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Linear and Nonlinear Seismic Rayleigh Waves with Damping: A Heuristic Direct Method

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Abstract: In this study, nonlinearity in earthquake is investigated for the propagating seismic waves instead of linear waves. Assuming the presence of nonlinear effects in this earthquake modeling, the Rayleigh waves are formed by incorporating the nonlinear sine-Gordon equation into the linear asymptotic governing equations for finding similarity reduction. The existence of reduction to the modified asymptotic governing equations is demonstrated and is consequently shown to give both the linear and nonlinear Rayleigh waves solutions. The related velocity and amplitude dependent Rayleigh waves are obtained and also the nonlinear form of Rayleigh waves. Multiple nonlinear surface displacements are identified. These nonlinear waves are shown to leave the trails of crucial surface displacements.

Key words: Linear rayleigh wave, nonlinear rayleigh wave, nonlinear damping, sine-gordon, surface displacements

INTRODUCTION

Earthquake modelling is important in rescue scenario. Understanding the displacements' profiles, the rescue activities are more manageable. Vafaeinezhad *et al.* (2009) developed a new approach for modeling spatio-temporal events in an earthquake rescue scenario. However, undestanding the seismic waves profiles are more important since the propagating of the seismic waves are more crucial in developing disastrous surface displacments. Balideh *et al.* (2009) studied the wave propagation in elastic environments analytically and numerically. Whilst, the linear and nonlinear seismic waves will be studied in this study for the wave propagation.

The year 1885 marked the glorious year for earthquake studies whereby seismic Rayleigh waves were named after Lord Rayleigh. Lord Rayleigh realized the key part that surface elastic waves play in creating the earth surface displacement. The quest for exact Rayleigh waves was clearly begun then and under suitable conditions as discussed by Ben and Ari (1981), depth dependent Rayleigh waves are approximated by the dimensionless wave equation:

$$\mu\nabla^2 u+(\lambda+\mu)\nabla(\nabla.u)+a_z\frac{d\lambda}{dz}\nabla.u+\frac{d\mu}{dz}\left(2\frac{\partial u}{\partial z}+a_z\times\nabla\times u\right)-\rho\frac{\partial^2 u}{\partial t^2}=0 \tag{1.1}$$

where, subscript z is used to denote the depth of the medium with mass density ρ, while μ and λ are the Lamb parameters, a_x, a_z are the unit vectors. The corresponding displacement is:

$$u=\left[A(a_x-i\gamma_\alpha a_z)e^{-\gamma_\alpha kz}+B(i\gamma_\beta a_x+a_z)e^{-\gamma_\beta kz}\right]e^{ik(ct-x)},\ \gamma_\delta=\sqrt{1-\frac{c^2}{\delta^2}},\ \delta=\alpha,\beta \tag{1.2}$$

where, A and B are the amplitudes of the P waves and S waves, c is the velocity and k is the wave number. By utilizing the boundary conditions for P waves and S waves, the Rayleigh wave's dependency on its velocities are obtained as:

$$\left(2-\frac{c^2}{\beta^2}\right)^2-4\left(1-\frac{c^2}{\alpha^2}\right)^{0.5}\left(1-\frac{c^2}{\beta^2}\right)^{0.5}=0 \tag{1.3}$$

such that α, β are the velocities for P and S waves, respectively. Equation 1.3 has now become the reference model for Rayleigh waves in earthquake modelling (Pujol, 2003).

Fan (2004) has successfully introduced, without loss of generality, a method of deriving Eq. 1.3 through asymptotic governing equations. The damping mechanism is introduced to the governing equations and, in particular, one notes that a variety of outcomes are possible depending upon one's view of the task at hand. In this paper, the method introduced by Fan (2004) for deriving the Rayleigh waves is extended for the velocity and amplitude such that these waves can then be shown for nonlinearity.

GOVERNING EQUATIONS AND DISCUSSIONS

Fan (2004) gave a comprehensive account of the dimensionless governing equations for dynamic rupture

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with damping. The idea of damping is then introduced and used effectively with the asymptotic governing equations and consequently the surface seismic Rayleigh waves with damping in dimensionless form were derived. Fan (2004) generated his asymptotic governing equation which consists of the damping factor and the fact that the governing equations can be deduced to Rayleigh waves similar to Eq. 1.3. In this study, the condition of Cauchy-Riemann is introduced to Fan (2004) ways of deriving Rayleigh waves Eq. 1.3. The horizontal and vertical components of the Rayleigh waves are introduced according to the stress components. These would give the condition of Cauchy-Riemann before the Eq. 1.3 is derived soon after.

The implementation of damping method starts by writing the functions of P and S waves asymptotically and this yields:

$$\phi = \phi_0 + a_P\phi_1 + a_P^2\phi_2 + \ldots \tag{2.1a}$$

$$\psi = \psi_0 + a_S\psi_1 + a_S^2\psi_2 + \ldots \tag{2.1b}$$

If a trial form of complex analytic function is $f(z) = \phi + i\psi$, the necessary condition that f(x) is analytic is that the Cauchy-Riemann equations be satisfied:

$$-\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}, \quad \frac{\partial u}{\partial x} = \frac{\partial w}{\partial z} \tag{2.2}$$

