Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 11 (14): 2585-2591, 2011
ISSN 1812-5654 / DOI: 10.3923/jas.2011.2585.2591

Operational Tau Approximation for Neutral Delay Differential Systems

J. Sedighi Hafshejani, S. Karimi Vanani and J. Esmaily
Department of Mathematics, Islamic Azad University, Shahrekord Branch, P.O. Box 166, Shahrekord, Iran

Abstract: Neutral Delay Differential Systems (NDDSs) arise in many areas of various mathematical modeling. Infectious diseases, population dynamics, physiological and pharmaceutical kinetics and chemical kinetics, the navigational control of ships and mechanical systems, chemical process simulation and optimal control are the main field concerning with NDDSs. The purpose of this study was to present an extension of the algebraic formulation of the Operational Tau Method (OTM) for the numerical solution of NDDSs. The proposed method converts the delay parts of the desired NDDS to some operational matrices. Then the NDDS reduces to a set of algebraic equations. Some orthogonal bases including shift Chebyshev and shifted Legendre polynomials are used to decrease the volume of computations. Two illustrative linear and nonlinear experiments are included to show the high accuracy and efficiency of the proposed method.

Key words: Spectral methods, tau method, delay differential equations, neutral delay differential systems

INTRODUCTION

Spectral methods and spectral modelings provide a computational approach which achieved substantial popularity in the last three decades (Ghafarian *et al*., 2011; Taiwo and Abubakar, 2011; Tasci, 2003). Tau method is one of the most important spectral methods which is extensively applied for numerical solution of many problems. This method was invented by Lanczos (1938) for solving Ordinary Differential Equations (ODEs) and then the expansion of the method were done for many different problems such as Partial Differential Equations (PDEs) (Matos *et al*., 2004; Kong and Wu, 2009; Doha and Abd-Elhameed, 2005), Integral Equations (IEs) (El-Daou and Khajah, 1997), Integro-Differential Equations (IDEs) (Pour-Mahmoud *et al*., 2005) and etc. (Rahimi-Ardabili and Shahmorad, 2007; Garcia-Olivares, 2002; Parand and Razzaghi, 2004).

A time delay phenomenon is encountered in a wide variety of scientific and engineering applications, including circuit analysis, computer-aided design, real-time simulation of mechanical systems, chemical process simulation, optimal control, population dynamics and vibrating masses attached to an elastic bar (Hale and Verduyn Lunel, 1993; Taiwo and Odetunde, 2010; Rao *et al*., 2011; Vanani and Aminataei, 2009, 2010; Kolmanovskii and Myshkis, 1992; Salamon, 1984).

In this study, we are interested in solving NDDSs with an operational approach of the Tau method. Because in the Tau method, we are dealing with a system of equations wherein the matrix of unknown coefficients is sparse and can be easily invertible. Also, the delay parts appearing in the equation are replaced by their operational Tau representation. Then, we obtain a system of algebraic equations wherein its solution is easy.

Operational tau method: In this section, we state some preliminaries and notations using in this study.

For any integrable functions Ψ (x) and ϕ (x) on (a, b), we define the scalar product <, > by:

$$<\psi(x),\varphi(x)>_w=\int_a^b\psi(x)\varphi(x)\omega(x)dx,$$

where, $\|\psi\|_w^2=<\psi(x),\psi(x)>_\omega$ and ω (x) is a weight function. Let L^2_ω [a, b] be the space of all functions f: [a, b] →R with $\|f\|^2_\omega<\infty$.

The main idea of the method is to seek a polynomial to approximate u (x) $\in L^2_\omega$ [a,b]. Let $\varphi_x=\{\varphi_i(x)\}_{i=0}^{\infty}=\Phi X$ be a set of arbitrary orthogonal polynomial bases defined by a lower triangular matrix Φ and $X_x = [1, x, x^2, ...]^T$.

Lemma 1: Suppose that u (x) is a polynomial as $u(x)=\sum_{i=0}^{\infty}u_ix^i=uX_x$, then we have:

$$D^r(x)=\frac{d^r}{dx^r}u(x)=uM^rX_x,\quad r=0,1,2,\ldots, \tag{1}$$

$$x^su(x)=uN^sX_x,\quad s=0,1,2,\ldots, \tag{2}$$

Corresponding Author: J. Sedighi Hafshejani, Department of Mathematics, Islamic Azad University, Shahrekord Branch, P.O. Box 166, Shahrekord, Iran Tel: 00989131858762

and:

$$\int_a^x u(t)dt = uPX_x - uPX_a, \quad (3)$$

where, $u = [u_0, u_1, ..., u_n, ...]$, $x_a = [1, a, a^2, ...]^T$, $a \in \Re$ and M, N and P are infinite matrices with only nonzero elements:

$$M_{i+1,i} = i+1, N_{i,i+1} = 1, P_{i,i+1} = \frac{1}{i+1}, i = 0,1,2,\ldots$$

Proof: (Liu and Pan, 1999).

