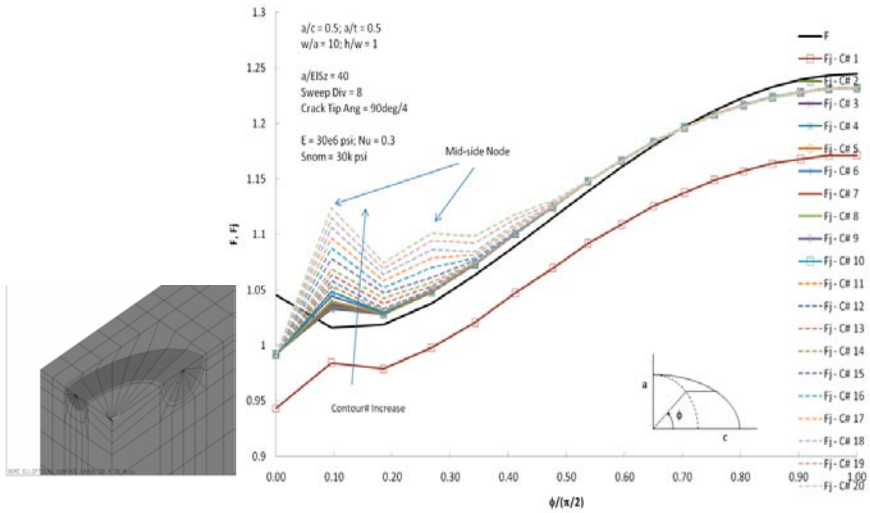
Baseline model	Values
Plate width to crack depth ratio (w/a)	1
Plate height to width ratio (h/w)	1
Ratio of singular element size to crack depth (EISz/a)	1/40
Sweep division	8
Crack tip angle	90°/4
Crack depth width ratio (a/c)	0.5
Crack depth to plate thickness ratio (a/t)	0.5
Elasticity modulus (E)	207 GPa
Poisson ratio (nu)	0.3
Yield stress (S_y)	207 Mpa

Almost all baseline parameters had been in fine arrangement (result I n minimum difference), except the J-integral dependence to the contour was found in distorted sweep scheme. The differences increased as the contour increase, especially for the midside nodes Fig. 3a. It implies that element distortion will produce significant error. Based on the results, the undistorted sweep scheme along with baseline parameters was recommended to be used as base mesh design along with BS4 modeling approach.

CONSTRAINT PARAMETER AND ITS IMPLEMENTATION ON FAD

Even though the single-parameter J-based fracture mechanics has long been regarded as a material property into industry testing standards by American Society of Testing and Materials (ASTM) (Rice, 1968; Hutchinson, 1968; Hutchinson, 1999), it has been found that fracture toughness indeed depends on specimen size, thickness, crack depth, geometry and loading condition, which are attributed to different crack-front constraints. Fracture constraint at crack front means the resistance against the plastic deformation. It is questionable to apply the fracture toughness value determined from small laboratory specimens to integrity assessment of large defected structures. The level of constraint at crack front play important role and can be revealed by examining accurately the details of the crack-front stress and deformation fields. Normally, the plain-strain state exhibits the highest constraint, generates the highest triaxiality of stresses, whereas the plane-stress state yields the lowest limit.

Some approaches have been proposed to predict this fracture toughness dependence and quantify the crack-front constraint against the plastic flow. Among them, two representatives are the J-T theory proposed by Betegon and Hancock (1991), J-Q theory developed by O'Dowd and Shih (1991, 1992) and the J-A2 three-term solution developed by Yang *et al.* (1993a, b) and Chao *et al.* (1994).



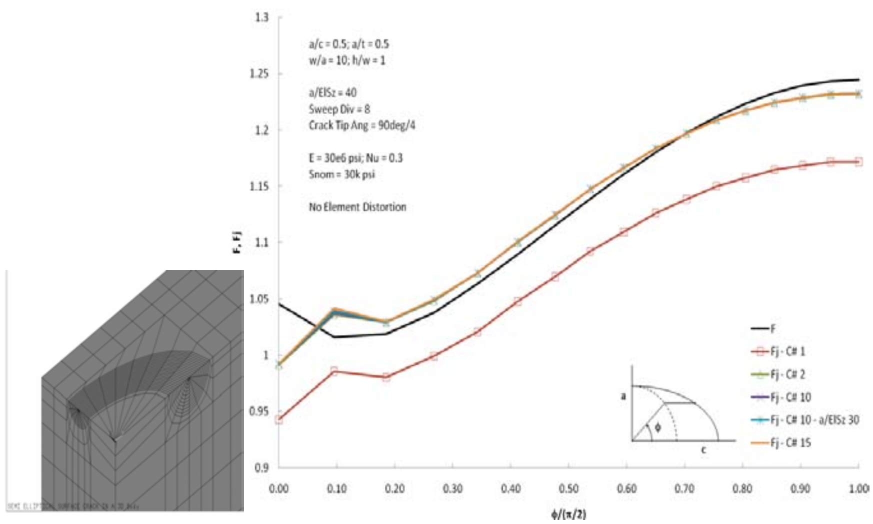


Fig. 3: Convergence study of the non dimensional J-integral result for two different sweep schemes: (a) distorted (b) undistorted