The two potential functions for the Rayleigh waves which flow in the medium are:
Stream function

$$\psi_x = w, \quad \psi_z = -u \tag{2.3a}$$

Velocity function

$$\phi_x = -u, \quad \phi_x = -w \tag{2.3b}$$

The dimensionless asymptotic governing equation is similar to:

$$\left.\begin{aligned} \nabla^2\phi_0 - \frac{1}{c^2}\frac{\partial^2\phi_0}{\partial t^2} &= a_P\frac{\partial\phi_0}{\partial t} \\ \nabla^2\psi_0 - \frac{1}{c^2}\frac{\partial^2\psi_0}{\partial t^2} &= a_S\frac{\partial\psi_0}{\partial t} \end{aligned}\right\} \tag{2.4}$$

subject to the Cauchy-Riemann conditions Eq. 2.2 and which yield:

$$\left(\frac{\partial w_0}{\partial z} - \frac{\partial u_0}{\partial x}\right)_{z=0} = 0 \tag{2.5}$$

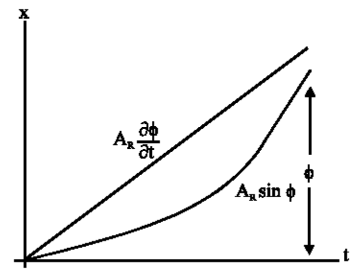

Fig. 1: Linear and nonlinear damping

$$\left(\frac{\partial w_0}{\partial x} + \frac{\partial u_0}{\partial z}\right)_{z=0} = 0 \tag{2.6}$$

The linear and nonlinear damping are illustrated in Fig. 1. For this purposes, it is sufficient to look for solutions of Eq. 2.4 in the form of:

$$\phi_0(x,z,t) = f(z)e^{i(\omega t - \alpha x)}, \quad \psi_0(x,z,t) = g(z)e^{i(\omega t - \alpha x)} \tag{2.7}$$

where, $\omega = \alpha_{p,s}c$. Solving Eq. 2.4 are entirely algorithmic; it often involves a large amount of tedious algebra and auxiliary calculations which can become virtually unmanageable if attempted for higher orders. Our objective is to reduce Eq. 2.4 into a single equation by heuristically introduce instead a sine component in order to merge Eq. 2.4 and with some amendments to give the amplitude dependent characteristic. Using this, we mean solutions of the partial differential equation for Eq. 2.4 will only be shown here.

Substituting Eq. 2.7 into 2.4 yields:

$$\frac{\partial^2 f(z)}{\partial z^2} - \left(\alpha^2 - ia_p - k_p^2\right) = 0, \qquad \frac{\partial^2 g(z)}{\partial z^2} - \left(\alpha^2 - ia_s - k_s^2\right) = 0 \tag{2.8}$$

These equations are solvable by the method of characteristics which lead to:

$$f(z) = L\exp\left[\pm\sqrt{\left(\alpha^2 - ia_p - k_p^2\right)}z\right], \quad g(z) = M\exp\left[\pm\sqrt{\left(\alpha^2 - ia_S - k_S^2\right)}z\right] \tag{2.9}$$

For our future purposes, we introduce the following new dependent relations:

$$k_P = \xi_P + i\lambda_P, \quad k_S = \xi_S + i\lambda_S \tag{2.10}$$

Inserting Eq. 2.9 into 2.7 one gets:

$$\left.\begin{aligned}\phi_0(x,z,t) &= L\exp\left[-\sqrt{\left(\alpha^2-\left(\lambda_P^2-\xi_P^2\right)^2\right)}z\right]\exp\left[i(\omega t-\alpha x)\right]\\ \psi_0(x,z,t) &= M\exp\left[-\sqrt{\left(\alpha^2-\left(\lambda_S^2-\xi_S^2\right)^2\right)}z\right]\exp\left[i(\omega t-\alpha x)\right]\\ a_P = 2\lambda_P\xi_P \qquad & a_S = 2\lambda_S\xi_S\end{aligned}\right\} \quad (2.11)$$

for $z\rightarrow\infty$, the amplitudes L,M vanish. The Rayleigh waves are assembled by horizontal component and vertical component. Here, we write the horizontal component and vertical component from the two potential functions Eq. 2.3a and b. The components that fulfill conditions Eq. 2.2 are:

$$\left(\phi_{zz}-\phi_{xx}\right)L+\left(\phi_{xz}+\phi_{zx}\right)M=0 \quad \text{(Horizontal component)} \quad (2.12a)$$

$$\left(-\psi_{xz}+\psi_{zx}\right)L+\left(\psi_{xx}-\psi_{zz}\right)M=0 \quad \text{(Vertical component)} \quad (2.12b)$$

The horizontal component is chosen according to Pujol (2003) implication on Rayleigh waves. The Rayleigh waves that propagate horizontally would generate stress in both the horizontal and vertical directions of the medium while the Rayleigh waves which propagate in vertical direction would only create stress vertically. We now propose another form of the horizontal and vertical components for Rayleigh waves, though equations Eq. 2.12a and b must again fulfill the Cauchy-Riemann conditions.