Let us consider:

$$u(x) = \sum_{i=0}^{\infty} u_i \varphi_i(x) = u\phi X_x, \quad (4)$$

To be an orthogonal series expansion of the exact solution where, $u = \{u_i\}_{i=0}^{\infty}$ is a vector of unknown coefficients, ΦX_x is an orthogonal basis for polynomials in $\Re$.

In the Tau method, the aim is to convert the linear and nonlinear terms to an algebraic system using some operational matrices. Therefore, we state the following lemma.

Lemma 2: Let $X_x = [1, x, x^2, ...]^T$, $u = [u_0, u_1, u_2, ...]$ be infinite vectors and $\Phi = [\phi_0|\phi_1|\phi_2|...]$, ϕ_i are infinite columns of matrix Φ. Then, we have:

$$X_x u \phi X_x = U X_x \quad (5)$$

where, U is an upper triangular matrix as:

$$U_{i,j} = \begin{cases} \sum_{k=0}^{\infty} u_k \varphi_{k,j-i}, & j \ge i,\ i,j = 0,1,\ldots, \\ 0, & j < i,\ i,j = 0,1,\ldots. \end{cases} \quad (6)$$

In addition, if we suppose that $u(x) = u\Phi X_x$ represents a polynomial, then for any positive integer p, the relation:

$$u^p(x) = u\Phi U^{p-1} X_x \quad (7)$$

is valid.

Proof: We have:

$$X_x u\phi = [1, x, x^2, ...]^T [u\varphi_0 | u\varphi_1 | u\varphi_2 | \ldots] = \begin{bmatrix} u\varphi_0 & u\varphi_1 & u\varphi_2 & \cdots \\ u\varphi_0 x & u\varphi_1 x & u\varphi_2 x & \cdots \\ u\varphi_0 x^2 & u\varphi_1 x^2 & u\varphi_2 x^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Therefore:

$$X_x u\phi X_x = \begin{bmatrix} u\varphi_0 & u\varphi_1 & u\varphi_2 & \cdots \\ u\varphi_0 x & u\varphi_1 x & u\varphi_2 x & \cdots \\ u\varphi_0 x^2 & u\varphi_1 x^2 & u\varphi_2 x^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} [1, x, x^2, ...]^T = \begin{bmatrix} \sum_{i=0}^{\infty} u\varphi_i x^{i+0} \\ \sum_{i=0}^{\infty} u\varphi_i x^{i+1} \\ \sum_{i=0}^{\infty} u\varphi_i x^{i+2} \\ \vdots \end{bmatrix}$$

$$= \begin{bmatrix} u\varphi_0 & u\varphi_1 & u\varphi_2 & \cdots \\ 0 & u\varphi_0 & u\varphi_1 & \cdots \\ 0 & 0 & u\varphi_0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 \\ x \\ x^2 \\ \vdots \end{bmatrix},$$

If we call the last upper triangular coefficient matrix as U, then we have:

$$U_{i,j} = \begin{cases} u\varphi_{j-i}, & j \ge i,\ i,j = 0,1,\ldots, \\ 0, & j < i,\ i,j = 0,1,\ldots, \end{cases} = \begin{cases} \sum_{k=0}^{\infty} u_k \varphi_{k,j-i}, & j \ge i,\ i,j = 0,1,\ldots, \\ 0, & j < i,\ i,j = 0,1,\ldots. \end{cases}$$

Now, in order to prove Eq. 7, we apply induction. For p = 1, it is obvious that $u(x) = u\Phi X_x$. For p = 2 we rewrite $u^2(x) = u\Phi X_x u\Phi X_x = u\Phi(X_x u\Phi X_x)$ and using Eq. 5, we have:

$$u^2(x) = u\phi u X_x$$

Therefore, Eq. 7 is hold for p = 2. Now, suppose that Eq. 7 is hold for p = k, then we must prove that the relation is valid for s = k + 1 . Thus:

$$u^{k+1}(x) = u^k(x)u(x) = (u\phi U^{k-1}X_x)(u\phi X_x) = u\phi U^{k-1}(X_x u\phi X_x) = u\phi U^k X_x,$$

So, Eq. 7 is proved.