In the LEFM, it has been found that a second term, T-stress or A_3 can represent the affect of specimen geometry and loading condition on crack-front stress fields while the stress intensity factor K represent the applied loading. Using William's series solution for the crack-front fields, one has:

$$\sigma_{ij}(r,\partial)=\frac{K_I}{\sqrt{2\pi r}}f_{ij}(\theta)+T\delta_{1i}\delta_1 \qquad (1)$$

where, (r, θ) are polar coordinates with the origin located at the crack-front, K_I is the Mode-I stress intensity factor, $f_{ij}(\theta)$ are non-dimensional angular functions, T is the second term and is a uniform stress parallel to the crack face commonly referred to as T-stress, δ_{1i} and δ_{1j} are the Kronecker delta. The indexes i, j have the range of 1-2.

In EPFM, J is used to represent the loading level. A second parameter Q is defined as hydrostatic stress, which is extension of parameter T for elastoplastic material. After simplification, the crack-front stress field is:

$$\sigma_{ij}(r,\theta)=\sigma_{ij}(r,\theta)\|_{\tau=0}Q\sigma_0\delta_{ij} \qquad (2a)$$

where, the first term is a standard ssy solution with T = 0 that replaces the HRR field, σ_0 is reference stress that is generally equal to yield stress. The parameter Q is defined as hydrostatic stress by the difference of the HRR stress field and the full-field stress field:

$$Q = \frac{\sigma_{\theta\theta}(\sigma_{\theta\theta})HRR}{\sigma_0} \tag{2b}$$

parameter Q is engineering definition and depend of the location. The location of $r/(J/\sigma 0) = 2$ and $\theta = 0$ is generally used for the determination of Q.

A more rigorous analysis of higher-order crack-front fields in power-law hardening materials is the J-A_2 three-term asymptotic stress field:

$$\frac{\sigma_{\theta\theta}(r,\theta)}{\sigma_0} = A_1\left[\left(\frac{r}{L}\right)^{S1}\partial_{ij}^{(1)}(\theta) + A_2\left(\frac{r}{L}\right)^{S2}\partial_{ij}^{(2)}(\theta) + A_2^2\left(\frac{r}{L}\right)^{S3}\partial_{ij}^{(3)}(\theta)\right] \tag{3a}$$

where the angular functions $\partial_{ij}^{(k)}$ with k = 1, 2, 3, the stress power exponents s_k ($s_1<s_2<s_3$) are only dependent of the hardening exponent n and independent of other material constants and applied loads. L is characteristic length parameter which can be chosen as the crack length a, the specimen width W, the thickness B, or unity. The parameter A_1 and s_1 are given by:

$$A_2 = \left(\frac{1}{\alpha\epsilon_0\sigma_0 I_n L}\right)^{-S1};\ S_1 = \frac{1}{n+1} \tag{3b}$$

and $s_3 = 2s_2 - s_1$ for n = 3. A_2 is an undetermined parameter and may be related to the loading and geometry of specimen. The angular function and s_2 in (3a) are given in a report written by Chao *et al*. (1994).

As the elastic T-stress requires only elastic calculations, it is recommended for initial evaluations. The hydrostatic Q-stress is expected to provide more accurate assessments, particularly when plasticity becomes widespread and should be used when more refined estimates of load margins are required or as part of sensitivity studies. On contrary, there are only a few works that study the J-A_2 three-term asymptotic stress field. Further study is needed to explore particularly in its application on FAD.

To address the constraint affect on FAD, some design code and standard, such as SINTAP/FITNET, employ a special structural constraint factor β hat can be obtained from elastic T-stress and the hydrostatic Q-stress:

$$\beta = \frac{T}{L_r\sigma_0};\ \beta = \frac{Q}{L_r} \tag{4}$$

The β factor is then used to determine a constraint dependent fracture toughness designated as K^c_{mat}. The increase in fracture toughness in both the brittle and ductile regimes may be represented by an expression of the form:

$$K^c_{mat} - \begin{cases} K_{mat} & \text{for } \beta L_r > 0 \\ K_{mat}\,[1+\alpha(-\beta L_r)^K] & \text{for } \beta L_r \le 0 \end{cases} \tag{5}$$

Using finite element analysis, the J-integral (in elastic and in plastic property) and the constraint parameter can be calculated. Both parameter is used to generate the Kr and Fr as SINTAP/FITNET employs constraint parameter β to incorporate the constraint effect on the FAD. This parameter can be obtained from elastic T-stress and hydrostatic Q-stress.

CONCLUSION

The cost-effective meshing and modeling of crack under uniaxial load had been developed. The convergence study for single-parameter linear-elastic fracture mechanic had been done and cost-effective mesh design had been provided.

Critical review of fracture parameter that considered the constraint conditions had been done briefly. Works on implementing the two-parameter fracture mechanic and on developing failure assessment diagram are planned to be done further.

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