Introducing Eq. 2.11 into 2.12a and b, this yields:

$$\left[\left(2\alpha^2-\left(\lambda_S^2-\xi_S^2\right)^2\right)\right]L-\left[2i\alpha\sqrt{\left(\alpha^2-\left(\lambda_P^2-\xi_P^2\right)^2\right)}\right]M=0 \quad (2.13a)$$

$$\left[2i\alpha\sqrt{\left(\alpha^2-\left(\lambda_S^2-\xi_S^2\right)^2\right)}\right]L+\left[\left(2\alpha^2-\left(\lambda_S^2-\xi_S^2\right)^2\right)\right]M=0 \quad (2.13b)$$

Equation (2.13a, b) are similar to that obtained in Fan (2004). Referring to Fan (2004), the condition that L and M have nontrivial solution is that the determinant of the coefficients is zero. This yields the frequency equation or the polarization equation:

$$\left(2\alpha^2-\left(\lambda_S^2-\xi_S^2\right)^2\right)^2-4\alpha^2\left[\sqrt{\left(\alpha^2-\left(\lambda_S^2-\xi_S^2\right)^2\right)}\right]\left[\sqrt{\left(\alpha^2-\left(\lambda_P^2-\xi_P^2\right)^2\right)}\right]=0 \quad (2.15)$$

For $\alpha = 1$, Eq. 2.15 is exactly similar to Eq. 1.3 which represents velocity dependent Rayleigh waves. In next section, we use Rayleigh waves in terms of velocity and amplitude and subsequently obtain their linear and nonlinear characteristics. We are of the opinion that these nonlinear Rayleigh waves are the ones which left the trails of crucial surface displacements such as sand volcanoes and earth crack on the earth. The actual figures for sand volcanoes and earth crack can be referred to Tarbuck and Lutgens (1994).

LINEAR AND NONLINEAR RAYLEIGH WAVES

The results of the Rayleigh wave's modelling can be interesting if the quivers of the Rayleigh waves are revealed from simulation. The ignition of Rayleigh waves till their annihilation are shown later in this section for the generation of soil displacements i.e., land subsidence, sand volcanoes and earth crack. Motivated by the fact that the oscillation of a damped spring-mass system around the equilibrium can be represented by a sine wave, we heuristically develop an alternative method of reduction for Eq. 2.4. Thus without loss of generality, the Eq. 2.4 are merged together and yielding:

$$\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=\frac{1}{\xi}\sin\zeta\phi, \quad \phi=\phi_{P,S}=\phi_0+a_{P,S}\phi_1+a_{P,S}^2\phi_2+\dots\dots \quad (3.1)$$

Equation 3.1 is nearly similar as sine-Gordon equation in the course of investigation of surfaces displacement by soliton. By focusing on the characteristics of the Rayleigh waves that will point out to the amplitude, one has to derive the initial characteristic of these waves by introducing the tangent line or characteristic line of the initial Rayleigh waves into Eq. 3.1 to denote the solutions dependent on characteristic (amplitude). The corresponding tangent line from Eq. 2.7 and 2.9 is:

$$\tan\phi=\frac{\phi_0(x,z,t)\big|_{t=0}}{\psi_0(x,z,t)\big|_{x=0}}\Bigg|_{z=0}=\frac{L\exp\left[i(-\alpha x)\right]}{M\exp\left[i(\omega t)\right]} \quad (3.2)$$

By using Eq. 3.2, it is sufficient to work out Eq. 3.1 so as to generate the Rayleigh waves. However, we would like to reintroduce the variables u and wto represent Eq. 3.2 such that:

$$\tan\phi=\frac{u}{w} \quad (3.3)$$

The inverse of Eq. 3.3 is:

$$\phi=\arctan\frac{u}{w} \quad (3.4)$$

Similar to solving initial value problem, inserting Eq. 3.4 into 3.2 yields:

$$\left(u^2+w^2\right)\left(\frac{1}{u}\frac{\partial^2 u}{\partial x^2}-\frac{1}{w}\frac{\partial^2 w}{\partial t^2}\right)-2\left[\left(\frac{\partial u}{\partial x}\right)^2-\left(\frac{\partial w}{\partial t}\right)^2\right]=w^2-u^2 \quad (3.5)$$

Differentiating Eq. 3.5 with respect to x and t, we obtain the respective expressions as:

$$2u\frac{\partial u}{\partial x}\left(\frac{1}{u}\frac{\partial^2 u}{\partial x^2}-\frac{1}{w}\frac{\partial^2 w}{\partial t^2}\right)+\left(u^2+w^2\right)\frac{\partial}{\partial x}\left(\frac{1}{u}\frac{\partial^2 u}{\partial x^2}\right)-4\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}=-2u\frac{\partial u}{\partial x} \quad (3.6a)$$

$$2w\frac{\partial w}{\partial t}\left(\frac{1}{u}\frac{\partial^2 u}{\partial x^2}-\frac{1}{w}\frac{\partial^2 w}{\partial t^2}\right)+\left(u^2+w^2\right)\frac{\partial}{\partial t}\left(-\frac{1}{w}\frac{\partial^2 w}{\partial t^2}\right)+4\frac{\partial w}{\partial t}\frac{\partial^2 w}{\partial t^2}=2w\frac{\partial w}{\partial t} \quad (3.6b)$$

Rearranging Eq. 3.6a and b, we obtain:

$$\left[\left(\frac{1}{u\frac{\partial u}{\partial x}}\right)\frac{\partial}{\partial x}\left(\frac{1}{u}\frac{\partial^2 u}{\partial x^2}\right)\right]=\frac{-2}{\left(u^2+w^2\right)}-\frac{2}{\left(u^2+w^2\right)}\left(\frac{1}{u}\frac{\partial^2 u}{\partial x^2}-\frac{1}{w}\frac{\partial^2 w}{\partial t^2}\right)+\frac{4}{u\left(u^2+w^2\right)}\frac{\partial^2 u}{\partial x^2} \quad (3.7a)$$

$$\left[\frac{1}{w\frac{\partial w}{\partial t}}\frac{\partial}{\partial t}\left(-\frac{1}{w}\frac{\partial^2 w}{\partial t^2}\right)\right]=\frac{2}{\left(u^2+w^2\right)}+\frac{2}{\left(u^2+w^2\right)}\left(\frac{1}{u}\frac{\partial^2 u}{\partial x^2}--\frac{1}{w}\frac{\partial^2 w}{\partial t^2}\right)-\frac{4}{w\left(u^2+w^2\right)}\frac{\partial^2 w}{\partial t^2} \quad (3.7b)$$