Application on NDDSs: Let us consider the following NDDS:

$$\begin{aligned} &\dot{U}(x) = A(x)U(x) + B(x)U(\alpha(x)) + C(x)\dot{U}(\beta(x)) + F(x),\ a \le x \le b, \\ &U(x) = \Psi(x),\ x \le a, \end{aligned} \quad (8)$$

Where:

$$U(x) = [u_1(x), u_2(x), ..., u_m(x)]^T,\ U_k(x) \in C,\ k = 1, 2..., m \quad (9)$$

is the state vector and:

$$\begin{aligned} &U(\alpha(x)) = [u_1(\alpha_1(x)), u_2(\alpha_2(x)), \ldots, u_m(\alpha_m(x))]^T, \\ &\dot{U}(\beta(x)) = [\dot{u}_1(\beta_1(x)), \dot{u}_2(\beta_2(x)), \ldots, \dot{u}_m(\beta_m(x))]^T, \end{aligned} \quad (10)$$

Such that $\{\alpha_k(x) \le b\}_{k=1}^m$ and $\{\beta_k(x) \le b\}_{k=1}^m$ are delay functions; A (x), B (x) and C (x) are m-dimensional matrices which their entries are complex functions of x. Also:

$$\Psi(x) = [\psi_1(x), \psi_2(x), \ldots, \psi_m(x)]^T, \quad \psi_k(x) \in C, k = 1, 2, \ldots, m, \quad (11)$$

$$F(x) = [f_1(x), f_2(x), \ldots, f_m(x)]^T, \quad f_k(x) \in C, k = 1, 2, \ldots, m, \quad (12)$$

represent the initial vector function and known vector function, respectively.

Now the aim is to write $u_i(\alpha_i(x))$ and $\dot{u}_i(\beta_i(x))$ i = 0, 1..., m; in operational forms. Using Eq. 4, we have:

$$u_i(\alpha_i(x)) = u\,\Phi\, X_{\alpha i(x)}, \; i = 0, 1, \ldots, m \quad (13)$$

We know that $X_x = [1, x, x^2, \ldots]^T$, therefore:

$$X_{\alpha i(x)} = [1, \alpha_i(x), \alpha^2_i(x), \ldots]^T$$

By approximating each power of $\alpha_i(x)$ as $\alpha_i^j(x) = \sum_{k=0}^{\infty} \alpha_{ijk} x^k$, we obtain:

$$X_{\alpha_i(x)} = \begin{bmatrix} 1 \\ \alpha_{i10} + \alpha_{i11}x + \ldots + \alpha_{i1n}x^n + \ldots \\ \alpha_{i20} + \alpha_{i21}x + \ldots + \alpha_{i2n}x^n + \ldots \\ \vdots \\ \alpha_{in0} + \alpha_{in1}x + \ldots + \alpha_{inn}x^n + \ldots \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & 0 & \ldots & 0 & \ldots \\ \alpha_{i10} & \alpha_{i11} & \ldots & \alpha_{i1n} & \ldots \\ \alpha_{i20} & \alpha_{i21} & \ldots & \alpha_{i2n} & \ldots \\ \vdots & \vdots & \ldots & \vdots & \ldots \\ \alpha_{in0} & \alpha_{in1} & \ldots & \alpha_{inn} & \ldots \\ \vdots & \vdots & \ldots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 \\ x \\ x^2 \\ \vdots \\ x^n \\ \vdots \end{bmatrix}$$

If $\prod_i$ be the last coefficient matrix then we have:

$$X_{\alpha i(x)} = \prod\nolimits_i X_x \quad (14)$$

Substituting Eq. 14 in Eq. 13 we get:

$$u_i(\alpha_i(x)) = u_i \Phi \prod\nolimits_i X_x, \; i = 0, 1, \ldots, m \quad (15)$$

Also, from Eq. 1 and 4, it is obvious that:

$$\dot{u}_i(x) = u_i \phi M X_x, \quad i = 0, 1, \ldots, m.$$

In the same manner from Eq. 13 to 15, there exist the coefficient matrices Δ_i such that:

$$\dot{u}_i(\beta_i(x)) = u_i \phi M \Delta_i X_x, \; i = 0, 1, \ldots, m \quad (16)$$