Previously, we separated the Eq. 3.5 into 3.6a and 3.6b with the purpose of rearranging the mathematical structures to give Eq. 3.7a and b. The next step is to return its original formulation by adding together Eq. 3.7a and b. These results in:

$$\left[\frac{1}{w\frac{\partial w}{\partial t}}\frac{\partial}{\partial t}\left(-\frac{1}{w}\frac{\partial^2 w}{\partial t^2}\right)\right]+\left[\left(\frac{1}{u\frac{\partial u}{\partial x}}\right)\frac{\partial}{\partial x}\left(\frac{1}{u}\frac{\partial^2 u}{\partial x^2}\right)\right]=-4b^2+4b^2=0 \quad (3.8)$$

Eventually we write Eq. 3.8 as:

$$\left[\left(\frac{1}{u\frac{\partial u}{\partial x}}\right)\frac{\partial}{\partial x}\left(\frac{1}{u}\frac{\partial^2 u}{\partial x^2}\right)\right]=4b^2,\quad \left[\frac{1}{w\frac{\partial w}{\partial t}}\frac{\partial}{\partial t}\left(-\frac{1}{w}\frac{\partial^2 w}{\partial t^2}\right)\right]=-4b^2 \quad (3.9)$$

These PDEs are amenable to solution by integrating which leads to:

$$\frac{\partial^2 u}{\partial x^2}=2b^2u^3+c_1u \quad (3.10a)$$

$$\frac{\partial^2 w}{\partial t^2}=2b^2w^3+d_1w \quad (3.10b)$$

Multiplying $\partial u/\partial x$ to Eq. 3.10a and $\partial w/\partial t$ to Eq. 3.10b and the integration yields:

$$\left(\frac{\partial u}{\partial x}\right)^2=b^2u^4+\sqrt{c_1^2}u^2+e_1 \quad (3.11a)$$

$$\left(\frac{\partial w}{\partial t}\right)^2=b^2w^4+\sqrt{d_1^2}w^2+e_1 \quad (3.11b)$$

By using Eq. 3.10a and b, 3.11a and b, the Eq. 3.5 can now be written as:

$$(c_1-d_1)v^2+(c_1-d_1)u^2+2f_1=v^2-u^2+2e_1 \quad (3.12)$$

The Eq. 3.12 means that:

$$c_1-d_1=\pm 1,\quad e_1=f_1 \quad (3.13)$$

Relations Eq. 3.13 are important for limiting the solutions for Eq. 3.1 within the series; the Rayleigh waves emerge when c_1-d_1 = -1 and it ceases when c_1-d_1 = +1 or vice versa for $e_1 = f_1$ which will be plotted later.

Solutions for b = 0, e = 0: When b = 0, e = 0, Eq. 3.11a and b give:

$$\left(\frac{\partial u}{\partial x}\right)^2=\sqrt{c_1^2}u^2 \quad (3.14a)$$

$$\left(\frac{\partial w}{\partial t}\right)^2=\sqrt{d_1^2}w^2 \quad (3.14b)$$

These PDEs are amenable to solutions by the method of characteristics which leads to:

$$u=a_1\exp\left(-\sqrt{c_1}x\right) \quad (3.15a)$$

$$w=a_2\exp\left(-\sqrt{d_1}t\right) \quad (3.15b)$$

Since, the solution is proposed to be in the form of Eq. 3.4, without loss of generality Eq. 3.15a and b are inserted into Eq. 3.4 to give:

$$\phi = \arctan\left[\frac{a_1 \exp\left(-\sqrt{c_1}\,x\right)}{a_2 \exp\left(-\sqrt{c_1} \pm 1t\right)}\right], \quad c_1 = -1 \text{ to } 1 \qquad (3.16)$$

Figure 2 shows the evolution of rayleigh waves from $-1 \le c_1 \le 1$ according to Eq. 3.16. With initial amplitude for P waves and S waves are chosen equal to 1, the Rayleigh waves emerge as displacement of height 0.7854 from the surface according to Fig. 2a rather than at the height 1 for velocities depending on x and t such that x : -1 : 1 : and t = -1 : 0.1 : 1. Figure 2b and c explain the formation of Fig. 2c. Note that, the subsidence displacement emerges by Rayleigh waves become apparent from Fig. 2a-f. From Fig. 2f, the Rayleigh wave's type Eq. 3.16 creates the dip depth at around 0.2 while the peak height is at around 1.5 from initial displacement height 0.7854. Amazingly, this amplitude and velocity dependent Rayleigh waves Eq. 3.16 show that the displacement by Rayleigh waves did not emerged initially from the medium surface but at the height above the medium surface.