In next step we desire to approximate each elements of matrices A (x), B (x) and C (x) in operational forms. Since, each elements of A (x), B (x) and C (x) are smooth functions therefore we can approximate them as follows:

$$A_{ij}(x) = \sum_{k=0}^{n} a_{ijk} x^k, \; B_{ij}(x) = \sum_{k=0}^{n} b_{ijk} x^k, \; C_{ij}(x) = \sum_{k=0}^{n} c_{ijk} x^k, \; i, j = 0, 1, \ldots, m. \quad (17)$$

Substituting above equations in Eq. 8 and using Eq. 2, we obtain:

$$\begin{aligned} A(x)U(x) &= \left[\sum_{j=0}^{m} A_{0j} u_j(x), \sum_{j=0}^{m} A_{1j} u_j(x), \ldots, \sum_{j=0}^{m} A_{mj} u_j(x) \right]^T = \\ &\left[\sum_{j=0}^{m}\sum_{k=0}^{n} a_{0jk} x^k u_j(x), \sum_{j=0}^{m}\sum_{k=0}^{n} a_{1jk} x^k u_j(x), \ldots, \sum_{j=0}^{m}\sum_{k=0}^{n} a_{mjk} x^k u_j(x) \right]^T = \\ &\left[\sum_{j=0}^{m}\sum_{k=0}^{n} a_{0jk} u_j \phi N^k X_x, \sum_{j=0}^{m}\sum_{k=0}^{n} a_{1jk} u_j \phi N^k X_x, \ldots, \sum_{j=0}^{m}\sum_{k=0}^{n} a_{mjk} u_j \phi N^k X_x \right]^T = \\ &\left[\sum_{j=0}^{m}\sum_{k=0}^{n} a_{0jk} u_j \phi N^k, \sum_{j=0}^{m}\sum_{k=0}^{n} a_{1jk} u_j \phi N^k, \ldots, \sum_{j=0}^{m}\sum_{k=0}^{n} a_{mjk} u_j \phi N^k \right]^T X_x. \end{aligned} \quad (18)$$

Therefore:

$$A(x)U(x) = \tilde{A} X_x, \quad \tilde{A}_i = \sum_{j=0}^{m}\sum_{k=0}^{n} a_{ijk} u_j \phi N^k, \; i = 0, 1, \ldots, m. \quad (19)$$

In the same way, we have:

$$B(x)U(\alpha(x)) = \tilde{B} X_x, \quad \tilde{B}_i = \sum_{j=0}^{m}\sum_{k=0}^{n} a_{ijk} u_j \phi \Pi_j\, N^k \quad (20)$$

$$C(x)\dot{U}(\beta(x)) = \tilde{C} X_x, \quad \tilde{C}_i = \sum_{j=0}^{m}\sum_{k=0}^{n} a_{ijk} u_j \phi M \Delta_j N^k. \quad (21)$$

The vectors U (x) and F (x) also can be considered as:

$$U(x) = \tilde{U} \phi X_x, \; \tilde{U}_i = u_i \quad (22)$$

$$F(x) = \tilde{F} X_x, \; \tilde{F}_{i,j} = f_{ij} \quad (23)$$

Thus Eq. 8 is replaced by the following algebraic system:

$$\tilde{U}\phi X_x = \tilde{A}\phi^{-1}\phi X_x + \tilde{B}\phi^{-1}\phi X_x + \tilde{C}\phi^{-1}\phi X_x + \tilde{F}\phi^{-1}\phi X_x \quad (24)$$

So, the residual matrix R (x) of Eq. 8, can be written as:

$$R(x) = \left[\tilde{U} - \tilde{A}\phi^{-1} - \tilde{B}\phi^{-1} - \tilde{C}\phi^{-1} - \tilde{F}\phi^{-1} \right] \phi X_x = \tilde{R}\phi X_x \quad (25)$$

Where:

$$\tilde{R} = \left[\tilde{U} - \tilde{A}\phi^{-1} - \tilde{B}\phi^{-1} - \tilde{C}\phi^{-1} - \tilde{F}\phi^{-1} \right]$$

Now, we set the residual matrix $\tilde{R}=0$ or we use the following inner products:

$$\langle R_i(x),\varphi_k(x)\rangle_w = 0,\ i,k=0,1,\ldots \tag{26}$$

For supplementary conditions of Eq. 8 we have:

$$U(a)=\tilde{U}\phi X_a = \Psi(a) \tag{27}$$

Therefore, imposing supplementary conditions and setting $\tilde{R}=0$, a system of algebraic equations is obtained. Since, somewhere we require finite terms of approximation, then we must truncate the series solution to finite number of terms. This is the so-called operational Tau method which is applicable for finite, infinite, regular and irregular domains.

Some shifted orthogonal polynomials: We have considered OTM based on arbitrary orthogonal polynomials. Orthogonal functions can be used to obtain a good approximation for transcendental functions. Since shifted Chebyshev and Legendre polynomials are more applicable orthogonal functions for a wide range of problems therefore, we consider them, briefly.

Shifted chebyshev polynomials: The Chebyshev polynomials are defined on [-1, 1] as:

$$\begin{cases} T_0(x)=1, T_1(x)=x, \\ T_{i+1}(x)=2xT_i(x)-T_{i-1}(x), \quad i=1,2,3,\ldots, \end{cases} \tag{28}$$

or:

$$[T_0(x),T_1(x),T_2(x),\ldots]^T = TX_x, \quad \text{where} \quad T=\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ -1 & 0 & 2 & 0 & \cdots \\ 0 & -3 & 0 & 4 & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix},$$

and shifted Chebyshev polynomials are defined as:

$$\begin{cases} T_0^*(x)=1, T_1^*(x)=\dfrac{2x-(b+a)}{b-a}, & x\in[a,b], \\ T_{i+1}^*(x)=2(\dfrac{2x-(b+a)}{b-a})T_i^*(x)-T_{i-1}^*(x), & i=1,2,3,\ldots. \end{cases} \tag{29}$$

Now, we consider the following lemma.

Lemma 3:. Suppose that T and T* are coefficient matrices of Chebyshev polynomials $\{T_i(x)|\ x \in [-1, 1], i = 0, 1, 2,...\}$ and shifted Chebyshev polynomials $\{T^*_i(x)|\ x \in [a, b], i = 0, 1, 2,...\}$, respectively. Hence, we have:

$$T^* = TQ$$

Where:

$$Q_{i,j}=\begin{cases} \binom{i}{j} v^{i-j} w^j, & i\ge j,\ i,j=0,1,2,\ldots \\ 0, & i<j, \end{cases}$$

with v = 2/b-a and w = a + b/a-b.

Proof: Definition of T states that:

$$\begin{bmatrix} T_0(x) \\ T_1(x) \\ T_2(x) \\ \vdots \end{bmatrix} = T \begin{bmatrix} 1 \\ x \\ x^2 \\ \vdots \end{bmatrix} \Rightarrow \begin{bmatrix} T_0^*(x) \\ T_1^*(x) \\ T_2^*(x) \\ \vdots \end{bmatrix} = T \begin{bmatrix} 1 \\ \dfrac{2x-(b+a)}{b-a} \\ (\dfrac{2x-(b+a)}{b-a})^2 \\ \vdots \end{bmatrix} = T \begin{bmatrix} 1 \\ vx+w \\ (vx+w)^2 \\ \vdots \end{bmatrix}$$

We know that $(vx+w)^n = \sum_{i=0}^{n}\binom{n}{i} v^i w^{n-i} x^i$, thus:

$$\begin{bmatrix} 1 \\ vx+w \\ (vx+w)^2 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ v & w & 0 & 0 & \cdots \\ v^2 & 2vw & w^2 & 0 & \cdots \\ v^3 & 3vw^2 & 3v^2w & w^3 & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 \\ x \\ x^2 \\ \vdots \end{bmatrix}$$

If we let Q to be the last coefficient matrix, then:

$$[T_0^*(x),T_1^*(x),T_2^*(x)\ldots]^T = TQX_x, \quad \text{where} \quad Q_{i,j}=\begin{cases} \binom{i}{j} v^{i-j} w^j, & i\ge j,\ i,j=0,1,2,\ldots \\ 0, & i<j, \end{cases}$$

so

$$T^* = TQ$$

Therefore, the lemma is valid.

Shifted legendre polynomials: The Legendre polynomials on [-1, 1] are defined as:

$$\begin{cases} P_0(x)=1, P_1(x)=x, \\ P_i(x)=(2-\dfrac{1}{i})xP_{i-1}(x)-(1-\dfrac{1}{i})P_{i-2}(x), \quad i=2,3,4,\ldots, \end{cases} \tag{30}$$

and we define shifted Legendre polynomials as:

$$\begin{cases} P_0^*(x)=1, P_1^*(x)=\dfrac{2x-(b+a)}{b-a}, \quad x\in[a,b], \\ P_i^*(x)=(2-\dfrac{1}{i})(\dfrac{2x-(b+a)}{b-a})P_{i-1}^*(x)-(1-\dfrac{1}{i})P_{i-2}^*(x), \quad i=2,3,4,\ldots \end{cases} \tag{31}$$

In a similar manner with lemma 3 we can prove P* = PC, where, P and P* are coefficient matrices of Legendre and shifted Legendre polynomials, respectively.