Solutions for b = 0, e ≠ 0: When b = 0, e ≠ 0, Eq. 3.11a and b give:

$$\left(\frac{\partial u}{\partial x}\right)^2 = c_1 u^2 + e \qquad (3.17a)$$

$$\left(\frac{\partial w}{\partial t}\right)^2 = d_1 w^2 + e \qquad (3.17b)$$

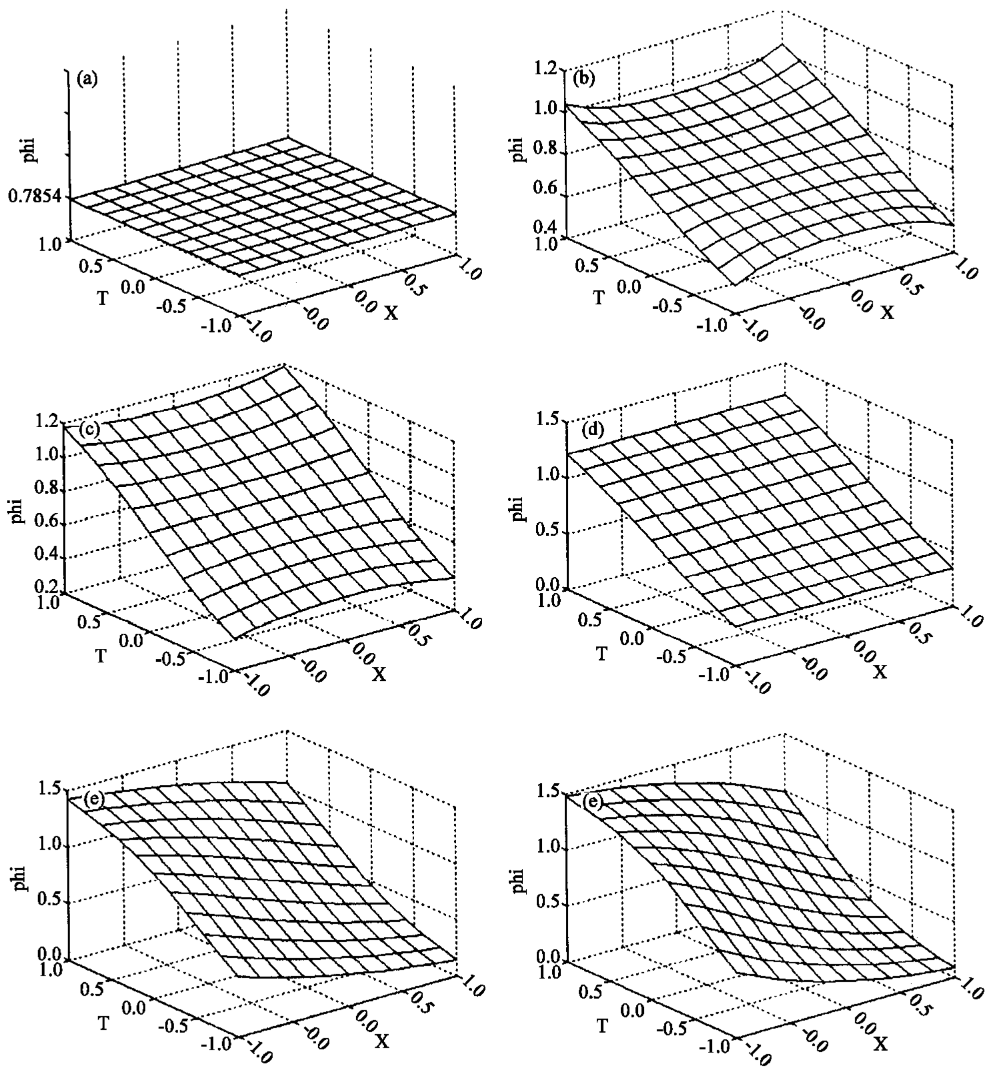

Fig. 2: The displacements by Rayleigh waves (3.16) at the surface, are plotted for $-1 \le c_1 \le 1$, a_1, a_2 = 1. (a) c_1 = -1, g(b) c_1 = -0.8 (c) c_1 = -0.5, (d) c_1 = 0, (e) c_1 = 0.5 and (f) c_1 = 0.1