Illustrative numerical experiments: In this section, two experiments of NDDSs are given to illustrate the efficiency of the method. In all experiments, we consider the shifted Chebyshev and Legendre polynomials as basis functions and have compared the obtained results with the exact solutions. The computations associated with the experiments discussed above were performed in Maple 14 on a PC with a CPU of 2.4 GHz.

Experiment 1: Consider the following NDDS (Vanani and Aminataei, 2010):

$$\begin{cases} \dot{u}_1(x)=-2(1-\cos(x))(x-\sin(x))u_2(x-\sin(x))+\dot{u}_3(\sin(x))-u_3(\sin(x)), & 0\le x\le 1, \\ \dot{u}_2(x)=-2xu_2(x)+\dot{u}_1(x)+2(1-\cos(x))(x-\sin(x))u_2(x-\sin(x)), & 0\le x\le 1, \\ \dot{u}_3(x)=-\dot{u}_2(\sqrt{x})-2\sqrt{x}u_2(\sqrt{x})+u_3(x), & 0\le x\le 1, \\ \\ u_1(x)=x^2+1, \quad x\le 0, \\ u_2(x)=1-x^3, \quad x\le 0, \\ u_3(x)=e^x, \quad x\le 0. \end{cases}$$

The exact solution in the interval [0, 1] is:

$$\begin{cases} u_1(x)=e^{-(x-\sin(x))^2} \\ u_2(x)=e^{-x^2} \\ u_3(x)=e^x \end{cases}$$

We have solved this experiment by OTM for n = 20 with shifted Chebyshev and Legendre bases and compared with the exact solution. Results are given in Table 1 and 2 for u_1 (x), u_2 (x) and u_3 (x), respectively.

From the numerical results in Table 1 and 2, it is easy to conclude that obtained results by OTM are in good agreement with the exact solution. Also, during the running of programs we find out the run time of OTM is 0.952 sec. Therefore, the algorithm of OTM is fast.

Experiment 2: Consider the following nonlinear NDDS (Vanani and Aminataei, 2009):

$$\begin{cases} \dot{u}_1(x)=xu_1(\frac{x}{2})-xu_2(\sin(\frac{x}{2}))+\cos(x)\dot{u}_2(\sin(x))+\frac{1}{2}u_3(\sqrt{x})-\dot{u}_3(\sqrt{x}), & 0\le x\le 1, \\ \dot{u}_2(x)=-u_1(\sqrt{x})+u_2(t)+\dot{u}_2(\sin(\sqrt{x})), & 0\le x\le 1, \\ \dot{u}_3(x)=u_1(x)-u_2(\sin(x))+\frac{1}{2}\dot{u}_2(\frac{x}{2})-\frac{x}{3}u_3(\frac{2x}{3})+x\dot{u}_4(x), & 0\le x\le 1, \\ \dot{u}_4(x)=-u_2(\frac{x}{2})+\frac{1}{3}\dot{u}_2(\frac{x}{3})+u_3(x), & 0\le x\le 1, \\ \\ u_1(x)=e^{\sin(x)}, \quad x\le 0 \\ u_2(x)=e^{x}, \quad x\le 0 \\ u_3(x)=e^{\frac{x}{2}}, \quad x\le 0 \\ u_4(x)=e^{\frac{x}{3}}, \quad x\le 0 \end{cases}$$

The exact solution is:

$$u_1(x)=e^{\sin(x)},\ u_2(x)=e^{x},\ u_3(x)=e^{\frac{x}{2}} \text{ and } u_4(x)=e^{\frac{x}{3}}.$$

We have solved this experiment by OTM for n = 20 with shifted Chebyshev and Legendre bases. Results are given in Table 3 and 4 for u_1 (x), u_2 (x), u_3 (x) and u_4 (x), respectively. Numerical results in Tables 3 and 4 illustrate a good agreement between OTM solutions and exact solutions. In this experiment, the run time of OTM is 1.607 sec. Again, we can conclude that OTM is a fast method.

Table 1: Exact and approximate solutions of u_1 (x) and u_2 (x) of experiment 1

	u_1 (x)			u_2 (x)		
x_i	u_{Che}	u_{Leg}	u_{Exact}	u_{Che}	u_{Leg}	u_{Exact}
0.0	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000
1.0	0.9999999722	0.9999999722	0.9999999722	0.9900498337	0.9900498337	0.9900498337
0.2	0.9999982293	0.9999982293	0.9999982293	0.9607894391	0.9607894391	0.9607894391
0.3	0.9999799316	0.9999799316	0.9999799316	0.9139311852	0.9139311852	0.9139311852
0.4	0.9998880347	0.9998880347	0.9998880347	0.8521437889	0.8521437889	0.8521437889
0.5	0.9995767811	0.9995767811	0.9995767811	0.7788007830	0.7788007830	0.7788007830
0.6	0.9987506264	0.9987506264	0.9987506264	0.6976763260	0.6976763260	0.6976763260
0.7	0.9968931697	0.9968931698	0.9968931698	0.6126263941	0.6126263941	0.6126263941
0.8	0.9931932556	0.9931932566	0.9931932568	0.5272924240	0.5272924240	0.5272924240
0.9	0.9864796226	0.9864796326	0.9864796325	0.4448580662	0.4448580662	0.4448580662
1.0	0.9751818168	0.9751818168	0.9751817172	0.3678794411	0.3678794411	0.3678794411

Table 2: Exact and approximate solution of u_3 (x) of experiment 1

	u_3 (x_i)		
x_i	u_{Che}	u_{Leg}	u_{Exact}
0.0	1.0000000000	1.0000000000	1.0000000000
0.1	1.1051709180	1.1051709180	1.1051709180
0.2	1.2214027581	1.2214027581	1.2214027581
0.3	1.3498588075	1.3498588075	1.3498588075
0.4	1.4918246976	1.4918246976	1.4918246976
0.5	1.6487212707	1.6487212707	1.6487212707
0.6	1.8221188003	1.8221188003	1.8221188007
0.7	2.0137527074	2.0137527074	2.0137527003
0.8	2.2255409284	2.2255409284	2.2255409274
0.9	2.4596031109	2.4596031109	2.4596031184
1.0	2.7182818270	2.7182818270	2.7182818284

Table 3: Exact and approximate solution of u_1 (x) and u_2 (x) of experiment 2

	u_1 (x_i)			u_2 (x_i)		
x_i	u_{Che}	u_{Leg}	u_{Exact}	u_{Che}	u_{Leg}	u_{Exact}
0.0	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000
0.1	1.1049868302	1.1049868302	1.1049868303	1.1051709179	1.1051709179	1.1051709180
0.2	1.2197785558	1.2197785558	1.2197785556	1.2214027578	1.2214027578	1.2214027581
0.3	1.3438252434	1.3438252434	1.3438252437	1.3498588070	1.3498588070	1.3498588075
0.4	1.4761219460	1.4761219460	1.4761219464	1.4918246969	1.4918246969	1.4918246976
0.5	1.6151462959	1.6151462959	1.6151462964	1.6487212698	1.6487212698	1.6487212708
0.6	1.7588188451	1.7588188451	1.7588188457	1.8221187993	1.8221187993	1.8221188003
0.7	1.9044965336	1.9044965336	1.9044965336	2.0137527062	2.0137527062	2.0137527074
0.8	2.0490086491	2.0490086491	2.0490086501	2.2255409271	2.2255409271	2.2255409284
0.9	2.1887419108	2.1887419108	2.1887419126	2.4596031094	2.4596031094	2.4596031111
1.0	2.3197768221	2.3197768221	2.3197768247	2.7182818250	2.7182818250	2.7182818284

Table 4: Exact and approximate solution of u_1 (x) and u_2 (x) of experiment 2

	u_1(x_i)			u_2 (x_i)		
x_i	u_{Che}	u_{Leg}	u_{Exact}	u_{Che}	u_{Leg}	u_{Exact}
0.0	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000
0.1	1.0512710963	1.0512710963	1.0512710963	1.0338951134	1.0338951134	1.0338951135
0.2	1.1051709179	1.1051709179	1.1051709180	1.06893910 56	1.0689391056	1.0689391057
0.3	1.1618342426	1.1618342426	1.1618342427	1.1051709179	1.1051709179	1.1051709180
0.4	1.2214027579	1.2214027579	1.2214027581	1.1426308115	1.1426308115	1.1426308117
0.5	1.2840254164	1.2840254164	1.2840254166	1.1813604125	1.1813604125	1.1813604128
0.6	1.3498588073	1.3498588073	1.3498588075	1.2214027578	1.2214027578	1.2214027581
0.7	1.4190675483	1.4190675483	1.4190675485	1.2628023428	1.2628023428	1.2628023432
0.8	1.4918246973	1.4918246973	1.4918246976	1.3056051715	1.3056051715	1.3056051720
0.9	1.5683121852	1.5683121852	1.5683121854	1.3498588069	1.3498588069	1.3498588075
1.0	1.6487212713	1.6487212713	1.6487212707	1.3956124237	1.3956124237	1.3956124250