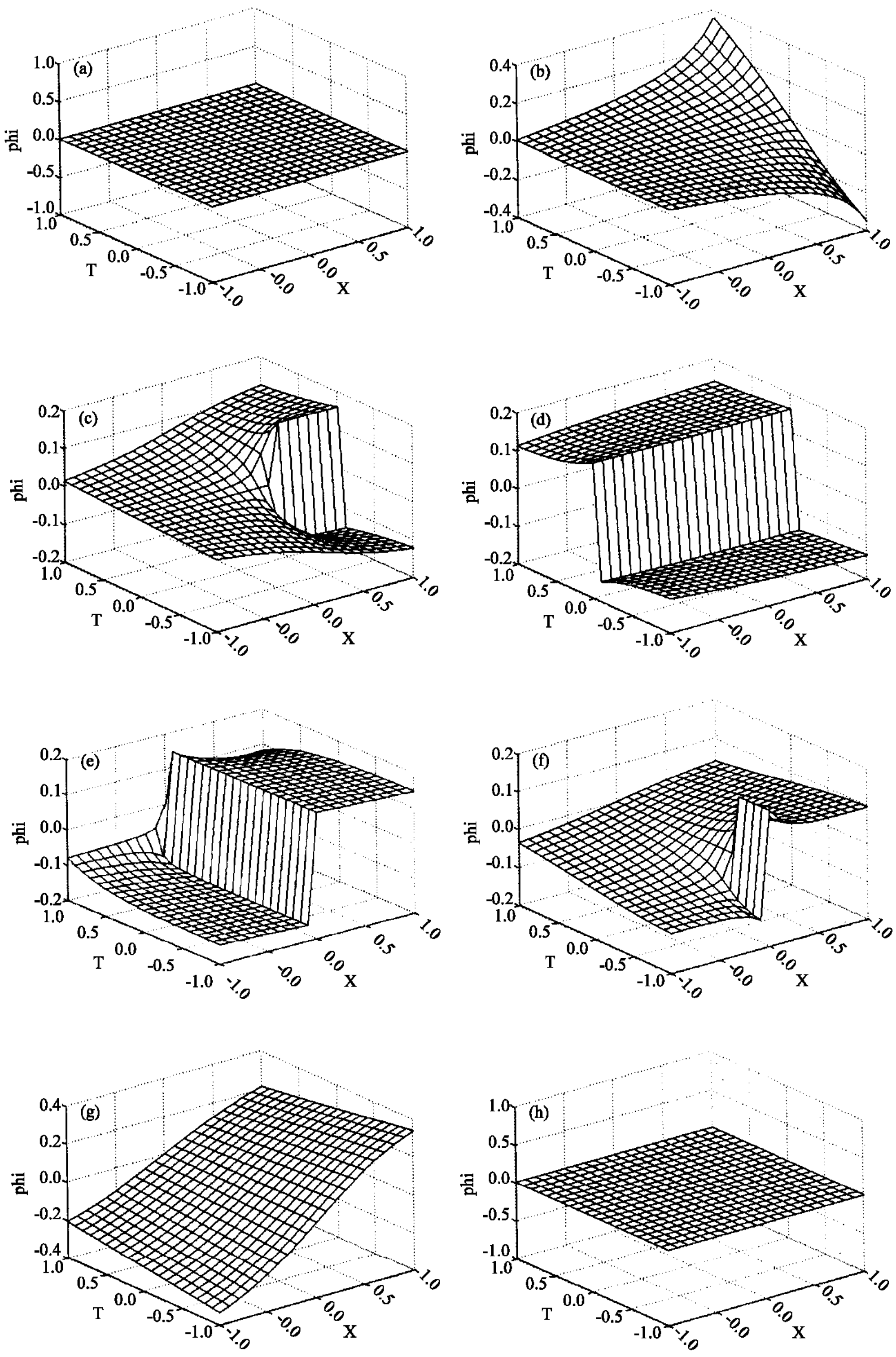

Fig. 3: The displacements (a-d) by Rayleigh waves (3.20a) at the surface are plotted for $1 \geq c_1 > 0$, f, g = 1 while the displacements (e-h) by Rayleigh waves (3.20b) at the surface are plotted for $-1 \leq c_1 < 0$, f, g = 1. (a) $c_1 = 1$, (b) $c_1 = 0.8$, (c) $c_1 = 0.5$, (d) $c_1 = 0.1$, (d) $c_1 = -0.1$, (e) $c_1 = -0.1$, (f) $c_1 = -0.5$, (g) $c_1 = -0.8$ and (g) $c_1 = -1$

These PDEs are amenable to solutions by the method of characteristics which leads to:

$$u=\sqrt{\frac{e}{c_1}}\sinh\left((\sqrt{c_1})x\pm f\right) \quad (3.18a)$$

$$w=\sqrt{\frac{e}{c_1\pm 1}}\cosh\left((\sqrt{c_1\pm 1})t+g\right) \quad (3.18b)$$

Since, the solution is proposed in the form of Eq. 3.4, without loss of generality (3.18a) and (3.18b) are inserted into Eq. 3.4 to give:

$$\phi=\arctan\left(\frac{\sqrt{c_1\pm 1}}{\sqrt{c_1}}\frac{\sinh\left((\sqrt{c_1})x+f\right)}{\cosh\left((\sqrt{c_1\pm 1})t+g\right)}\right) \quad (3.19)$$

The calculus requires $c_1 \neq 0$ for finite solution. Thus, Eq. (3.19) yields:

$$\phi=\arctan\left(\frac{\sqrt{c_1-1}}{\sqrt{c_1}}\frac{\sinh\left((\sqrt{c_1})x+f\right)}{\cosh\left((\sqrt{c_1-1})t+g\right)}\right),\quad c_1: 1 \text{ to } 0.1 \quad (3.20a)$$

$$\phi=\arctan\left(\frac{\sqrt{c_1+1}}{\sqrt{c_1}}\frac{\sinh\left((\sqrt{c_1})x+f\right)}{\cosh\left((\sqrt{c_1+1})t+g\right)}\right),\quad c_1: -0.1 \text{ to } -1 \quad (3.20b)$$

The Fig. 3 shows the displacement by Rayleigh waves Eq. 3.20a and b with velocities depending on x and t such that x, t : -1 : 1. Both Eq. 3.20a and b give Fig. 3a-h which show the process of earth crack emergence. By comparing Fig. 3 with Fig. 2, the Rayleigh waves Eq. 3.20a and 3.20b emerge at the surface, because it does not depend on the P and S wave's amplitudes. This type of Rayleigh waves Eq. 3.20a and b are not dependent on P and S wave's amplitude. Besides, we observe 2 stages of displacements. The first stage displacement is by the S waves in accordance to the direction showed in Fig. 3a-d. The S waves push the particles upwards. The second stage of displacement is by the P waves in accordance to Fig. 3e-h. Next we will show the roles of f and g in Eq. 3.20a and b.

From Fig. 4a and b, the Rayleigh waves make another type of displacement with f, g = 0. Only Eq. 3.20a is plotted since Eq. 3.20b will give similar figures but in the different direction as showed in Fig. 3. Here, we conclude that the Rayleigh waves Eq. 3.20a and 3.20b are dependent on P and S wave's coefficients.

Another question arises here, what eventually happen to the series of profiles shown in Fig. 3 and 4. We address the solution by plotting Eq. 3.20a and b for $c_1 = \pm 1.1$ as illustrated in Fig. 5.

From the graphical output, Fig. 5 shows the formation of the earth crack. The discussion above is referred to the amplitude and coefficient dependent Rayleigh waves. In the next section, we will be using another type of Rayleigh waves.

Solutions for $b \neq 0$, $e = 0$: When $b \neq 0$, $e = 0$, Eq. 3.11a and b give:

$$\left(\frac{\partial u}{\partial x}\right)^2=b^2u^4+c_1u^2,\quad \left(\frac{\partial w}{\partial t}\right)^2=b^2v^4+d_1v^2 \quad (3.21)$$

These PDEs are amenable to solutions by the method of characteristics which leads to:

$$u=\sqrt{c_1}\frac{1}{\sinh\left(\sqrt{c_1}x\right)},\quad w=\sqrt{(c_1\pm 1)}\frac{1}{\sinh\left(\sqrt{(c_1\pm 1)}t\right)} \quad (3.22)$$

Since, the solution is proposed in the form of Eq. 3.4, without loss of generality Eq. 3.22 is inserted into Eq. 3.4 to give:

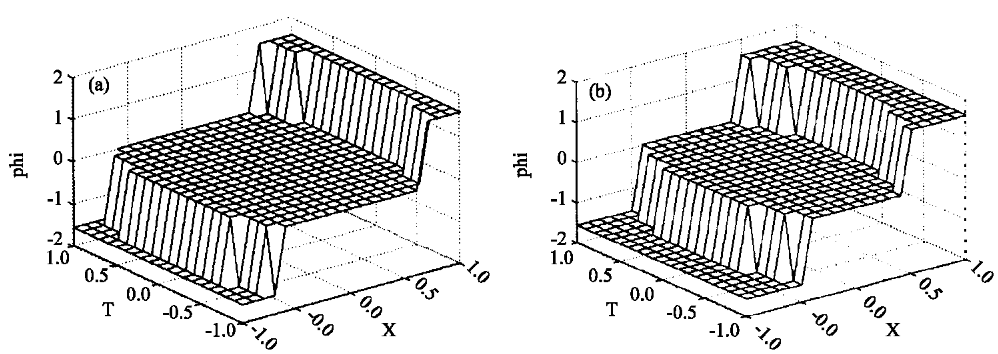

Fig. 4: The displacements (a)-(b) by Rayleigh waves (3.20a) at the surface are plotted for $1 \geq c_1d_1 > 0$, f, g = 0. (a) $c_1 = 0.5$ and (b) $c_1 = 0.1$