CONCLUSION

In the present study, OTM is proposed for solving NDDSs. Reducing the NDDSs to algebraic equations is the first characteristic of the proposed method. The main idea of the proposed method is to convert the NDDS including linear and nonlinear terms to an algebraic system to simplify the computations. Arbitrary orthogonal polynomial bases were applied as basis functions to reduce the volume of computations. Furthermore, this method yields the desired accuracy only in a few terms in a series form of the exact solution. All of these advantages of the OTM to solve nonlinear problems assert the method as a convenient, reliable and powerful tool.

REFERENCES

Doha, E.H. and W.M. Abd-Elhameed, 2005. Accurate spectral solutions for the parabolic and elliptic partial differential equations by the ultraspherical tau method. J. Comput. Applied Math., 181: 24-45.

El-Daou, M.K. and H.G. Khajah, 1997. Iterated solutions of linear operator equations with the Tau method. Math. Comput., 66: 207-213.

Garcia-Olivares, A., 2002. Analytical approximants of time-dependent partial differential equations with Tau methods. Math. Comput. Simul., 61: 35-45.

Ghafarian, P., M.T. Zamanian and T. Haghroosta, 2011. Design of barotropic spectral model for predicting of vorticity field. Res. J. Environ. Sci., 5: 386-393.

Hale, J. and S.M. Verduyn Lunel, 1993. Introduction to Functional Differential Equations. Springer, New York, USA., ISBN-13: 9780387940762, pp: 447.

Kolmanovskii, V.B. and A.D. Myshkis, 1992. Applied Theory of Functional Differential Equations. Kluwer Academic Publishers, Dordrecht, The Netherlands, ISBN-13: 9780792320135, pp: 234.

Kong, W. and X. Wu, 2009. Chebyshev tau matrix method for Poisson-type equations in irregular domain. J. Comput. Applied Math., 228: 158-167.

Lanczos, C., 1938. Trigonometric interpolation of empirical and analytic functions. J. Math. Phys., 17: 123-199.

Liu, K.M. and C.K. Pan, 1999. The automatic solution to systems of ordinary differential equations by the Tau method. Comput. Math. Appl., 38: 197-210.

Matos, J., M.J. Rodrigues and P.B. Vasconcelos, 2004. New implementation of the Tau method for PDEs. J. Comput. Applied Math., 164-165: 555-567.

Parand, K. and M. Razzaghi, 2004. Rational Chebyshev Tau method for solving volterra's population model. Applied Math. Comput., 149: 893-900.

Pour-Mahmoud, J., M.Y. Rahimi-Ardabili and S. Shahmorad, 2005. Numerical solution of the system of Fredholm integro-differential equations by the Tau method. Applied Math. Comput., 168: 465-478.

Rahimi-Ardabili, M.Y. and S. Shahmorad, 2007. Iterative numerical solution of non-linear integro-differential equations by the Tau method. Applied Math. Comput., 193: 514-522.

Rao, T.P., K.S. Rao and C.L. Usha, 2011. Stochastic modeling of blood glucose levels in type-2 diabetes mellitus. Asian J. Mathe. Stat., 4: 56-65.

Salamon, D., 1984. Control and Observation of Neutral Systems. Pitman Advanced Publishing Program, Boston, MA., USA., ISBN-13: 9780273086185, pp: 204.

Taiwo, O.A. and A. Abubakar, 2011. An application method for the solution of second order non linear ordinary differential equations by Chebyshev polynomials. Asian J. Applied Sci., 4: 255-262.

Taiwo, O.A. and O.S. Odetunde, 2010. On the numerical approximation of delay differential equations by a decomposition method. Asian J. Math. Stat., 3: 237-243.

Tasci, F., 2003. On the approximative solution of boundary value problems by collocation. J. Applied Sci., 3: 210-215.

Vanani, S.K. and A. Aminataei, 2009. Multiquadric approximation scheme on the numerical solution of delay differential systems of neutral type. Math. Comput. Modell., 49: 234-241.

Vanani, S.K. and A. Aminataei, 2010. On the numerical solution of delay differential systems. J. Applied Funct. Anal., 5: 169-176.