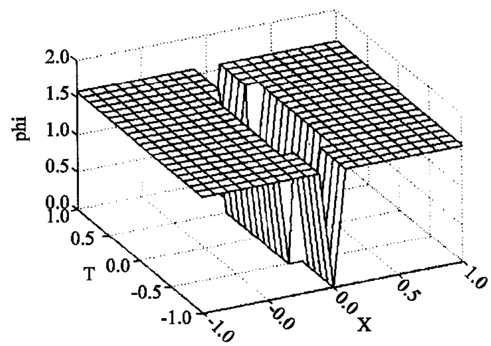

Fig. 5: $c_1 = \pm 1.1$

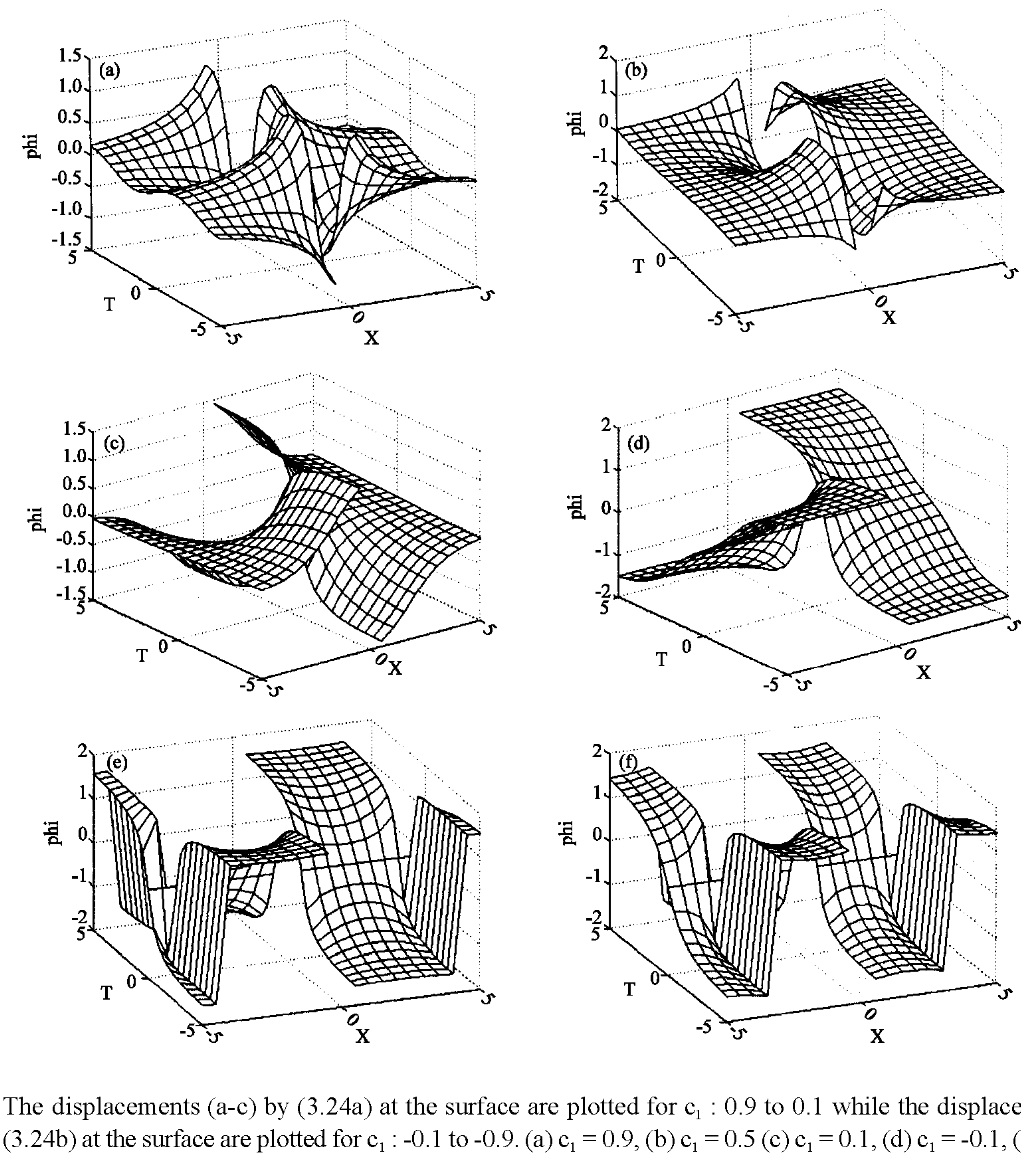

Fig. 6: The displacements (a-c) by (3.24a) at the surface are plotted for c_1 : 0.9 to 0.1 while the displacements (d-f) by (3.24b) at the surface are plotted for c_1 : -0.1 to -0.9. (a) $c_1 = 0.9$, (b) $c_1 = 0.5$ (c) $c_1 = 0.1$, (d) $c_1 = -0.1$, (e) $c_1 = -0.5$ and (f) $c_1 = -0.9$

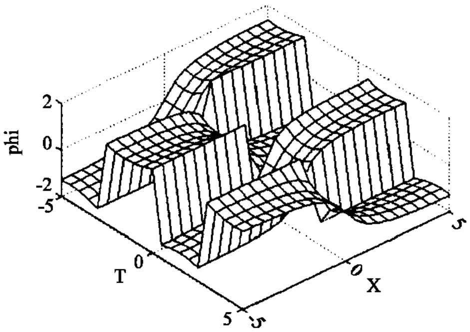

Fig. 7: $c_1 = \pm 1.1$

$$\phi = \arctan\left(\frac{\sqrt{c_1}}{\sqrt{c_1 \pm 1}} \frac{\sinh\left(\sqrt{c_1 \pm 1}t\right)}{\sinh\left(\sqrt{c_1}x\right)}\right), \quad c_1 \neq 0, \mp 1 \qquad (3.23)$$

In other words, we write Eq. 3.23 as:

$$\phi = \arctan\left(\frac{\sqrt{c_1}}{\sqrt{c_1 - 1}} \frac{\sinh\left(\sqrt{c_1 - 1}t\right)}{\sinh\left(\sqrt{c_1}x\right)}\right), \quad c_1 : 0.9 \text{ to } 0.1 \qquad (3.24a)$$

$$\phi = \arctan\left(\frac{\sqrt{c_1}}{\sqrt{c_1 + 1}} \frac{\sinh\left(\sqrt{c_1 + 1}t\right)}{\sinh\left(\sqrt{c_1}x\right)}\right), \quad c_1 : -0.1 \text{ to } -0.9 \qquad (3.24b)$$

Figure 6 shows the Rayleigh waves generated from Eq. 3.24a and b. Apparently, this is a single stage of displacement as compared to Fig. 4 and 5. It is shown that the two Rayleigh waves exist in accordance to Fig. 6. By plotting $c_1 = \pm 1.1$, we obtain Fig. 7. Clearly it is observed that Fig. 7 shows the formation of sand volcanoes as comparable to the sand volcanoes as seen at the Imperial Valley, California (Tarbuck and Lutgens, 1994).

CONCLUSIONS

In this study, the Rayleigh waves are explored deeper with the help of the well-known nonlinear sine-Gordon equation. Three different types of Rayleigh waves with displacements are shown; amplitude dependent Rayleigh waves, P and S waves coefficient dependent Rayleigh waves and dual Rayleigh waves. Amplitude dependent Rayleigh waves are resulted in land subsidence whiles the P and S wave's dependent Rayleigh waves resulted in the formation of earth cracks. The dual Rayleigh waves are seen to form the sand volcanoes. Obviously the nonlinearity factor can no longer be ignored, since we have shown here that the sine-Gordon equation can effectively model the nonlinear Rayleigh waves. Besides, we have shown the possibility of improving the Rayleigh waves as discussed in Fan (2004) by implementing the nonlinear wave equation into their framework.

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